AP® CALCULUS AB
2008 SCORING GUIDELINES (Form B)

Question 1

Let R be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the vertical line $x = -1$.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the y-axis are squares. Find the volume of this solid.

The graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$ intersect at the points $(0, 0)$ and $(9, 3)$.

(a) $\int_0^9 \left(\sqrt{x} - \frac{x}{3}\right) dx = 4.5$

OR

$\int_0^3 \left(3y - y^2\right) dy = 4.5$

$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $\pi \int_0^3 \left((3y+1)^2 - \left(y^2+1\right)^2\right) dy$

$= \frac{207\pi}{5} = 130.061 \text{ or } 130.062$

$4 : \begin{cases} 1 : \text{constant and limits} \\ 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) $\int_0^3 \left(3y - y^2\right)^2 dy = 8.1$

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and answer} \end{cases}$

Form B
AB1
1A,

CALCULUS AB

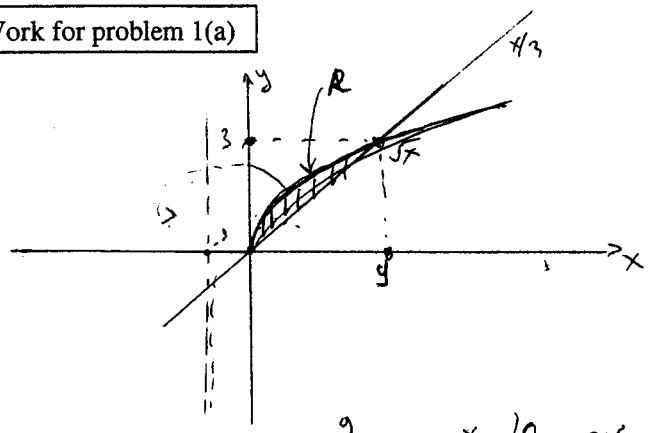
SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)



$\frac{x}{3} = \sqrt{x}$

$\frac{x^2}{9} = x$

$9x = x^2$

$x = 9$ ← intersection coordinate

$R = \int_0^9 (\sqrt{x} - \frac{x}{3})\,dx \approx 4.5$ (by calculator)

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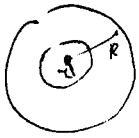
Continue problem 1 on page 5.

Work for problem 1(b)



$y = \frac{1}{3}x$

$x = 3y$

$y = \sqrt{x}$

$x = y^2$

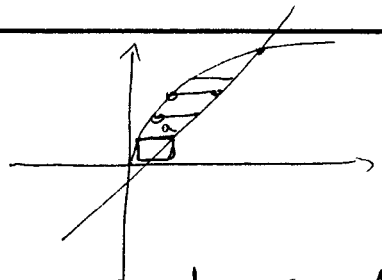
Use washer method.

R (outline) $= 3y + 1$

$r = y^2 + 1$

$$\text{Volume} = \pi \int_0^3 (R^2 - r^2)\,dy = \pi \int_0^3 \left[(3y+1)^2 - (y^2+1)^2\right] dy = 130.0619 \quad \text{(by calc.)}$$

Work for problem 1(c)



If a cross section is square, then its area $A = (3y - y^2)^2$

$$\text{Volume} = \int_0^3 A\,dy = \int_0^3 (3y - y^2)^2\,dy = 8.1 \quad \text{(by calc.)}$$

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Form B

AB1

1B1

CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

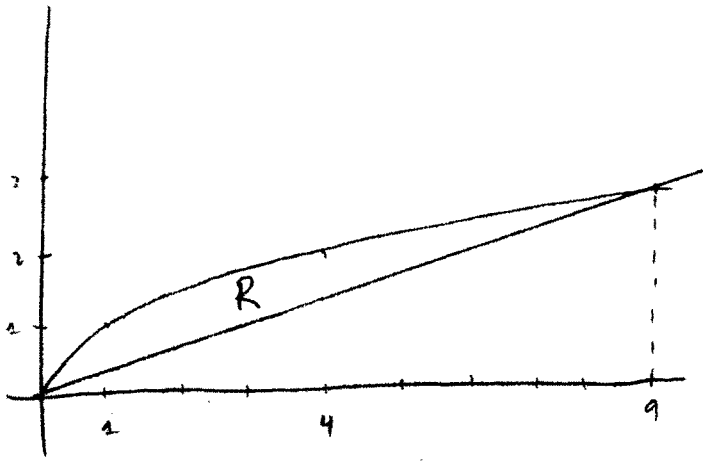
A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

Intersects between $\sqrt{x}$ and $\frac{x}{3}$ at $x = 9$ and $x = 0$

$$R = \int_0^9 \left(\sqrt{x} - \frac{x}{3}\right) dx = 4.5$$

the area is the area under the curve of $\sqrt{x}$ minus the area under the curve of $\frac{x}{3}$



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Continue problem 1 on page 5.

Work for problem 1(b)

R : big radius $R = 3y$

r : small radius $r = y^2$

$$V = \pi \int_0^3 (R)^2 - (r)^2 \, dy$$

$$V = \pi \int_0^3 (3y)^2 - (y^2)^2 \, dy = \frac{162}{5} = 32.4$$

Work for problem 1(c)

$$V = \int_0^3 (3y - y^2)^2 \, dy = \frac{81}{10} = 8.1$$

1 1 1 1 1 Form B

AB1
1C1

CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

$y = \sqrt{x}$ $y = \frac{x}{3}$

$\sqrt{x} = \frac{x}{3}$ $x^{1/2} = \frac{x}{3}$ $x = 9$

graphs intersect at $x = 9$

First Quadrant

$$\int_0^9 \left(\sqrt{x} - \frac{x}{3}\right) dx = 4.5 \text{ units}^2$$

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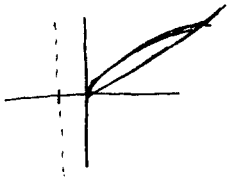
Work for problem 1(b)



$y = \sqrt{x}$, $y = \frac{x}{3}$

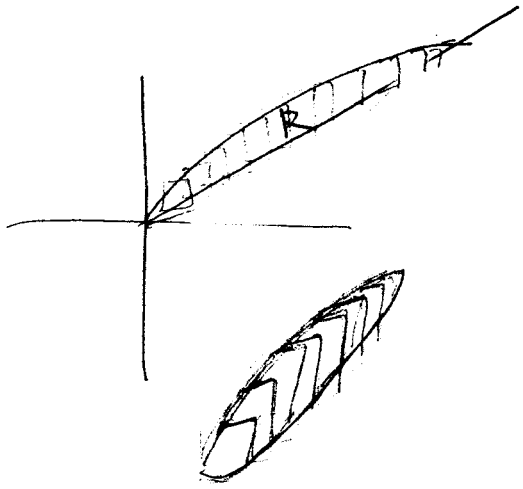
$x = y^2$ $x = 3y$

$dy = 2y$ $dy = 3$

~~$\pi \int_0^9 ((y^2)^2 - (3y)^2)\, dy = 30230.9$ units^3~~

$\pi \int_0^9 (1 - y^2)^2 - (1 - 3y)^2\, dy = 29467.5$ units^3

Work for problem 1(c)



area of $R = 4.5$ units^2

Volume $= \int_0^9 (\sqrt{x} - \frac{x}{3})^2\, dx$

$= 2.7$ units^3

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AP® CALCULUS AB
2008 SCORING COMMENTARY (Form B)

Question 1

Sample: 1A
Score: 9

The student earned all 9 points.

Sample: 1B
Score: 6

The student earned 6 points: 3 points in part (a), 1 point in part (b), and 2 points in part (c). The student presents correct work in parts (a) and (c). In part (b) the student has the correct limits and constant but rotates R about the line $x = 0$ instead of $x = -1$. As a result, the student earned only 1 point.

Sample: 1C
Score: 4

The student earned 4 points: 3 points in part (a), no points in part (b), and 1 point in part (c). The student presents correct work in part (a). In part (b) the student makes several errors. Although the constant is correct, the limits are incorrect, so the student did not earn the first point. The student attempts to rotate about $x = -1$ but has incorrect values in the integrand, so the response did not earn the integrand or answer points. In part (c) the student has the correct volume for cross sections drawn perpendicular to the x-axis and earned 1 point.