AP® CALCULUS AB
2008 SCORING GUIDELINES (Form B)

Question 6

Consider the closed curve in the xy-plane given by

$$x^2 + 2x + y^4 + 4y = 5.$$

(a) Show that $\dfrac{dy}{dx} = \dfrac{-(x+1)}{2\left(y^3+1\right)}$.

(b) Write an equation for the line tangent to the curve at the point $(-2, 1)$.

(c) Find the coordinates of the two points on the curve where the line tangent to the curve is vertical.

(d) Is it possible for this curve to have a horizontal tangent at points where it intersects the x-axis? Explain your reasoning.

(a) $2x + 2 + 4y^3\dfrac{dy}{dx} + 4\dfrac{dy}{dx} = 0$

$\left(4y^3 + 4\right)\dfrac{dy}{dx} = -2x - 2$

$\dfrac{dy}{dx} = \dfrac{-2(x+1)}{4\left(y^3+1\right)} = \dfrac{-(x+1)}{2\left(y^3+1\right)}$

$2 : \begin{cases} 1 : \text{implicit differentiation} \\ 1 : \text{verification} \end{cases}$

(b) $\left.\dfrac{dy}{dx}\right|_{(-2,\,1)} = \dfrac{-(-2+1)}{2(1+1)} = \dfrac{1}{4}$

Tangent line: $y = 1 + \dfrac{1}{4}(x+2)$

$2 : \begin{cases} 1 : \text{slope} \\ 1 : \text{tangent line equation} \end{cases}$

(c) Vertical tangent lines occur at points on the curve where $y^3 + 1 = 0$ (or $y = -1$) and $x \neq -1$.

On the curve, $y = -1$ implies that $x^2 + 2x + 1 - 4 = 5$, so $x = -4$ or $x = 2$.

Vertical tangent lines occur at the points $(-4, -1)$ and $(2, -1)$.

$3 : \begin{cases} 1 : y = -1 \\ 1 : \text{substitutes } y = -1 \text{ into the equation of the curve} \\ 1 : \text{answer} \end{cases}$

(d) Horizontal tangents occur at points on the curve where $x = -1$ and $y \neq -1$.

The curve crosses the x-axis where $y = 0$.

$(-1)^2 + 2(-1) + 0^4 + 4 \cdot 0 \neq 5$

No, the curve cannot have a horizontal tangent where it crosses the x-axis.

$2 : \begin{cases} 1 : \text{works with } x = -1 \text{ or } y = 0 \\ 1 : \text{answer with reason} \end{cases}$

Work for problem 6(a)

$$2x + 2 + 4y^3 y' + 4y' = 0$$

$$y'(4y^3+4) = -2x-2$$

$$y' = \frac{-2x-2}{4y^3+4} = \frac{-(x+1)}{2(y^3+1)}$$

Work for problem 6(b)

$$y' = \text{slope} = \frac{-(-2+1)}{2(1+1)} = \frac{1}{4}$$

$$y - y_0 = y'(x - x_0)$$

$$y - 1 = \frac{1}{4}(x+2)$$

$$y = \frac{1}{4}x + \frac{3}{2}$$

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Continue problem 6 on page 15.

Work for problem 6(c)

line tangent to the curve is vertical ⇒ slope is undefined

$2(y^3+1)=0$

$y^3+1=0$

$y^3=-1$

$y=-1$

$x^2+2x+1-4=5$

$x^2+2x-8=0$

$(x-2)(x+4)=0$

$x=2$ or $x=-4$

→ The two points are $(2,-1)$ and $(-4,-1)$

Work for problem 6(d)

intersects the x-axis ⇒ $y=0$

$\frac{dy}{dx}=\frac{-(x+1)}{2}$

horizontal tangent is when slope $=0$

$\frac{-(x+1)}{2}=0$

$x+1=0$

$x=-1$

$+1-2+0+4(0)=5$

$-1\neq 5$

→ No. It is not possible for this curve to have a horizontal tangent at points where it intersects the x-axis.

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Form B
AB 6
6 B1

NO CALCULATOR ALLOWED

Work for problem 6(a)

$$2x + 2 + 4y^3 y' + 4y' = 0$$

$$4y^3 y' + 4y' = -2x - 2$$

$$y'(4y^3 + 4) = -2x - 2$$

$$y' = \frac{-2x - 2}{4y^3 + 4}$$

$$y' = \frac{-2(x+1)}{2(y^3+1)}$$

$$y' = \frac{-(x+1)}{2(y^3+1)} = \frac{dy}{dx}$$

Work for problem 6(b)

$$y' = \frac{-(-2+1)}{2(1^3+1)} \Rightarrow \frac{+1}{4}$$

$$y - y_0 = m(x - x_0)$$

$$y - 1 = \frac{1}{4}(x + 2)$$

$$y - 1 = \frac{1}{4}x + \frac{1}{2}$$

$$y = \frac{1}{4}x + \frac{3}{2}$$

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Continue problem 6 on page 15.

Work for problem 6(c)

slope = ∞

$y' = -\frac{(x+1)}{2(y^2+1)} = \infty$

Work for problem 6(d)

horizontal pts $\Rightarrow y' = 0$

$\ast$ int-x'axis $\Rightarrow y = 0$

$x^2 + 2x + y^2 + y = 5$

$x^2 + 2x - 5 = 0$

$\frac{-(x+1)}{2(y^2+1)} = 0 \Rightarrow -(x+1) = 0$

$-x - 1 = 0$

$-x = 1$

$x = -1$

$\Rightarrow (-1)^2 + 2(-1) - 5 = 0$

$2 - 2 - 5 \neq 0$, it's not possible.

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Form B
AB6
6C1

NO CALCULATOR ALLOWED

Work for problem 6(a)

$$2x + 2 + 4y^3 \frac{dy}{dx} + 4\frac{dy}{dx} = 0$$

$$4y^3\frac{dy}{dx} + 4\frac{dy}{dx} = -2x - 2$$

$$\frac{dy}{dx}(4y^3 + 4) = -2x - 2$$

$$\frac{dy}{dx} = \frac{-2x-2}{(4y^3+4)}$$

$$\frac{dy}{dx} = \frac{-2(x+1)}{4(y^3+1)} = \boxed{-\frac{(x+1)}{2(y^3+1)}}$$

Work for problem 6(b)

Slope of tangent line $= \frac{dy}{dx}$ at $(-2,1)$

$$\frac{dy}{dx} = \frac{-2(-2+1)}{2(1^3+1)} = \frac{4-2}{4} = \frac{1}{2}$$

$$y - 1 = \frac{1}{2}(x+2)$$

$$y = \frac{x}{2} + 1 + 1$$

$$\boxed{y = \frac{x}{2} + 2}$$

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Continue problem 6 on page 15.

Work for problem 6(c)

$2(y^3+1) = 0$ means the vertical line

$2y^3 + 2 = 0$

$y^3 = -1$

$y = -1$

when $y = -1$ $x =$

Work for problem 6(d)

No, because it does not touch the x-axis

AP® CALCULUS AB
2008 SCORING COMMENTARY (Form B)

Question 6

Sample: 6A
Score: 9

The student earned all 9 points.

Sample: 6B
Score: 6

The student earned 6 points: 2 points in part (a), 2 points in part (b), no points in part (c), and 2 points in part (d). The student presents correct work in parts (a), (b), and (d). In part (c) the student does not present $y^3 = -1,$ so the response did not earn any points.

Sample: 6C
Score: 4

The student earned 4 points: 2 points in part (a), 1 point in part (b), 1 point in part (c), and no points in part (d). The student presents correct work in part (a). In part (b) the student makes an error in calculating the slope so did not earn the first point. The student uses the incorrect slope and gives a tangent line equation, which earned the second point. In part (c) the student earned 1 point for finding $y = -1,$ but the response does not substitute the value of y in the original equation, so no additional points were earned. In part (d) the student presents an answer without any supporting work, so no points were earned.