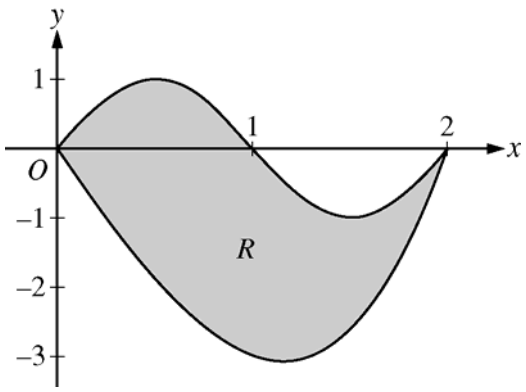
AP® CALCULUS AB
2008 SCORING GUIDELINES

Question 1



Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.

(a) Find the area of R.

(b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

(d) The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

(a) $\sin(\pi x) = x^3 - 4x$ at $x = 0$ and $x = 2$

Area $= \int_0^2 \left(\sin(\pi x) - \left(x^3 - 4x\right)\right) dx = 4$

3 : { 1 : limits; 1 : integrand; 1 : answer }

(b) $x^3 - 4x = -2$ at $r = 0.5391889$ and $s = 1.6751309$

The area of the stated region is $\int_r^s \left(-2 - \left(x^3 - 4x\right)\right) dx$

2 : { 1 : limits; 1 : integrand }

(c) Volume $= \int_0^2 \left(\sin(\pi x) - \left(x^3 - 4x\right)\right)^2 dx = 9.978$

2 : { 1 : integrand; 1 : answer }

(d) Volume $= \int_0^2 (3 - x)\left(\sin(\pi x) - \left(x^3 - 4x\right)\right) dx = 8.369$ or 8.370

2 : { 1 : integrand; 1 : answer }

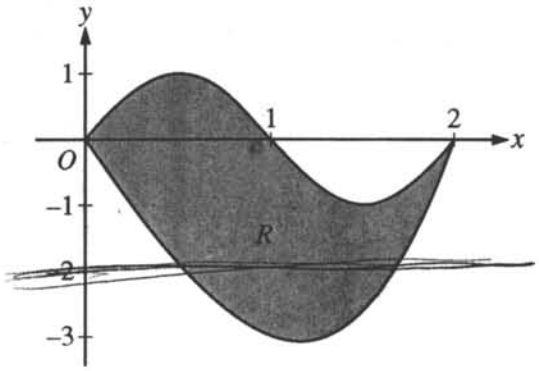
CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$\sin(\pi x) = x^3 - 4x$

$x = 2$

$$A = \int_0^2 \sin(\pi x) - (x^3 - 4x)\,dx$$

$A = 4$

Work for problem 1(b)

$-2 = x^3 - 4x$

$x = .5391887$, and 1.6751309

$$A = \int_{.5391887}^{1.6751309} (-2) - (x^3 - 4x)\,dx$$

$$= \int_{.5391887}^{1.6751309} -2 - x^3 + 4x\,dx$$

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Continue problem 1 on page 5

Work for problem 1(c)

$$V = \int_0^2 (\sin(\pi x) - (x^3 - 4x))^2 \, dx$$

$$V = 9.978344126$$

Work for problem 1(d)

$$V = \int_0^2 (\sin(\pi x) - (x^3 - 4x))(3 - x) \, dx$$

$$V = 8.369953106$$

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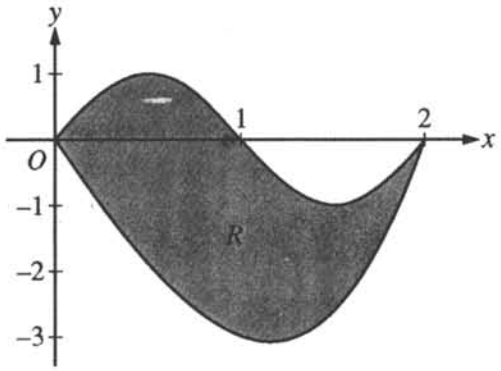
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$\sin(\pi x) = x^3 - 4x$

$x = -2 \quad x = 0 \quad x = 2$

$A = \sin(\pi x) - (x^3 - 4x)$

$\int_0^2 [\sin(\pi x) - (x^3 - 4x)]\,dx = 4$

Work for problem 1(b)

$\sin(\pi x) - (x^3 - 4x)$

$y = -2$

$[\sin(\pi x) - (x^3 - 4x)] - (-2)$

$-x^3 + 4x$

$\int_0^2 ([(\sin(\pi x) - x^3 + 4x)] + 2)\,dx$

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Continue problem 1 on page 5.

Work for problem 1(c)

cross section = square

$A = s^2$

$= [\sin \pi x - x^3 + 4x]^2$

$V = \int_0^2 [\sin \pi x - x^3 + 4x]^2 dx = 9.9783$

Work for problem 1(d)

$h(x) = 3 - x$ (depth)

$V = \pi \int_0^2 [\sin \pi x - x^3 + 4x](3 - x)\, dx$

26.2950

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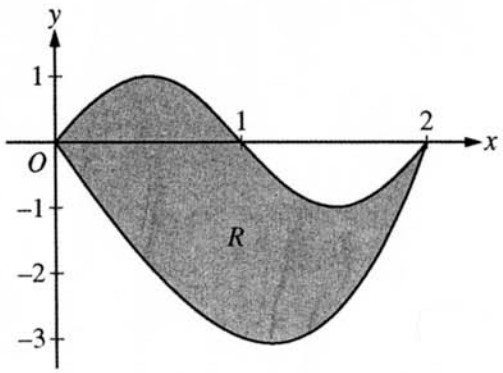
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$$\int_0^2 (\sin(\pi x)) - (x^3 - 4x)\, dx = 4$$

Work for problem 1(b)

$$\int_0^2 (-2) - (x^3 - 4x)\, dx$$

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Do not write beyond this border.

Continue problem 1 on page 5.

Work for problem 1(c)

$$V_{alone} = \int_0^2 (\sin(\pi x))^2 - (x^3 - 4x)^2 = 8.752$$

Work for problem 1(d)

$$V_{\text{water pond}} = 4\pi \int_0^2 \frac{1}{2}(3-x)^2 = 17.333\pi$$

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AP® CALCULUS AB
2008 SCORING COMMENTARY

Question 1

Overview

In this problem, students were given the graph of a region R bounded by two curves in the *xy*-plane. The points of intersection of the two curves were observable from the supplied graph. The formulas for the curves were given—a trigonometric function and a cubic polynomial—and students needed to match the appropriate functions to the upper and lower bounding curves. In each part, students had to set up and evaluate an appropriate integral. Part (a) asked for the area of R. Part (b) asked for the area of the portion of R below the line $y = -2$, so students needed to use a calculator to solve for the *x*-coordinates of the points of intersection of $y = -2$ and the lower curve to set up the appropriate integral. Part (c) asked for the volume of a solid with base R whose cross sections perpendicular to the *x*-axis are squares. In part (d) students were asked to find a volume in an applied setting. They had to determine that cross sections perpendicular to the *x*-axis are rectangles with one dimension in region R and the other dimension supplied by $h(x) = 3 - x$.

Sample: 1A
Score: 9

The student earned all 9 points. In part (a) the student has the correct limits, integrand, and answer and earned all 3 points. In part (b) the limits are correct to three decimal places, and the limits point was earned. The student earned the integrand point on the first presentation of the integral. The second, simplified integral is also correct. In part (c) the student earned the integrand point with a correct integrand and earned the answer point since the answer is correct to three decimal places. In part (d) the integrand is correct, and the answer is correct to three decimal places.

Sample: 1B
Score: 6

The student earned 6 points: 3 points in part (a), no points in part (b), 2 points in part (c), and 1 point in part (d). In part (a) the student has the correct limits, integrand, and answer and earned all 3 points. In part (b) the student does not find the intersection of the cubic curve with the line $y = -2$, and so the limits point was not earned. The integrand is not correct. In part (c) the integrand and answer are correct. The student earned both points. In part (d), although the integrand is correct, the student multiplies the integral by π, and so the answer point was not earned.

Sample: 1C
Score: 4

The student earned 4 points: 3 points in part (a), 1 point in part (b), no points in part (c), and no points in part (d). In part (a) the student has the correct limits, integrand, and answer and earned all 3 points. In part (b) the student earned the integrand point. The student does not find the intersection of the cubic curve with the line $y = -2$, and so the limits point was not earned. In part (c) the student integrates a difference of squares, rather than the square of the difference of the functions. The integrand point was not earned, and the student was not eligible for the answer point. In part (d) the integrand is not correct, and the student was not eligible for the answer point.