**Intent of Question**

The goals of this question are to assess a student’s ability to: (1) explain how a commonly used statistic measures variability; (2) use a graphical display to address the research question of interest in a simple comparative experiment; and (3) use a confidence interval to make an appropriate inference.

**Solution**

**Part (a):**

Roughly speaking, the standard deviation \( s = 2.141 \) measures a “typical” or “average” distance between the individual discoloration ratings and the mean discoloration rating for the strawberries in the control group.

**Part (b):**

The preservative does appear to have been effective in lowering the amount of discoloration in strawberries. The discoloration ratings for the strawberries that received the preservative, shown in the top dotplot, are clearly centered at a value that is lower than the center of the discoloration rating distribution for the control group, shown in the bottom dotplot. In addition, the dotplots can be used to find all five statistics in the five-number summary (min, Q1, median, Q3, and max) for both groups. In fact, four of the five statistics (the maximum is the only exception) are lower for the strawberries that received the preservative.

**Part (c):**

Since zero is not contained in the 95 percent confidence interval for the difference \( \mu_t - \mu_u \), we can conclude that there is a significant difference between the mean ratings for the two groups at the \( \alpha = 0.05 \) level. The population mean discoloration rating for untreated strawberries is estimated to be between 0.16 and 2.72 units higher than the population mean discoloration rating for treated strawberries. Thus, we think there would be a difference in the population mean discoloration ratings for treated and untreated strawberries.

**Scoring**

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

**Part (a) is scored as essentially correct (E) if**

the standard deviation is interpreted correctly in the context of this experiment.

Part (a) is scored as partially correct (P) if:

- a standard textbook description of the standard deviation is provided without any reference to context, e.g., the standard deviation is described as the square root of an average squared deviation, or a “typical” or “average” deviation from the mean;

- the student provides evidence that the distribution of discoloration ratings in the control group is approximately normal, then correctly applies the 68-95-99.7 rule.
Question 1 (continued)

Part (a) is scored as incorrect (I) if:
the formula for the standard deviation is copied from the formula sheet and no further explanation is provided;

OR
the student uses the 68-95-99.7 rule without justifying that the distribution of discoloration ratings in the control group is approximately normal.

Part (b) is scored as essentially correct (E) if the student indicates that the preservative appears to be effective and explicitly links this decision to comparison of a characteristic of relative standing from the dotplots for the two groups.

Part (b) is scored as partially correct (P) if:
the student says that the preservative appears to be effective because the discoloration ratings appear to be lower for the treatment group, but the student does not explicitly link this decision to comparison of a characteristic of relative standing for the two groups;

OR
the student correctly compares one or more characteristics of relative standing for the two groups but never states that the preservative was effective in lowering discoloration.

Part (b) is scored as incorrect (I) if:
the student says that the preservative is not effective because the centers of the two distributions are roughly the same;

OR
the student says that the preservative is effective, with incorrect or no justification.

Part (c) is scored as essentially correct (E) if the student indicates that zero is not included in the confidence interval, so there is a difference (in population means), AND states the conclusion in the context of this experiment.

Part (c) is scored as partially correct (P) if:
the student indicates that zero is not included in the confidence interval, so there is a difference (in population means), but does not state the conclusion in the context of this experiment;

OR
the student correctly interprets the 95 percent confidence interval in context and indicates that there is a difference (in population means), without indicating that zero is not included in the confidence interval.

Part (c) is scored as incorrect (I) if the student concludes that the preservative is not effective OR says that no conclusion can be made based on the confidence interval, OR the student states a conclusion that refers to sample means instead of population means.

Notes:
• The student is not required to specify the significance level in part (c), but if it is specified, it must be correct.
Question 1 (continued)

- An adjustment could be made to formally conduct a one-sided test, but in general, confidence intervals are used to conduct two-sided tests. The fact that the lower endpoint of the confidence interval is positive does provide evidence that the preservative is effective in lowering the amount of discoloration in strawberries. The correct formal statement is: The 97.5 percent lower confidence bound for the difference in the means is above zero (0.16), so at the 0.025 level we would conclude that the mean rating for the treated berries is significantly lower than the mean for the untreated strawberries.

4  Complete Response

   All three parts essentially correct

3  Substantial Response

   Two parts essentially correct and one part partially correct

2  Developing Response

   Two parts essentially correct and no parts partially correct

   OR  One part essentially correct and two parts partially correct

   OR  Three parts partially correct

1  Minimal Response

   One part essentially correct and either zero or one part partially correct

   OR  No parts essentially correct and two parts partially correct
STATISTICS
SECTION II
Part A
Questions 1-5
Spend about 65 minutes on this part of the exam.
Percent of Section II grade—75

Directions: Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

1. The department of agriculture at a university was interested in determining whether a preservative was effective in reducing discoloration in frozen strawberries. A sample of 50 ripe strawberries was prepared for freezing. Then the sample was randomly divided into two groups of 25 strawberries each. Each strawberry was placed into a small plastic bag.

The 25 bags in the control group were sealed. The preservative was added to the 25 bags containing strawberries in the treatment group, and then those bags were sealed. All bags were stored at 0°C for a period of 6 months. At the end of this time, after the strawberries were thawed, a technician rated each strawberry’s discoloration from 1 to 10, with a low score indicating little discoloration.

The dotplots below show the distributions of discoloration rating for the control and treatment groups.

(a) The standard deviation of ratings for the control group is 2.141. Explain how this value summarizes variability in the control group.

The average distance from the mean discoloration rating is 2.141 in the control group.

GO ON TO THE NEXT PAGE.
(b) Based on the dotplots, comment on the effectiveness of the preservative in lowering the amount of
discoloration in strawberries. (No calculations are necessary.)

The preservative was fairly effective in
lowering the amount of discoloration
because the mean rating of the treatment
group - 5.16 - is less than the control
group - 6.36- indicating there was generally
less discoloration. Also, the median of the
preservative group was 5, which was less
than the median of the control group - 7.

(c) Researchers at the university decided to calculate a 95 percent confidence interval for the difference in mean
discoloration rating between strawberries that were not treated with preservative and those that were treated
with preservative. The confidence interval they obtained was (0.16, 2.72). Assume that the conditions
necessary for the t-confidence interval are met.

Based on the confidence interval, comment on whether there would be a difference in the population mean
discoloration ratings for the treated and untreated strawberries.

There would be a difference in the population
mean discoloration ratings for the treated
and untreated strawberries because we are
95% confident that the difference is
between .16 and 2.72; indicating a difference
in the means because zero is not within
this interval.
STATISTICS
SECTION II
Part A
Questions 1-5
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The 25 bags in the control group were sealed. The preservative was added to the 25 bags containing strawberries in the treatment group, and then those bags were sealed. All bags were stored at 0°C for a period of 6 months. At the end of this time, after the strawberries were thawed, a technician rated each strawberry’s discoloration from 1 to 10, with a low score indicating little discoloration.

The dotplots below show the distributions of discoloration rating for the control and treatment groups.

(a) The standard deviation of ratings for the control group is 2.141. Explain how this value summarizes variability in the control group.

The standard deviation is a measure of the typical amount by which data varies from the mean.

Distributions with higher standard deviations exhibit greater variation than those with lower standard deviations.

(standard deviation = \sqrt{variance})

GO ON TO THE NEXT PAGE.
(b) Based on the dotplots, comment on the effectiveness of the preservative in lowering the amount of discoloration in strawberries. (No calculations are necessary.)

While the overall range for the data of the treatment group is greater than the range for the control group, this is only because the maximum value stayed the same (max = 10) while the minimum value decreased (min = 1); this is an indication that there is some improvement. In addition, both the mean and the median values for the treatment group were lower than their corresponding values for the control group. Lower ratings for discoloration mean improvement with the treatment.

(c) Researchers at the university decided to calculate a 95 percent confidence interval for the difference in mean discoloration rating between strawberries that were not treated with preservative and those that were treated with preservative. The confidence interval they obtained was (0.16, 2.72). Assume that the conditions necessary for the $t$-confidence interval are met.

Based on the confidence interval, comment on whether there would be a difference in the population mean discoloration ratings for the treated and untreated strawberries.

Because a value of 0 is not contained within the interval, we can assume that there would be a difference in the population mean discoloration ratings for the treated and untreated strawberries with 95% confidence.
STATISTICS
SECTION II
Part A
Questions 1-5
Spend about 65 minutes on this part of the exam.
Percent of Section II grade—75

Directions: Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

1. The department of agriculture at a university was interested in determining whether a preservative was effective in reducing discoloration in frozen strawberries. A sample of 50 ripe strawberries was prepared for freezing. Then the sample was randomly divided into two groups of 25 strawberries each. Each strawberry was placed into a small plastic bag.

The 25 bags in the control group were sealed. The preservative was added to the 25 bags containing strawberries in the treatment group, and then those bags were sealed. All bags were stored at 0°C for a period of 6 months. At the end of this time, after the strawberries were thawed, a technician rated each strawberry’s discoloration from 1 to 10, with a low score indicating little discoloration.

The dotplots below show the distributions of discoloration rating for the control and treatment groups.

(a) The standard deviation of ratings for the control group is 2.141. Explain how this value summarizes variability in the control group.

The standard deviation of 2.141 is the measure of the variability of the observations away from the mean discoloration rating of 6.60.

GO ON TO THE NEXT PAGE.
(b) Based on the dotplots, comment on the effectiveness of the preservative in lowering the amount of discoloration in strawberries. (No calculations are necessary.)

Based on the dotplots, the mean discoloration rating of the Treatment group of strawberries is lower than that of the control group. The median of the treatment group is lower as well with a median of five, whereas the control group has a median of seven. For the treatment group, the middle 50% of the data lies between 3.5 and 6.5 and the middle 50% of the data from the control group lies between 5.5 and 8.

Based on the data, the treatment is rather effective in lowering the amount of discoloration.

(c) Researchers at the university decided to calculate a 95 percent confidence interval for the difference in mean discoloration rating between strawberries that were not treated with preservative and those that were treated with preservative. The confidence interval they obtained was (0.16, 2.72). Assume that the conditions necessary for the t-confidence interval are met.

Based on the confidence interval, comment on whether there would be a difference in the population mean discoloration ratings for the treated and untreated strawberries.

Based on the confidence interval, there would be a difference in the population mean discoloration ratings for the treated and untreated strawberries since the confidence interval does not contain zero.
Question 1

Overview

The goals of this question were to assess students’ abilities to: (1) explain how a commonly used statistic measures variability; (2) use a graphical display to address the research question of interest in a simple comparative experiment; and (3) use a confidence interval to make an appropriate inference.

Sample: 1A
Score: 4

In part (a) the response gives a clear, concise explanation of the standard deviation in context. In part (b) the response provides a comparison of two measures of center, mean, and median, as justification for the effectiveness of the preservative. Note that the incorrect value of the control group mean (6.36 instead of 6.6) was viewed as a minor error since the problem specifies that no calculations are necessary. In part (c) the response demonstrates sound inferential reasoning about the difference in population means based on the 95 percent confidence interval, which does not include zero as a plausible value.

Sample: 1B
Score: 3

The response correctly defines the standard deviation in part (a) as “the typical amount by which data varies from the mean” but fails to include context. In the first sentence of part (b) the response suggests that the lower minimum discoloration rating in the treatment group gives evidence of “some improvement” due to the preservative. (This weak argument is enhanced by the subsequent comparison of the mean and median ratings in the two groups as justification for the preservative’s effectiveness.) In part (c) the response makes a clear, correct conclusion about the difference in population means using the confidence interval provided.

Sample: 1C
Score: 2

The response’s comment about standard deviation in part (a) doesn’t go far enough in clarifying that standard deviation measures the typical or average distance of the observations from the mean discoloration rating. In part (b) the response provides three valid comparisons of characteristics of relative standing for the treatment and control groups as evidence for the effectiveness of the preservative. The efficient response in part (c) conveys an understanding of the link between zero not being in the confidence interval and the difference in population mean discoloration ratings for treated and untreated strawberries.