CollegeBoard
Advanced Placement Program

AP® Calculus

Teacher's Guide

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connect to college success™
www.collegeboard.com

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The College Board: Connecting Students to College Success

The College Board is a not-for-profit membership association whose mission is to connect students to college success and opportunity. Founded in 1900, the association is composed of more than 5,000 schools, colleges, universities, and other educational organizations. Each year, the College Board serves seven million students and their parents, 23,000 high schools, and 3,500 colleges through major programs and services in college admissions, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT®, the PSAT/NMSQT®, and the Advanced Placement Program® (AP®). The College Board is committed to the principles of excellence and equity, and that commitment is embodied in all of its programs, services, activities, and concerns.

For further information, visit www.collegeboard.com.

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Welcome Letter from the College Board

Dear AP® Teacher:

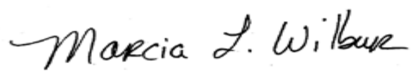
Whether you are a new AP teacher, using this AP Teacher's Guide to assist in developing a syllabus for the first AP course you will ever teach, or an experienced AP teacher simply wanting to compare the teaching strategies you use with those employed by other expert AP teachers, we are confident you will find this resource valuable. We urge you to make good use of the ideas, advice, classroom strategies, and sample syllabi contained in this Teacher's Guide.

You deserve tremendous credit for all that you do to fortify students for college success. The nurturing environment in which you help your students master a college-level curriculum—a much better atmosphere for one's first exposure to college-level expectations than the often large classes in which many first-year college courses are taught—seems to translate directly into lasting benefits as students head off to college. An array of research studies, from the classic 1999 U.S. Department of Education study *Answers in the Tool Box* to new research from the University of Texas and the University of California, demonstrate that when students enter high school with equivalent academic abilities and socioeconomic status, those who develop the content knowledge to demonstrate college-level mastery of an AP Exam (a grade of 3 or higher) have much higher rates of college completion and have higher grades in college. The 2005 National Center for Educational Accountability (NCEA) study shows that students who take AP have much higher college graduation rates than students with the *same* academic abilities who do not have that valuable AP experience in high school. Furthermore, a Trends in International Mathematics and Science Study (TIMSS, formerly known as the Third International Mathematics and Science Study) found that even AP Calculus students who score a 1 on the AP Exam are significantly outperforming other advanced mathematics students in the United States, and they compare favorably to students from the top-performing nations in an international assessment of mathematics achievement. (Visit AP Central® at apcentral.collegeboard.com for details about these and other AP-related studies.)

For these reasons, the AP teacher plays a significant role in a student's academic journey. Your AP classroom may be the only taste of college rigor your students will have before they enter higher education. It is important to note that such benefits cannot be demonstrated among AP courses that are AP courses in name only, rather than in quality of content. For AP courses to meaningfully prepare students for college success, courses must meet standards that enable students to replicate the content of the comparable college class. Using this AP Teacher's Guide is one of the keys to ensuring that your AP course is as good as (or even better than) the course the student would otherwise be taking in college. While the AP Program does not mandate the use of any one syllabus or textbook and emphasizes that AP teachers should be granted the creativity and flexibility to develop their own curriculum, it is beneficial for AP teachers to compare their syllabi not just to the course outline in the official AP Course Description and in chapter 3 of this guide, but also to the syllabi presented on AP Central, to ensure that each course labeled AP meets the standards of a college-level course. Visit AP Central® at apcentral.collegeboard.com for details about the AP Course Audit, course-specific Curricular Requirements, and how to submit your syllabus for AP Course Audit authorization.

As the Advanced Placement Program® continues to experience tremendous growth in the twenty-first century, it is heartening to see that in every U.S. state and the District of Columbia, a growing proportion of high school graduates have earned at least one grade of 3 or higher on an AP Exam. In some states, more than 20 percent of graduating seniors have accomplished this goal. The incredible efforts of AP teachers are paying off, producing ever greater numbers of college-bound seniors who are prepared to succeed in college. Please accept my admiration and congratulations for all that you are doing and achieving.

Sincerely,



Marcia Wilbur
Director, Curriculum and Content Development
Advanced Placement Program

Equity and Access

In the following section, the College Board describes its commitment to achieving equity in the AP Program.

Why are equitable preparation and inclusion important?

Currently, 40 percent of students entering four-year colleges and universities and 63 percent of students at two-year institutions require some remedial education. This is a significant concern because a student is less likely to obtain a bachelor's degree if he or she has taken one or more remedial courses.[1]

Nationwide, secondary school educators are increasingly committed not just to helping students complete high school but also to helping them develop the habits of mind necessary for managing the rigors of college. As *Educational Leadership* reported in 2004:

> The dramatic changes taking place in the U.S. economy jeopardize the economic future of students who leave high school without the problem-solving and communication skills essential to success in postsecondary education and in the growing number of high-paying jobs in the economy. To back away from education reforms that help all students master these skills is to give up on the commitment to equal opportunity for all.[2]

Numerous research studies have shown that engaging a student in a rigorous high school curriculum such as is found in AP courses is one of the best ways that educators can help that student persist and complete a bachelor's degree.[3] However, while 57 percent of the class of 2004 in U.S. public high schools enrolled in higher education in fall 2004, only 13 percent had been boosted with a successful AP experience in high school.[4] Although AP courses are not the only examples of rigorous curricula, there is still a significant gap between students with college aspirations and students with adequate high school preparation to fulfill those aspirations.

Strong correlations exist between AP success and college success.[5] Educators attest that this is partly because AP enables students to receive a taste of college while still in an environment that provides more support and resources for students than do typical college courses. Effective AP teachers work closely with their students, giving them the opportunity to reason, analyze, and understand for themselves. As a result, AP students frequently find themselves developing new confidence in their academic abilities and discovering their previously unknown capacities for college studies and academic success.

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1. Andrea Venezia, Michael W. Kirst, and Anthony L. Antonio, *Betraying the College Dream: How Disconnected K–12 and Postsecondary Education Systems Undermine Student Aspirations* (Palo Alto, Calif.: The Bridge Project, 2003), 8.

2. Frank Levy and Richard J. Murnane, "Education and the Changing Job Market." *Educational Leadership* 62 (2) (October 2004): 83.

3. In addition to studies from University of California–Berkeley and the National Center for Educational Accountability (2005), see the classic study on the subject of rigor and college persistence: Clifford Adelman, *Answers in the Tool Box: Academic Intensity, Attendance Patterns, and Bachelor's Degree Attainment* (Washington, D.C.: U.S. Department of Education, 1999).

4. *Advanced Placement Report to the Nation* (New York: College Board, 2005).

5. Wayne Camara, "College Persistence, Graduation, and Remediation," *College Board Research Notes* (RN-19) (New York: College Board, 2003).

Which students should be encouraged to register for AP courses?

Any student willing and ready to do the work should be considered for an AP course. The College Board actively endorses the principles set forth in the following Equity Policy Statement and encourages schools to support this policy.

> The College Board and the Advanced Placement Program encourage teachers, AP Coordinators, and school administrators to make equitable access a guiding principle for their AP programs. The College Board is committed to the principle that all students deserve an opportunity to participate in rigorous and academically challenging courses and programs. All students who are willing to accept the challenge of a rigorous academic curriculum should be considered for admission to AP courses. The Board encourages the elimination of barriers that restrict access to AP courses for students from ethnic, racial, and socioeconomic groups that have been traditionally underrepresented in the AP Program. Schools should make every effort to ensure that their AP classes reflect the diversity of their student population.

The fundamental objective that schools should strive to accomplish is to create a stimulating AP program that academically challenges students and has the same ethnic, gender, and socioeconomic demographics as the overall student population in the school. African American and Native American students are severely underrepresented in AP classrooms nationwide; Latino student participation has increased tremendously, but in many AP courses Latino students remain underrepresented. To prevent a willing, motivated student from having the opportunity to engage in AP courses is to deny that student the possibility of a better future.

Knowing what we know about the impact a rigorous curriculum can have on a student's future, it is not enough for us simply to leave it to motivated students to seek out these courses. Instead, we must reach out to students and encourage them to take on this challenge. With this in mind, there are two factors to consider when counseling a student regarding an AP opportunity:

1. Student motivation

Many potentially successful AP students would never enroll if the decision were left to their own initiative. They may not have peers who value rigorous academics, or they may have had prior academic experiences that damaged their confidence or belief in their college potential. They may simply lack an understanding of the benefits that such courses can offer them. Accordingly, it is essential that we not gauge a student's motivation to take AP until that student has had the opportunity to understand the advantages—not just the challenges—of such course work.

Educators committed to equity provide all students in a school with an understanding of the benefits of rigorous curricula. Such educators conduct student assemblies and/or presentations to parents that clearly describe the advantages of taking an AP course and outline the work expected of students. Perhaps most important, they have one-on-one conversations with the students in which advantages and expectations are placed side by side. These educators realize that many students, lacking confidence in their abilities, will be listening for any indication that they should not take an AP course. Accordingly, such educators, while frankly describing the amount of homework to be anticipated, also offer words of encouragement and support, assuring the students that if they are willing to do the work, they are wanted in the course.

The College Board has created a free online tool, AP Potential™, to help educators reach out to students who previously might not have been considered for participation in an AP course. Drawing upon data based on correlations between student performance on specific sections of the PSAT/NMSQT® and performance on specific AP Exams, AP Potential generates rosters of students at your school who have a strong likelihood of success in a particular AP course. Schools nationwide have successfully enrolled many more students in AP than ever before by using these rosters to help students (and their parents) see themselves as having potential to succeed in college-level studies. For more information, visit http://appotential.collegeboard.com.

Actively recruiting students for AP and sustaining enrollment can also be enhanced by offering incentives for both students and teachers. While the College Board does not formally endorse any one incentive for boosting AP participation, we encourage school administrators to develop policies that will best serve an overarching goal to expand participation and improve performance in AP courses. When such incentives are implemented, educators should ensure that quality verification measures such as the AP Exam are embedded in the program so that courses are rigorous enough to merit the added benefits.

Many schools offer the following incentives for students who enroll in AP:

- Extra weighting of AP course grades when determining class rank
- Full or partial payment of AP Exam fees
- On-site exam administration

Additionally, some schools offer the following incentives for teachers to reward them for their efforts to include and support traditionally underserved students:

- Extra preparation periods
- Reduced class size
- Reduced duty periods
- Additional classroom funds
- Extra salary

2. Student preparation

Because AP courses should be the equivalent of courses taught in colleges and universities, it is important that a student be prepared for such rigor. The types of preparation a student should have before entering an AP course vary from course to course and are described in the official AP Course Description book for each subject (available as a free download at apcentral.collegeboard.com).

Unfortunately, many schools have developed a set of gatekeeping or screening requirements that go far beyond what is appropriate to ensure that an individual student has had sufficient preparation to succeed in an AP course. Schools should make every effort to eliminate the gatekeeping process for AP enrollment. Because research has not been able to establish meaningful correlations between gatekeeping devices and actual success on an AP Exam, the College Board **strongly discourages** the use of the following factors as thresholds or requirements for admission to an AP course:

- Grade point average
- Grade in a required prerequisite course
- Recommendation from a teacher

- AP teacher's discretion
- Standardized test scores
- Course-specific entrance exam or essay

Additionally, schools should be wary of the following concerns regarding the misuse of AP:

- Creating "Pre-AP courses" to establish a limited, exclusive track for access to AP
- Rushing to install AP courses without simultaneously implementing a plan to prepare students and teachers in lower grades for the rigor of the program

How can I ensure that I am not watering down the quality of my course as I admit more students?

Students in AP courses should take the AP Exam, which provides an external verification of the extent to which college-level mastery of an AP course is taking place. While it is likely that the percentage of students who receive a grade of 3 or higher may dip as more students take the exam, that is not an indication that the quality of a course is being watered down. Instead of looking at percentages, educators should be looking at raw numbers, since each number represents an individual student. If the raw number of students receiving a grade of 3 or higher on the AP Exam is not decreasing as more students take the exam, there is no indication that the quality of learning in your course has decreased as more students have enrolled.

What are schools doing to expand access and improve AP performance?

Districts and schools that successfully improve both participation and performance in AP have implemented a multipronged approach to expanding an AP program. These schools offer AP as capstone courses, providing professional development for AP teachers and additional incentives and support for the teachers and students participating at this top level of the curriculum. The high standards of the AP courses are used as anchors that influence the 6–12 curriculum from the "top down." Simultaneously, these educators are investing in the training of teachers in the pre-AP years and are building a vertically articulated, sequential curriculum from middle school to high school that culminates in AP courses—a broad pipeline that prepares students step-by-step for the rigors of AP so that they will have a fair shot at success in an AP course once they reach that stage. An effective and demanding AP program necessitates cooperation and communication between high schools and middle schools. Effective teaming among members of all educational levels ensures rigorous standards for students across years and provides them with the skills needed to succeed in AP. For more information about Pre-AP® professional development, including workshops designed to facilitate the creation of AP Vertical Teams® of middle school and high school teachers, visit AP Central.

Advanced Placement Program
The College Board

Participating in the AP Course Audit

Overview

The AP Course Audit is a collaborative effort among secondary schools, colleges and universities, and the College Board. For their part, schools deliver college-level instruction to students and complete and return AP Course Audit materials. Colleges and universities work with the College Board to define elements common to college courses in each AP subject, help develop materials to support AP teaching, and receive a roster of schools and their authorized AP courses. The College Board fosters dialogue about the AP Course Audit requirements and recommendations, and reviews syllabi.

Starting in the 2007-08 academic year, all schools wishing to label a course “AP” on student transcripts, course listings, or any school publications must complete and return the subject-specific AP Course Audit form, along with the course syllabus, for all sections of their AP courses. Approximately two months after submitting AP Course Audit materials, schools will receive a legal agreement authorizing the use of the “AP” trademark on qualifying courses. Colleges and universities will receive a roster of schools listing the courses authorized to use the “AP” trademark at each school.

Purpose

College Board member schools at both the secondary and college levels requested an annual AP Course Audit in order to provide teachers and administrators with clear guidelines on curricular and resource requirements that must be in place for AP courses and to help colleges and universities better interpret secondary school courses marked “AP” on students’ transcripts.

The AP Course Audit form identifies common, essential elements of effective college courses, including subject matter and classroom resources such as college-level textbooks and laboratory equipment. Schools and individual teachers will continue to develop their own curricula for AP courses they offer—the AP Course Audit will simply ask them to indicate inclusion of these elements in their AP syllabi or describe how their courses nonetheless deliver college-level course content.

AP Exam performance is not factored into the AP Course Audit. A program that audited only those schools with seemingly unsatisfactory exam performance might cause some schools to limit access to AP courses and exams. In addition, because AP Exams are taken and exam grades reported after college admissions decisions are already made, AP course participation has become a relevant factor in the college admissions process. On the AP Course Audit form, teachers and administrators attest that their course includes elements commonly taught in effective college courses. Colleges and universities reviewing students’ transcripts can thus be reasonably assured that courses labeled “AP” provide an appropriate level and range of college-level course content, along with the classroom resources to best deliver that content.

For more information

You should discuss the AP Course Audit with your department head and principal. For more information, including a timeline, frequently asked questions, and downloadable AP Course Audit forms, visit apcentral.collegeboard.com/courseaudit.

Preface

Congratulations and welcome! You are about to undertake a challenge that is at once wonderful and daunting: being charged with guiding your students on their journey of discovery through AP Calculus. Calculus is filled with ideas and applications that are rich and varied, and that differ dramatically from those studied in mathematics classes that come before it. Students find its mysteries new and intriguing and its concepts captivating but difficult. Teachers usually share all of these feelings. Teaching an AP Calculus class is indeed a challenge. Calculus teachers have the unparalleled pleasure of watching their students' eyes light up and spirits soar when they grasp a concept that had been elusive. They share the sweat and frustration when insights seem slippery or beyond reach. I always tell my students that calculus is hard, that it takes supreme effort and tenacity to fully understand its mysteries. I would venture to say the same thing to new calculus teachers, although the effort required is often multiplied by the number of students in your care!

The purpose of this Teacher's Guide is to help new teachers prepare for and teach an AP Calculus course. We have tried to anticipate and answer many of your questions and to point you to various resources. Sprinkled throughout the Guide are tips from seasoned AP Calculus teachers who have generously shared the lessons they have learned over time and through hard work.

An essential step in preparing to teach AP Calculus is to acquire and carefully read the *AP Calculus Course Description*. The core materials from that publication are contained here in Chapter 1. You should refer to the Course Description regularly as you work through the rest of this Teacher's Guide and prepare your syllabus.

Chapter 1 begins with an overview from David Bressoud from Macalester College, chair of the AP Calculus Development Committee from 1999 to 2005. Following the overview, you will find a section on the *AP Calculus Course Description*, which has reprints of several important parts of that publication. Then there is an outline of the major concepts in AP Calculus and a few suggestions on how to cover them with students.

Chapter 2 contains an assortment of suggestions from experienced, successful AP Calculus teachers. These seasoned veterans have kindly contributed a collection of tips covering a wide variety of subjects. Much of that advice will be of interest not only to new teachers, but also to more experienced teachers of calculus.

Chapter 3 offers guidance as you create your own syllabus out of the details of the Course Description. In addition, we've included six complete sample syllabi from experienced calculus teachers: four high school teachers and two higher education faculty. Nestled throughout those syllabi are many pearls of wisdom, so I encourage you to read them all, even if the contributor's primary textbook or other circumstances differ from your own.

Chapter 4 is all about the AP Calculus Exams, including strategies to prepare your students. More advice from experienced teachers is included in this section.

Chapter 5 tells you where to find additional resources for teaching AP Calculus. There is a vast array of materials to help you and your students on your journey. A section on professional development opportunities sponsored by the College Board is also included. We hope this chapter helps you make sense of the many resources available to you and serves as a reliable companion as you begin your teaching task.

The calculus you studied in school could well have been quite different from the AP Calculus of the twenty-first century. In particular, calculus instruction has evolved to emphasize a variety of representational settings for concepts. Exploring concepts graphically, numerically, and symbolically has become something of a mantra for modern calculus instruction. More and more, students are expected to verbalize their understanding or interpretation of calculus ideas. A richer and more diverse sampling of applications is a welcome feature. You'll see slope fields used to visualize solutions of differential equations, Riemann sums used to approximate definite integrals, a fantastic diversity of applications of the definite integral, and Euler's method and logistic differential equations in Calculus BC. Calculus is truly one of the most amazing and useful accomplishments in human history. Hard work should lead you to a fulfilling teaching experience!

Mark Howell

Mark Howell is a teacher of AP Calculus and computer science at his alma mater, Gonzaga College High School in Washington, D.C. He has served the AP community as a member of the AP Calculus Development Committee and as an AP Reader, Table Leader, and Question Leader. His efforts in the use of technology to enhance the teaching and learning of mathematics have taken him throughout the continental United States, as well as to Puerto Rico, Hawaii, Singapore, Thailand, and Australia.

Chapter 1
About AP® Calculus

AP Calculus: Past, Present, and Future

The AP Calculus Exams have gone through some major changes in the past decade. The first exams to require the use of graphing calculators appeared in 1995. A redefinition of the topic outlines and goals for AP Calculus was undertaken at the same time. This included increased emphasis on tabular and graphical representations of functions as well as the need to justify answers in the free-response section of the exams. These new expectations and the Calculus AB subscore for Calculus BC were introduced for the first time in the 1998 exams. Other changes included the move, beginning with the 2000 exams, to separate the free-response sections into two parts—three questions that students may solve with the help of graphing calculators and three questions that do not allow the use of calculators—and beginning with the 2004 exam, the introduction of slope fields into Calculus AB.

The AP Calculus Development Committee keeps informed of innovations in calculus curricula and pedagogy. Its mission is to maintain a curriculum that is equivalent to that taught at most colleges and universities. In this sense, it can never be in front of the pack. The committee aims to continually incorporate best practices and keep abreast of the college community regarding improvements to calculus instruction. There are no anticipated changes for the near future, but some of the issues that the committee is following include:

- **Technology.** Graphing calculator use at colleges and universities seems to be peaking (37 percent in 1995, compared to 52 percent in 2000[1]). Computers are increasingly being integrated into the college curriculum (18 percent in 1995; 31 percent in 2000). At some time in the future, AP Calculus may need to decide how to incorporate the use of computers. Because of security concerns, the issue is complex, but we are already trying to envision what the future might be.

- **Changing content in second-semester calculus.** Some colleges and universities have moved away from the introduction of series in second-semester calculus. In its place, they introduce topics of several variable calculus or study differential equations in greater depth. We are following and measuring this trend. If it accelerates, we will need to respond.

- **Specialized calculus.** Many fields, biology foremost among them, have come to realize that their students need some but not all of the skills taught in a full year of college-level calculus. These students also require additional mathematical topics that include statistics, linear algebra, and dynamical systems. Mathematics courses that package the skills their students need within a two-course sequence are becoming increasingly popular. It is not clear how AP Calculus could articulate with such a program, but we are following this development.

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1. David J. Lutzer et al., *Statistical Abstract of Undergraduate Programs in the Mathematical Sciences in the United States. Fall 2000 CBMS Survey* (Providence, R.I.: American Mathematical Society, 2002). These are percentages of the total number of students taking first-semester calculus within a four-year undergraduate program.

- **Post-BC Calculus.** Many students now take and gain credit for Calculus BC before their senior year of high school. Many of them have options for continuing to study mathematics; many do not. Even when those options exist, they may not be optimal. These students are among our most mathematically talented. They need to be challenged. They also need to have their mathematical understanding deepened and broadened. The committee has begun discussions with some of the leading mathematics educators to understand the nature of the materials and teacher training that would be needed to support these students.

Finally, the sheer number of students taking part in AP Calculus is having effects that the committee is watching. In 2004, approximately 225,000 students took an AP Calculus Exam. This number will likely continue to grow by about 25,000 students per year. It compares with approximately 240,000 students in all two- and four-year colleges and universities in the United States who take first-semester mainstream calculus in the fall, a number that has been steady or slightly decreasing over the past 20 years. The size of the program has important repercussions in high schools that are under increasing pressure to offer it and to channel students into calculus so that they can compete for admission to the most selective colleges and universities. The size of the program also has important repercussions in colleges and universities, where many if not most of the best students, especially at selective institutions, arrive with credit for at least one semester of calculus. Two articles that discuss the impact of these numbers, "The Changing Face of Calculus: First-Semester Calculus as a High School Course" and "The Changing Face of Calculus: First- and Second-Semester Calculus as College Courses," are available on the Mathematical Association of America Web site: www.maa.org.

David M. Bressoud
Chair, AP Calculus Development Committee, 1999–2005
DeWitt Wallace Professor
Mathematics and Computer Science Department
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Saint Paul, Minnesota

Course Description Essentials

Aside from the primary textbook, the *AP Calculus Course Description* is the most essential resource for any teacher of AP Calculus. Every new teacher should read it thoroughly, probably twice, and check it annually to see what changes, if any, are planned for the future.

You will find a vast repository of information about every AP course, including Calculus AB and Calculus BC, on AP Central® (apcentral.collegeboard.com). You can also download the Course Description from this Web site.

For ease of reference, we have included major portions of the *AP Calculus Course Description* in this chapter:

- Philosophy and goals for the courses
- Prerequisites for students
- Topic outlines for Calculus AB and Calculus BC, which detail what is covered on the AP Exams
- Information on graphing calculator use and restrictions

Other important information found in the Course Description includes:

- Information on the exam format (different sections and how much time is allocated for each)
- Sample multiple-choice questions and answers
- Instructions for the free-response sections and sample free-response questions

The AP Calculus Development Committee, a group of secondary school, college, and university calculus teachers, is responsible for setting the direction of the program and keeping the Course Description up to date. In consultation with mathematics assessment specialists at ETS, they are also responsible for creating the exams. In addition, the Chief Reader, a college faculty member, aids in the development process. He or she attends the meetings of the Development Committee to ensure that the free-response questions selected for the exams can be scored reliably. The Chief Reader coordinates the scoring of the free-response questions at the annual AP Reading in June. (You can apply to be an AP Reader at the AP Central Web site.)

It takes at least two years to develop each AP Calculus Exam. Committee members independently write a selection of multiple-choice and free-response questions. Other high school and college faculty members also write questions. These are reviewed and revised by the entire committee and eventually assembled into complete exams. Some multiple-choice questions are pretested in college classes to obtain an estimate of each question's level of difficulty. The exams are assembled with an eye toward producing a reliable assessment instrument that covers the appropriate content and contains questions at different levels of difficulty. Chapter 1 of the *2003 AP Calculus AB & AP Calculus BC Released Exams* publication contains detailed information on the AP process and how the exams are developed.

As you review the Course Description, bear in mind several important points:

- It is important to read the philosophy and goals sections carefully. It might be tempting to skim over them and rush headlong to the topic outlines, but since the statements in these sections articulate ideas that are at the heart and soul of the AP Calculus courses, they deserve special scrutiny.
- Watch for specific mention of the multirepresentational (graphical, numerical, analytical, and verbal) approaches in the courses.
- Note that the courses are described not as a collection of recipes or algorithms meant to solve specific types of problems, but rather as a "cohesive whole" unified by the themes of derivatives, integrals, limits, approximation, and applications and modeling.
- Students are expected to use technology throughout the course, except perhaps for certain targeted "noncalculator" assessments. The goal is not to teach technology per se, but to use technology to enhance the learning of mathematics.
- Ensuring your students' success on the AP Calculus Exams requires you to balance your teaching between "traditional" and "reformed" approaches. The noncalculator portions of the exams seek to measure a student's understanding of calculus skills and concepts—an understanding that must be demonstrated without the help of technology. Students are expected to take derivatives and evaluate integrals "by hand." They should know basic "recipes" for solving calculus problems and, in some cases, students need to know values of trigonometric functions without using a calculator. Copying the AP Exam format, many teachers give "two-part tests" in their classes—one part where no calculator use is allowed and the other part where a graphing calculator is required.

- The topic outlines in the Course Description give the scope of coverage of material on the AP Calculus Exams. You are free to enrich your course with additional topics that aren't included on the exams. In addition, keep in mind that your calculus textbook is a *guide* for your course—it's not the course. It is important to supplement your primary textbook with other resources so that your students experience a multirepresentational approach to calculus.

Philosophy

Calculus AB and Calculus BC are primarily concerned with developing the students' understanding of the concepts of calculus and providing experience with its methods and applications. The courses emphasize a multirepresentational approach to calculus, with concepts, results, and problems being expressed graphically, numerically, analytically, and verbally. The connections among these representations also are important.

Calculus BC is an extension of Calculus AB rather than an enhancement; common topics require a similar depth of understanding. Both courses are intended to be challenging and demanding.

Broad concepts and widely applicable methods are emphasized. The focus of the courses is neither manipulation nor memorization of an extensive taxonomy of functions, curves, theorems, or problem types. Thus, although facility with manipulation and computational competence are important outcomes, they are not the core of these courses.

Technology should be used regularly by students and teachers to reinforce the relationships among the multiple representations of functions, to confirm written work, to implement experimentation, and to assist in interpreting results.

Through the use of the unifying themes of derivatives, integrals, limits, approximation, and applications and modeling, the course becomes a cohesive whole rather than a collection of unrelated topics. These themes are developed using all the functions listed in the prerequisites.

Goals

- Students should be able to work with functions represented in a variety of ways: graphical, numerical, analytical, or verbal. They should understand the connections among these representations.

- Students should understand the meaning of the derivative in terms of a rate of change and local linear approximation and should be able to use derivatives to solve a variety of problems.

- Students should understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of change and should be able to use integrals to solve a variety of problems.

- Students should understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus.

- Students should be able to communicate mathematics both orally and in well-written sentences and should be able to explain solutions to problems.

- Students should be able to model a written description of a physical situation with a function, a differential equation, or an integral.

- Students should be able to use technology to help solve problems, experiment, interpret results, and verify conclusions.
- Students should be able to determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement.
- Students should develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.

Prerequisites

Before studying calculus, all students should complete four years of secondary mathematics designed for college-bound students: courses in which they study algebra, geometry, trigonometry, analytic geometry, and elementary functions. These functions include those that are linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric, and piecewise defined. In particular, before studying calculus, students must be familiar with the properties of functions, the algebra of functions, and the graphs of functions. Students must also understand the language of functions (domain and range, odd and even, periodic, symmetry, zeros, intercepts, and so on) and know the values of the trigonometric functions of the numbers 0, $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$, and their multiples.

Topic Outline for Calculus AB

This topic outline is intended to indicate the scope of the course, but it is not necessarily the order in which the topics need to be taught. Teachers may find that topics are best taught in different orders. (See AP Central and chapter 3 in this Teacher's Guide for sample syllabi.) Although the exam is based on the topics listed here, teachers may wish to enrich their courses with additional topics. (Note: the Topic Outline below is from the May 2006, May 2007 *AP Calculus Course Description*.)

I. Functions, Graphs, and Limits

Analysis of graphs With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Limits of functions (including one-sided limits)

- An intuitive understanding of the limiting process
- Calculating limits using algebra
- Estimating limits from graphs or tables of data

Asymptotic and unbounded behavior

- Understanding asymptotes in terms of graphical behavior
- Describing asymptotic behavior in terms of limits involving infinity
- Comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth)

Continuity as a property of functions

- An intuitive understanding of continuity (The function values can be made as close as desired by taking sufficiently close values of the domain.)
- Understanding continuity in terms of limits
- Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem)

II. Derivatives

Concept of the derivative

- Derivative presented graphically, numerically, and analytically
- Derivative interpreted as an instantaneous rate of change
- Derivative defined as the limit of the difference quotient
- Relationship between differentiability and continuity

Derivative at a point

- Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
- Tangent line to a curve at a point and local linear approximation
- Instantaneous rate of change as the limit of average rate of change
- Approximate rate of change from graphs and tables of values

Derivative as a function

- Corresponding characteristics of graphs of f and f'
- Relationship between the increasing and decreasing behavior of f and the sign of f'
- The Mean Value Theorem and its geometric consequences
- Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Second derivatives

- Corresponding characteristics of the graphs of f, f', and f''
- Relationship between the concavity of f and the sign of f''
- Points of inflection as places where concavity changes

Applications of derivatives

- Analysis of curves, including the notions of monotonicity and concavity
- Optimization, both absolute (global) and relative (local) extrema
- Modeling rates of change, including related rates problems
- Use of implicit differentiation to find the derivative of an inverse function
- Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration
- Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations

Computation of derivatives

- Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions
- Basic rules for the derivative of sums, products, and quotients of functions
- Chain rule and implicit differentiation

III. Integrals

Interpretations and properties of definite integrals

- Definite integral as a limit of Riemann sums
- Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

- Basic properties of definite integrals (examples include additivity and linearity)

Applications of integrals Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include using the integral of a rate of change to give accumulated change, finding the area of a region, the volume of a solid with known cross sections, the average value of a function, and the distance traveled by a particle along a line.

Fundamental Theorem of Calculus

- Use of the Fundamental Theorem to evaluate definite integrals
- Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined

Techniques of antidifferentiation

- Antiderivatives following directly from derivatives of basic functions
- Antiderivatives by substitution of variables (including change of limits for definite integrals)

Applications of antidifferentiation

- Finding specific antiderivatives using initial conditions, including applications to motion along a line
- Solving separable differential equations and using them in modeling (in particular, studying the equation $y' = ky$ and exponential growth)

Numerical approximations to definite integrals Use of Riemann sums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values.

Topic Outline for Calculus BC

The topic outline for Calculus BC includes all Calculus AB topics. Additional topics are found in paragraphs that are marked with a plus sign (+) or an asterisk (*). The additional topics can be taught anywhere in the course that the instructor wishes. Some topics will naturally fit immediately after their Calculus AB counterparts. Other topics may fit best after the completion of the Calculus AB topic outline. (See AP Central and chapter 3 in this Teacher's Guide for sample syllabi.) Although the exam is based on the topics listed here, teachers may wish to enrich their courses with additional topics. (Note: the Topic Outline below is from the May 2006, May 2007 *AP Calculus Course Description*.)

I. Functions, Graphs, and Limits

Analysis of graphs With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Limits of functions (including one-sided limits)

- An intuitive understanding of the limiting process
- Calculating limits using algebra
- Estimating limits from graphs or tables of data

Asymptotic and unbounded behavior

- Understanding asymptotes in terms of graphical behavior
- Describing asymptotic behavior in terms of limits involving infinity
- Comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth)

Continuity as a property of functions

- An intuitive understanding of continuity (The function values can be made as close as desired by taking sufficiently close values of the domain.)
- Understanding continuity in terms of limits
- Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem)

***Parametric, polar, and vector functions** The analysis of planar curves includes those given in parametric form, polar form, and vector form.

II. Derivatives

Concept of the derivative

- Derivative presented graphically, numerically, and analytically
- Derivative interpreted as an instantaneous rate of change
- Derivative defined as the limit of the difference quotient
- Relationship between differentiability and continuity

Derivative at a point

- Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
- Tangent line to a curve at a point and local linear approximation
- Instantaneous rate of change as the limit of average rate of change
- Approximate rate of change from graphs and tables of values

Derivative as a function

- Corresponding characteristics of graphs of f and f'
- Relationship between the increasing and decreasing behavior of f and the sign of f'

- The Mean Value Theorem and its geometric consequences
- Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Second derivatives

- Corresponding characteristics of the graphs of f, f', and f''
- Relationship between the concavity of f and the sign of f''
- Points of inflection as places where concavity changes

Applications of derivatives

- Analysis of curves, including the notions of monotonicity and concavity

+ Analysis of planar curves given in parametric form, polar form, and vector form, including velocity and acceleration

- Optimization, both absolute (global) and relative (local) extrema
- Modeling rates of change, including related rates problems
- Use of implicit differentiation to find the derivative of an inverse function
- Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration
- Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations

+ Numerical solution of differential equations using Euler's method
+ L'Hospital's Rule, including its use in determining limits and convergence of improper integrals and series

Computation of derivatives

- Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions
- Basic rules for the derivative of sums, products, and quotients of functions
- Chain rule and implicit differentiation

+ Derivatives of parametric, polar, and vector functions

III. Integrals

Interpretations and properties of definite integrals

- Definite integral as a limit of Riemann sums
- Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

- Basic properties of definite integrals (examples include additivity and linearity)

***Applications of integrals** Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include using the integral of a rate of change to give accumulated change, finding the area of a region (including a region bounded by polar curves), the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, and the length of a curve (including a curve given in parametric form).

Fundamental Theorem of Calculus

- Use of the Fundamental Theorem to evaluate definite integrals
- Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined

Techniques of antidifferentiation

- Antiderivatives following directly from derivatives of basic functions

\+ Antiderivatives by substitution of variables (including change of limits for definite integrals), parts, and simple partial fractions (nonrepeating linear factors only)

\+ Improper integrals (as limits of definite integrals)

Applications of antidifferentiation

- Finding specific antiderivatives using initial conditions, including applications to motion along a line
- Solving separable differential equations and using them in modeling (in particular, studying the equation $y' = ky$ and exponential growth)

\+ Solving logistic differential equations and using them in modeling

Numerical approximations to definite integrals Use of Riemann sums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values

*IV. Polynomial Approximations and Series

* **Concept of series** A series is defined as a sequence of partial sums, and convergence is defined in terms of the limit of the sequence of partial sums. Technology can be used to explore convergence or divergence.

* **Series of constants**

  + Motivating examples, including decimal expansion
  + Geometric series with applications
  + The harmonic series
  + Alternating series with error bound
  + Terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of p-series
  + The ratio test for convergence and divergence
  + Comparing series to test for convergence or divergence

* **Taylor series**

  + Taylor polynomial approximation with graphical demonstration of convergence (for example, viewing graphs of various Taylor polynomials of the sine function approximating the sine curve)
  + Maclaurin series and the general Taylor series centered at $x = a$
  + Maclaurin series for the functions e^x, $\sin x$, $\cos x$, and $\frac{1}{1-x}$
  + Formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, antidifferentiation, and the formation of new series from known series
  + Functions defined by power series
  + Radius and interval of convergence of power series
  + Lagrange error bound for Taylor polynomials

Use of Graphing Calculators

Professional mathematics organizations such as the National Council of Teachers of Mathematics, the Mathematical Association of America, and the Mathematical Sciences Education Board of the National Academy of Sciences have strongly endorsed the use of calculators in mathematics instruction and testing.

The use of a graphing calculator in AP Calculus is considered an integral part of the course. Students should be using this technology on a regular basis so that they become adept at using their graphing calculators. Students should also have experience with the basic paper-and-pencil techniques of calculus and be able to apply them when technological tools are unavailable or inappropriate.

The AP Calculus Development Committee understands that new calculators and computers, capable of enhancing the teaching of calculus, continue to be developed. There are two main concerns that the committee considers when deciding what level of technology should be required for the exams: equity issues and teacher development.

Over time, the range of capabilities of graphing calculators has increased significantly. Some calculators are much more powerful than first-generation graphing calculators and may include symbolic algebra features. Other graphing calculators are, by design, intended for students studying mathematics at lower levels than calculus. The committee can develop exams that are appropriate for any given level of technology, but it cannot develop exams that are fair to all students if the spread in the capabilities of the technology is too wide. Therefore, the committee has found it necessary to make certain requirements of the technology that will help ensure that all students have sufficient computational tools for the AP Calculus Exams. Exam restrictions should not be interpreted as restrictions on classroom activities. The committee will continue to monitor the developments of technology and will reassess the testing policy regularly.

Graphing Calculator Capabilities for the Exams

The committee develops exams based on the assumption that all students have access to four basic calculator capabilities used extensively in calculus. A graphing calculator appropriate for use on the exams is expected to have the built-in capability to:

1) plot the graph of a function within an arbitrary viewing window,

2) find the zeros of functions (solve equations numerically),

3) numerically calculate the derivative of a function, and

4) numerically calculate the value of a definite integral.

One or more of these capabilities should provide the sufficient computational tools for successful development of a solution to any exam question that requires the use of a calculator. Care is taken to ensure that the exam questions do not favor students who use graphing calculators with more extensive built-in features.

Students are expected to bring a calculator with the capabilities listed above to the exams. AP teachers should check their own students' calculators to ensure that the required conditions are met. A list of acceptable calculators can be found at AP Central. If a student wishes to use a calculator that is not on the list, the teacher must contact the AP Program (609 771-7300) before April 1 of the testing year to request written permission for the student to use the calculator on AP Exams.

Technology Restrictions on the Exams

Nongraphing scientific calculators, computers, devices with a QWERTY keyboard, and pen-input/stylus-driven devices, or electronic writing pads are not permitted for use on the AP Calculus Exams.

Test administrators are required to check calculators before the exam. Therefore, it is important for each student to have an approved calculator. The student should be thoroughly familiar with the operation of the calculator he or she plans to use. Calculators may not be shared, and communication between calculators is prohibited during the exam. Students may bring to the exam one or two (but no more than two) graphing calculators from the approved list.

Calculator memories will not be cleared. Students are allowed to bring calculators containing whatever programs they want.

Students must not use calculator memories to take test materials out of the room. Students should be warned that their grades will be invalidated if they attempt to remove test materials by any method.

Showing Work on the Free-Response Sections

Students are expected to show enough of their work for AP Exam Readers to follow their line of reasoning. To obtain full credit for the solution to a free-response problem, students must communicate their methods and conclusions clearly. Answers should show enough work so that the reasoning process can be followed throughout the solution. This is particularly important for assessing partial credit. Students may also be asked to use complete sentences to explain their methods or the reasonableness of their answers, or to interpret their results.

For results obtained using one of the four required calculator capabilities listed on page 13, students are required to write the setup (e.g., the equation being solved, or the derivative or definite integral being evaluated) that leads to the solution, along with the result produced by the calculator. For example, if the student is asked to find the area of a region, the student is expected to show a definite integral (i.e., the setup) and the answer. The student need not compute the antiderivative; the calculator may be used to calculate the value of the definite integral without further explanation. For solutions obtained using a calculator capability other than one of the four required ones, students must also show the mathematical steps necessary to produce their results; a calculator result alone is not sufficient. For example, if the student is asked to find a relative minimum value of a function, the student is expected to use calculus and show the mathematical steps that lead to the answer. It is not sufficient to graph the function or use a built-in minimum finder.

When a student is asked to justify an answer, the justification must include mathematical (noncalculator) reasons, not merely calculator results. Functions, graphs, tables, or other objects that are used in a justification should be clearly labeled.

A graphing calculator is a powerful tool for exploration, but students must be cautioned that exploration is not a mathematical solution. Exploration with a graphing calculator can lead a student toward an analytical solution, and after a solution is found, a graphing calculator can often be used to check the reasonableness of the solution.

As on previous AP Exams, if a calculation is given as a decimal approximation, it should be correct to three places after the decimal point unless otherwise indicated. Students should be cautioned against rounding values in intermediate steps before a final calculation is made. Students should also be aware that there are limitations inherent in graphing calculator technology; for example, answers obtained by tracing along a graph to find roots or points of intersection might not produce the required accuracy.

For more information on the instructions for the free-response sections, read the "Calculus FRQ Instruction Commentary" written by the AP Calculus Development Committee and the Chief Reader. It is available on the home pages for Calculus AB and Calculus BC at AP Central.

Beginning with the 2005 exams, sign charts by themselves are not accepted as a sufficient response when a free-response question requires a justification for the existence of either a local or an absolute extremum of a function at a particular point in its domain. For more detailed information on this policy, read the article "On the Role of Sign Charts in AP Calculus Exams for Justifying Local or Absolute

Extrema" written by the AP Calculus Development Committee chair and the Chief Reader. It is available on the home pages for Calculus AB and Calculus BC at AP Central.

The Major Concepts of Calculus

Technology has brought to the classroom new opportunities to engage students in the process of learning calculus. Teaching students to use technology is not an end in itself, but rather a means by which new investigations into calculus concepts are made feasible. In this section, we will look at a few ways to explore the "big ideas" of calculus. Although this is by no means a comprehensive listing of the important concepts of calculus, it should give you a start.

I. Limits. The idea of limit is one of the most elusive in calculus. Since it appears in so many different contexts, students need a solid understanding of this fundamental idea.

One of the first activities I use is an investigation of $f(x)=\frac{\sin x}{x}$. In particular, we look at the behavior of this function for values of x close to 0 in order to gather evidence about $\lim_{x\to 0}\frac{\sin x}{x}$. Start in a window centered at the origin and trace on the graph at (0, 0). Students can see that the function is undefined at $x = 0$. Then adjust the viewing window so that the inputs are closer to 0, (leave the vertical scale unchanged). You might start in a window of [−4.7, 4.7] for x and [−3.1, 3.1] for y, and then change the horizontal view to [−0.47, 0.47], thus making the function inputs closer to 0. You should see the graph flatten out, with the outputs close to 1. Repeat until the graph looks flat. Then investigate the behavior of $\frac{\sin x}{x}$ near 0 in a numeric representation by looking at a table of values. Start the table at 0, and make the step size of the table about 0.1, then 0.01, and then 0.0001. You can make the values of $\frac{\sin x}{x}$ arbitrarily close to 1 (which is the value of the limit) by making the inputs sufficiently close to (but not equal to) 0. Essentially, this describes the idea of limit.

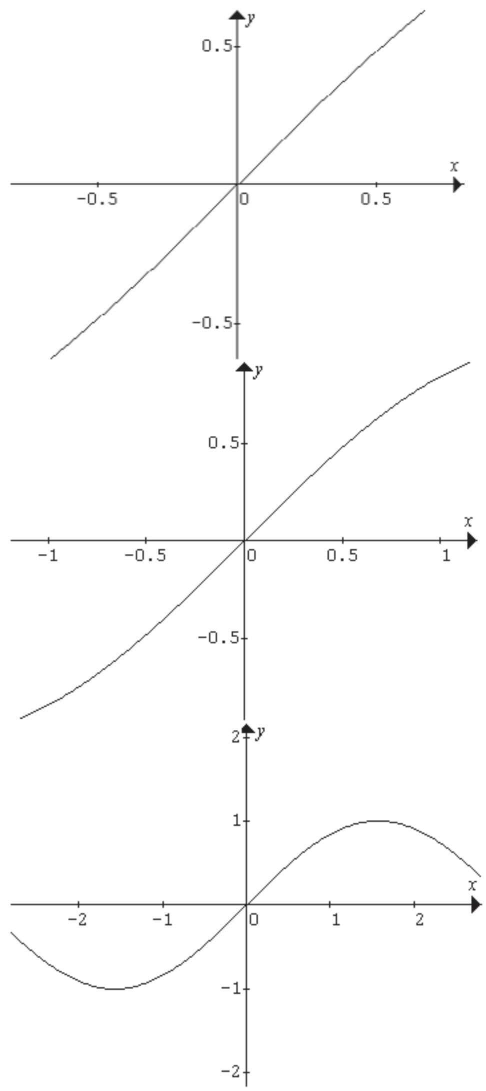
Sometimes students have the mistaken idea that any quotient where the numerator and denominator both approach 0 must have a limiting value of 1. To convince them this is not so, I show them the following limits:

$\lim_{x\to 0}\frac{ax}{x}$ for different values of a. In every case, the limit is a, yet numerator and denominator both approach 0.

$\lim_{x\to 0}\frac{x}{x^2}$. In this case, the limit doesn't exist, even though numerator and denominator both approach 0.

II. Definition of Derivative. The graphs of differentiable functions are "locally linear" when you look at them close up. The first time to discuss local linearity is when you introduce the derivative. It should come up again when you cover Riemann sums (in particular, the midpoint rule), slope fields, and, for BC students, Euler's method and L'Hospital's Rule.

One approach I've used is to show students a graph of $y=\sin x$ in a window centered at the origin, but "zoomed in" with equal scaling horizontally and vertically, to the point where the graph is indistinguishable from that of $y = x$. I then zoom out slowly (with zoom factors slightly greater than 1, to dramatize the effect) until the full graph of $y=\sin x$ is revealed.



We then discuss how to find the slope of $y = \sin x$ at $x = 0$ by using the line we had when it was "zoomed in." Students know to set up a difference quotient to find the slope of a line. So we go back to that zoomed-in graph, trace to a point (a, b) near $(0, 0)$, and evaluate the difference quotient, $\frac{b-0}{a-0}$. The result should be close to 1, which is the slope of the line tangent to the graph of $y = \sin x$ at $x = 0$. Zoom in a little closer, and repeat the calculation of the difference quotient. It shouldn't take long to convince students that the limiting value of the slope is indeed 1. We go on to define the derivative as the limit of a difference quotient, using both forms: $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ and $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$.

A few days later, I give every student in the class a different point to zoom in on as described above for a simple quadratic function like $y = \frac{x^2}{4}$. Each student zooms in centered on the point I assign and calculates and records the slope of the graph by forming a difference quotient. We then collect the slopes from everyone, making a table of the x-coordinate of the point and the corresponding slope. You probably won't need to encourage your students to propose a rule that tells slope as a function of x. This activity is a nice way to make the transition to the derivative as a function. We go on to define $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$.

III. The Integral. One application of the derivative that you will certainly cover is the idea that velocity is the instantaneous rate of change of position with respect to time. If you know position as a function of time, its derivative at a certain time tells you velocity at that time. The integral lets you reverse this process. Knowing velocity as a function of time, you use the integral to recover information about position.

One of my favorite calculus activities is the "car lab." I ask students to go for a drive with their parent or guardian as the driver of the car. The student records velocity readings from the speedometer every 30 seconds for 15 minutes. They also use the odometer to find their actual distance traveled, to the nearest tenth of a mile. They then use the Midpoint and Trapezoidal rules with the velocity readings to approximate the distance traveled. They hand in a lab report, where they respond to several questions.

You'll want students to draw left-hand, right-hand, and midpoint rectangles and calculate the sum of their areas early in your coverage of the integral. This should be done "by hand," to give students direct experience with the calculation of the sum of products (which is what every integral really is). Technology can be used with wonderful results to automate that process with large numbers of subintervals. Only then can students get a good feel for what it means for a Riemann sum to converge. A calculator program or online Java applet is a good way to accomplish this.

IV. Differential Equations. A differential equation is an equation with derivatives. The fact that the solution of a differential equation is a function, not a number, will be new to students and should be emphasized from the outset.

When I introduce differential equations to students, I start by referring back to finding the equation of a line given a point on the line (which is like the initial condition of a differential equation) and the slope of the line (here, given as a derivative). That is, this calculus problem:

"Solve the differential equation $\frac{dy}{dx}=2$, with $y(3)=-1$" is really the same as this algebra 1 problem:

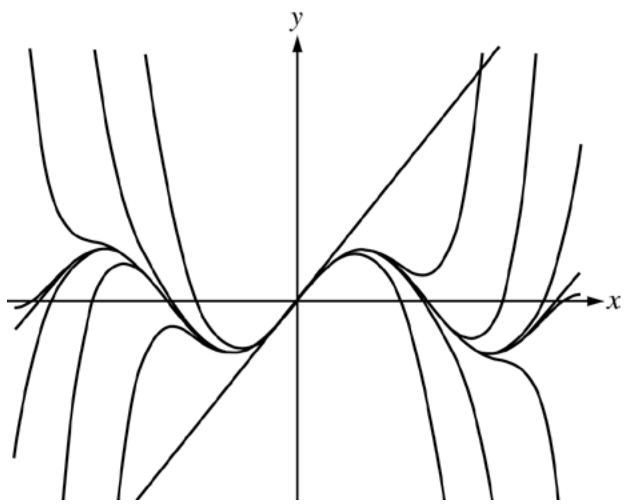
"Write an equation for the line passing through (3, −1) with slope 2"

The only difference is language and notation.

Solving differential equations offers a unique opportunity to exploit the multirepresentational approach. Graphically, the solution to a differential equation can be visualized from its slope field. Symbolically, differential equations in AP Calculus are solved using separation of variables. Though Euler's method is a Calculus BC-only topic, some AB teachers cover it precisely because it fills in the numerical representation of the solution of a differential equation. Solving the same differential equation by all three methods, and seeing the agreement among the three approaches, is a powerful pedagogic tool. The "skydiver" problem, which appeared on the 1997 AP Calculus Exams as free-response question AB 6/BC 6, offers a nice setting to investigate one problem using all three representations.

V. Series. Probably the most challenging topic covered in AP Calculus, series also offers rich opportunities for exploration. It is Calculus BC-only in its entirety, though AB students do touch on series. In fact, it's helpful for BC students to recognize that the tangent line is nothing more than the degree 1 Taylor polynomial for a function at the point of tangency.

Technology offers fantastic visualizations that allow students to witness first-hand the convergence of successively higher degree Taylor polynomials. Every Calculus BC student should create graphs like the ones below, which depict several Taylor polynomials for $y = \sin x$ at $x = 0$.



What the Philosophy Means for AP Calculus Exam Questions

The philosophy statement in the Course Description is more than window dressing. It guides the Development Committee during the creative process of writing questions for the AP Exams. If you look carefully at a couple of free-response questions from recent exams, you can see the manifestation of the course philosophy.

The following is free-response question AB 1/BC 1 from the 2004 AP Calculus Exams:

Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4\sin\left(\frac{t}{2}\right) \text{ for } 0 \le t \le 30,$$

where $F(t)$ is measured in cars per minute and t is measured in minutes.

(a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period?

(b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.

(c) What is the average value of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

(d) What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

This problem defines traffic flow as the rate at which cars pass through an intersection in cars per minute. For purposes of the following discussion, we'll call G the function that measures the number of cars that have passed through the intersection as a function of time t. In simple terms, since a derivative is a rate, students are given the derivative of $G(t)$. That is, in the problem, students are given $F(t) = G'(t)$.

In part (a), students are asked to determine how many cars pass through the intersection over the 30-minute period. This is a classic example of the "new" applications of the definite integral: The integral of a rate of change tells you net change. To solve it, students use the Fundamental Theorem of Calculus; that is, $\int_0^{30} G'(t)\,dt = G(30) - G(0)$. In words, the net change in the number of cars that have passed through the intersection between $t = 0$ and $t = 30$ (that is, $G(30) - G(0)$) is given by the integral of the rate of change of G from $t = 0$ to $t = 30$. Of course, since the problem gives $F(t) = G'(t)$, students must integrate the given function F from 0 to 30.

In part (b), students need to focus on the increasing/decreasing behavior for the given rate function F. The sign of $F'(7)$ determines if the rate is increasing or decreasing.

Parts (c) and (d) focus on the distinction between average value of a function and average rate of change of a function. The fact that both questions concern the time interval from $t = 10$ to $t = 15$ underlines this distinction. Part (c) is a "traditional" average value question, simply asking students to use the definite integral to find the average value of traffic flow, defined as F. The average rate of change asked for in part (d) is another way of asking for the slope of the line segment joining the ordered pairs $(10, F(10))$ and $(15, F(15))$ on the graph of F.

You can see how the rich context of this one question is used to ask students about a wide variety of calculus concepts. The setting reflects the commitment to a variety of applications.

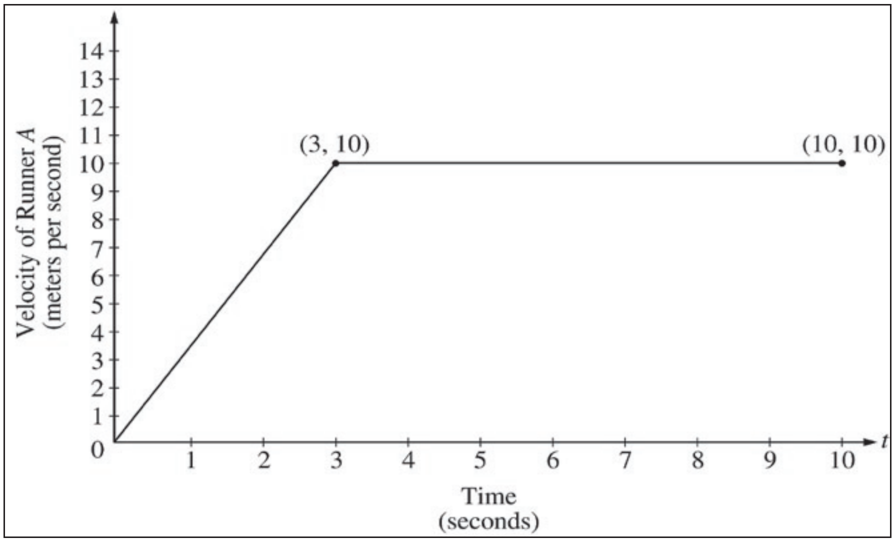
The question below illustrates the commitment to multiple representations of functions. Here, velocity functions for two runners are given—one graphically, the other with a formula. Students are asked to find the velocity of each runner at $t = 2$ in part (a) and the acceleration of each at the same time in part (b). In part (a), to get the velocity of Runner B is simply a matter of substituting $t = 2$ into the velocity function. For Runner A, students had several methods available but had to work with the graphical presentation in any case. Most students found an equation for the first line segment and then substituted $t = 2$.

In part (b), students had to find the derivative of each velocity function at $t = 2$. For Runner B, students could use their graphing calculator to evaluate the derivative. For Runner A, students needed the slope of the first line segment (which they may have already calculated when they found the equation of the segment).

Finally, part (c) asks for the total distance traveled by each runner from $t = 0$ to $t = 10$. For Runner A, this involved finding the area under the velocity graph by adding the areas of a triangle and a rectangle. For Runner B, students had to evaluate the integral of the given velocity function. The simplest way was to use the calculator.

Notice how this question emphasizes the connections between graphical and symbolic representations of the velocity function. The exact same calculus questions are asked for each runner, but the way you think about the answers depends on the representation of the function.

The following is free-response question AB 2/BC 2 from the 2000 AP Calculus Exams:



Two runners, A and B, run on a straight racetrack for $0 \le t \le 10$ seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner A. The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \dfrac{24t}{2t+3}$.

(a) Find the velocity of Runner A and the velocity of Runner B at time $t = 2$ seconds. Indicate units of measure.

(b) Find the acceleration of Runner A and the acceleration of Runner B at time $t = 2$ seconds. Indicate units of measure.

(c) Find the total distance run by Runner A and the total distance run by Runner B over the time interval $0 \le t \le 10$ seconds. Indicate units of measure.

Chapter 2
Advice for AP Calculus Teachers

Tips from Professionals in the Field

In this chapter, we offer some practical advice as you prepare to teach AP Calculus for the first time. You may find it helpful to look back over this chapter as a refresher after your first year teaching the course.

- **Stay true to the philosophy and goals.** Take time during the school year to reread the section in the Course Description pertaining to philosophy and goals. Make sure that what you are doing with students from day to day remains faithful to the philosophy and is geared toward achieving the goals. In particular, be attentive to pursuing a multirepresentational approach. When studying the derivative, for example, your students should zoom in to a graph of a differentiable function at a point and observe that graph "straightening out." The slope of that line you get is the value of the derivative of the function whose graph you are zooming in on. Students should also calculate difference quotients from data presented in a table. Past exam questions have asked that they do just that (see the 1998 AP Calculus Exam, free-response question AB 3, part (c); 2001 exam, question AB 2, part (a); and 2003 exam, question AB 3, part (a)). Students should evaluate difference quotients on their calculators using a small value for *h*. For example, calculating $\frac{\tan\left(\frac{\pi}{3}+0.001\right)-\tan\left(\frac{\pi}{3}\right)}{0.001}$ should result in a number very close to $4\left(\text{which is } \sec^2\left(\frac{\pi}{3}\right)\right)$. You should ask them to enter the function $y(x)=\frac{\sin(x+0.0001)-\sin(x)}{0.0001}$ and predict what its graph will look like before pressing the graph button on their calculator. Many students are somewhat baffled when the graph of $y=\cos x$ appears. Pointing out that $\frac{d}{dx}\sin x=\cos x\approx\frac{\sin(x+h)-\sin x}{h}$ for small values of *h*, such as .0001, removes the mystery.

- **Use conceptual touchstones.** So many ideas in calculus come up over and over again throughout the course. You can use this to your advantage in several ways. First, seeing an old concept in a new place is comforting to students. They'll see that calculus really is a cohesive body of knowledge. Second, seeing an old topic in a new setting offers an opportunity to review that topic in two different ways: as it was applied the first time around *and* how it applies to a new context.

  For example, as the integral is defined in terms of *limit*, remind students how limits were used to define the derivative. Limits are also used in defining continuity and in describing asymptotic behavior of functions.

The idea of *local linearity* comes up in many places in a calculus course. Refer to it when you introduce the derivative. As you zoom in on a point of a differentiable function with equal scaling horizontally and vertically, the graph straightens out, and the slope of the line that appears is the derivative of the function at the point where the zoom is centered. Local linearity comes up again when you cover the Trapezoidal and Midpoint rules for approximating integrals when you introduce slope fields and, in a Calculus BC class, with Euler's method and L'Hospital's Rule. In fact, there's a lovely argument justifying L'Hospital's Rule that relies on local linearity. It can be found in *Calculus— Single Variable* (Deborah Hughes-Hallett et al., 3rd ed.), section 3.10. Conceptual touchstones are a source of comfort and security to students and provide a convenient foundation upon which they can build their understanding.

I find myself repeatedly referring to simple ideas from algebra 1 in my calculus class. I reinterpret the slope of a line as the average rate of change of the dependent variable with respect to the independent variable. I point out to students that finding the equation of a tangent line is really an algebra 1 problem with calculus serving as a tool to find a needed quantity: the slope of the line. Calculus is less threatening to students when you use simple examples! Making liberal use of these opportunities allows students to believe they have a chance to succeed in a subject that is unquestionably challenging. It's important to reinforce that belief, particularly for students who are weaker in the subject.

- **Regularly emphasize the big concepts.** Some of the most important concepts in calculus are:
  - interpreting the derivative as instantaneous rate of change;
  - interpreting the definite integral as the accumulation of a rate of change (resulting in net change);
  - the role of limits at the heart of the big ideas (definition of derivative, definition of integral, definition of continuity); and
  - the relationship between the integral and the derivative as expressed in the Fundamental Theorem of Calculus.

  Students need to hear these interpretations time and again.

- **Stay in touch with exam trends.** AP Calculus is not a stagnant subject. Calculus does not change, but the way it is assessed does. Different kinds of questions come and go on the exam. Look at the last few years and try to sort out any patterns or trends. Certain kinds of questions appear regularly. For example, questions about functions defined by a definite integral, like $f(x)=\int_a^x g(t)\,dt$, have been common in recent years. Often, the graph of the integrand is given and, for example, questions are asked about maximum and minimum points on the graph of *f*, or about tangent lines to the graph of *f*. All of these are assessing two ideas at once: the Fundamental Theorem (recognizing that $f'(x)=g(x)$) along with interpretations and applications of the derivative. Area-volume free-response problems are typical on the Calculus AB Exam, and particle motion problems (involving position, velocity, and acceleration) are also common. Use them on your own tests and quizzes, as homework assignments, or for "in-class" group work. These exam questions are excellent, and you can often extend them. Don't wait until review time to use them—there are too many good ones to fit in.

- **Finding derivatives and antiderivatives is part of the course, but it's easy to overemphasize these mechanical skills.** Textbooks, particularly older ones, are brimming with exercises that require students to find derivatives and antiderivatives. In many ways, technology has freed us from the computational burdens of the past, and the current AP course reflects that. Students still need to be capable of finding basic derivatives and antiderivatives as described in the Course Description, but problems that are tricky or complex are rare on today's exams.

- **Teach students to evaluate integrals in a variety of ways.** Evaluating integrals using the Fundamental Theorem is part of the course, but so are evaluating integrals using a calculator, geometry, and Riemann sums. Students should understand that a definite integral is really just another way to write down a *number*. For example, just as 1 + 2 is an expression with the value 3, $\int_0^3 2x\,dx$ is an expression with the value 9. Students should have experience with evaluating such an integral in a variety of ways: calculating Riemann sums with more and more points in the partition, using the Fundamental Theorem, finding the area of a triangle with base 3 and altitude 6, and entering the integral into their calculator (using a numerical integration function, such as fnInt). In fact, seeing the same result appear through all the various methods is comforting!

- **Definite integrals and indefinite integrals are *very different*.** It's unfortunate in a way that we use the same symbol both for a definite integral and for an indefinite integral. Again, $\int_0^3 2x\,dx$ is a number; $\int 2x\,dx$ represents a *family of functions*. The derivative of every function in that family is $2x$. Emphasize this distinction with your students.

- **Review time is crucial.** Plan your course carefully so that you have at least three weeks to review before the AP Exams. For many students, this time will make all the difference. Encourage your students to work hard and believe in themselves. I tell them that I believe they can all get grades of 4 or 5 on the exam—and they can if they work hard at it. Notice how so many other veteran calculus teachers keep emphasizing the importance of review in the tips throughout this Teacher's Guide.

- **Emphasize communication.** Use written and oral reports from students to emphasize communication. Most likely your students will find this difficult at first, especially if they have not been asked to express mathematics in plain English until now. Persevere! Verbalizing concepts helps cement them, and more and more questions on the AP Exams require students to interpret and explain the meaning of a variety of expressions or numbers, or to justify their answers. See chapter 4 for a list of questions that do just that.

- **Use activities and group work to engage students in discovery lessons.** Using collaborative learning is a sound, time-tested teaching method. For example, when I'm making the transition from the derivative defined as slope at a point to the derivative as a function, I have each student zoom in on a different point on the graph of $y=\frac{x^2}{4}$ and find the slope at their point. We then collate the class results in a table and make a scatterplot of the resulting slopes. It soon becomes apparent that at any point the slope is given by $slope(x)=\frac{x}{2}$. Technology offers many opportunities for activities like this.

- **Use technology intelligently and with forethought.** Give tests that are partly closed-calculator when necessary. Don't ever ask a test question that gives some students an unfair advantage over others. For example, if you ask students to solve a variables separable differential equation like $\frac{dy}{dx}=xe^{-x^2}$ with $y(0) = 3$, and some students have a calculator equipped with a computer algebra system (CAS), such as the TI-89, but others do not, it's unfair. The students with the CAS have an advantage. Such questions should be asked in a "calculator-closed" setting.

  Students tend to place too much faith in calculator results. There is a collection of examples called "Lies My Calculator Told Me" on AP Central to shake their confidence a little in calculator results. To access the article, click on the link under "using Calculators" on the Calculus AB or Calculus BC home pages.

- **Engage your students' parents or guardians as allies.** The demands of any AP course are stressful for students. Consequently, teachers should try to develop an alliance with students' parents or guardians. Communication is a key component of that alliance. Provide parents with a copy of the course expectations and syllabus. Give students "homework calendars" with all the assignments and tests and quizzes scheduled during a specific time interval (probably no longer than one or two weeks, to allow you to make adjustments to the schedule). If possible, publish the homework calendar and course syllabus on a Web site so that parents can see what their daughters and sons are doing in your class.

  Consider involving parents in student lab activities. The "car lab" described in chapter 1 offers a natural opportunity to get students and parents to talk with each other about calculus.

- **Develop collaborative relationships with teachers of prerequisite courses.** Students' preparation for AP Calculus does not start with the idea of limit. Developing students' mathematical reasoning, and a willingness to think hard and deeply about tough concepts, is the responsibility of every mathematics teacher. The AP teacher can encourage the teachers of the prerequisite courses to:

  - explore mathematical concepts and connections using a multirepresentational approach to functions (graphical, numerical, algebraic, verbal);
  - make intelligent use of technology in their teaching;
  - encourage exploration by students;
  - develop students' verbal skills by requiring journal writing; and
  - use some of the language of calculus (such as describing "slope" as a "constant rate of change").

  Preparation for AP Calculus goes beyond simple mastery of a laundry list of facts and skills. The College Board's Pre-AP programs and the use of AP Vertical Teams can provide a formal mechanism to foster communication across grade levels. Go to AP Central for more details (see below).

- Visit AP Central regularly for updated information on the *AP Calculus Course Description* and exams; interesting articles; reviews of books and other media; information on workshops, conferences, and other professional development opportunities; the AP Calculus electronic discussion group (EDG); and other resources.

See the boxes in this chapter for other bits of advice from the classrooms in which many successful and seasoned calculus teachers have toiled over the years.

> One of the more valuable things you can do for yourself is to find a "buddy"—another teacher who is teaching calculus in a situation similar to your own. This person will be able to answer many of your questions and give some advice that will provide you with peace of mind as you make that journey through the first year of AP Calculus.
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> Attending a College Board weeklong workshop is valuable in many ways. It's a great opportunity to network. It's also a great opportunity to learn from other teachers. The workshop leaders share valuable teaching strategies and provide insights into how the AP Exam is scored. I have found that when I attend these workshops, I gain a new level of understanding and learn new problem-solving techniques.
>
> —Kathleen Goto, Iolani School,
> Honolulu, Hawaii

College Board Questions and Answers for New Teachers

- **How can I start to prepare during the spring and summer to teach my first course in the fall?**

  To prepare for teaching in the fall, you should review the Course Description, join the electronic discussion group (EDG), and review materials available on AP Central.

- **What are electronic discussion groups?**

  The AP EDGs are Web-based, threaded discussion groups that allow users to post messages online to be viewed by an entire community. Read more about the EDGs in the Professional Development section in chapter 5.

- **What training sessions should I attend as a beginning AP teacher?**

  Although the College Board has no official requirements for teaching an AP course, it is strongly recommended that you attend an AP Summer Institute the summer prior to teaching the course for the first time and then participate in one-day workshops and conferences available in your region throughout the academic year. For information geared to beginning teachers, look on AP Central for new teacher workshops that are offered in specific subject areas.

- **What is the difference between a summer institute and a workshop?**

  College Board workshops are developed and delivered by the College Board specifically for teachers, administrators, and counselors. They are taught by experienced College Board–approved instructors and available for every AP subject. College Board workshops are scheduled year-round in almost every state. See the Professional Development section at the end of chapter 5 for more information about these training opportunities.

- **What is the College Board Fellows Program?**

  The College Board AP Fellows program is a competitive grant program that provides stipends for secondary school teachers planning to teach AP courses in schools that serve minority or low-income students who have been traditionally underrepresented in AP courses. The stipends assist teachers with the cost of attending an AP Summer Institute. To qualify, a school must have 50 percent or more underrepresented minority students and/or be located in an area where the average income level is equivalent to, or below, the national annual average for a low-income family of four (approximately $36,000). Approximately 250 awards are distributed each year.

  The College Board Pre-AP Fellows Program is a competitive grant program that provides funding to AP Vertical Teams from schools in minority dominant and/or economically disadvantaged areas with few or no AP courses. The aim is to provide training in the following areas: English, mathematics, music theory, social studies, and studio art. Grants go toward funding the team's attendance at an endorsed Pre-AP Summer Institute. Grants of $10,000 each are awarded to AP Vertical Teams that best satisfy the eligibility requirements. Ten awards are available each grant year.

- **How can I apply for a stipend?**

  Application forms are available each September on AP Central. Hard copies can be obtained through your regional office or by e-mailing apequity@collegeboard.org.

- **Who pays for training if I am not eligible for the College Board Fellows Program?**

  Many school districts or school sites fund the professional development of their teachers. Check with your principal or district supervisor about available funding.

- **What resources are available to assist me as the school year progresses?**

  AP Central, the Calculus EDG, the *AP Calculus Teacher's Guide*, AP Released Exams, and the *AP Calculus Course Description* are all excellent resources available year-round.

- Before you leave for summer break, find out
  - o what students are registered for your AP Calculus class; and
  - o what the math course history is for each of them.
- If you can gather your class before the last day of school, send them off with cumulative review exercises that include
  - o algebra;
  - o functions and function notation; and
  - o trigonometry.
- Register at AP Central (apcentral.collegeboard.com).
- Register for and attend an AP Summer Institute, where you will, among other things,
  - o review the AP course in a collegial environment; and
  - o obtain helpful teacher resource materials.

—María Pérez Randle, Bishop Kenny High School,
Jacksonville, Florida

1. Know your calculus—if you have forgotten it, work through your textbook *before* you teach it.
2. Do all problems before you assign them—if not, you may assign problems that are entirely too complicated, or too long, or (heaven forbid) you may not be able to do them yourself.
3. Find a mentor—an experienced calculus teacher at your building or someone you can contact quickly with questions.
4. Do "reverse" planning—start with the date that you would like to be done with all the topics and work backwards to create a basic schedule organized by grading periods or months.
5. Don't be afraid to say "I don't know" to your students' questions, but then be sure to find the answer and come back the next day to complete the discussion.
6. Don't be afraid to let the students teach you some things, too. They will come up with some insights that had not occurred to you, which is a good thing. Calling a method or theorem "Jason's method" or "Jennifer's theorem" will help everyone remember it.
7. Enjoy teaching calculus and let it show. It will rub off on your students and they will enjoy calculus, too.
8. Communicate with other calculus teachers any way you can. Join the National Council of Teachers of Mathematics (NCTM), go to their meetings and read their monthly journal *Mathematics Teacher*, join the AP Calculus electronic discussion group, start a local calculus teachers' group.
9. Go to an AP workshop—a week-long Summer Institute is ideal, but if that is not possible, at least attend a workshop that is one or two days long.
10. Look at this opportunity to teach calculus as the best challenge in your professional career. You will work with the brightest and the best students in your school—and your life will never be the same!

—Martha Montgomery, Fremont Ross High School,
Fremont, Ohio

Chapter 3
Course Organization

Create Your Own Syllabus

You should start the process of translating the *AP Calculus Course Description* into a working syllabus by reading that publication carefully, if you have not already done so. Keep the Course Description close at hand while formulating your own syllabus. What follows are some general guidelines to help you.

Prerequisites

You'll get some help here from the Course Description, which includes a list of prerequisite skills needed by AP Calculus students (see page 5). Note that BC students should have a working knowledge of vectors before attempting the course. Every AP Calculus student should also have covered, in some form, these four high school mathematics classes: algebra 1, geometry, algebra 2, and precalculus (elementary functions/trigonometry). Here are some common strategies for students to ensure such coverage before a course in AP Calculus:

- begin algebra in grade 8;
- double up on early courses (optimally algebra 1 and geometry, although algebra 2 and geometry is another option);
- attend summer school; and
- telescope four courses into three years by reorganizing topics.

The second prerequisite is for students to be motivated to take on college-level work in high school. AP Calculus should not be seen as a "warm-up calculus course" that will make the college course easier. This sort of motivation compromises both the high school course and the college course. Students should be studying calculus to learn calculus, and once they've learned it, they should move on. That premise is the foundation of an AP course.

The College Board's Equity and Access policy is discussed in detail at the beginning of this Teacher's Guide. As is true for all AP classes, students taking AP Calculus should reflect the diversity of a school's overall population. The College Board discourages schools from screening students based on test scores or grades achieved in prerequisite courses for calculus.

Demonstration of strong motivation and commitment to excellence by a student should be given considerable weight in making a decision to admit the student to an AP class. The challenges of a rigorous syllabus can inspire students to achieve at a high level. For evidence in support of this position, one need look no farther than the experiences of Jaime Escalante's students in Los Angeles as documented in the film *Stand and Deliver*.

Schools might also consider a more proactive policy to increase enrollment in AP courses by students with diverse backgrounds. The goal of breaking down whatever barriers might exist to broad participation in the AP Program is both worthy and achievable (College Board studies demonstrate that one of the best indicators of success in college courses is the completion of an AP course in high school). Developing strategies to that end deserves special attention nationwide.

> Some schools have placement tests [for students wishing to take AP courses], some make decisions based on the grades students received in prerequisite courses, and others favor an open admissions policy based on passing a prerequisite course. The rationale for such an open admissions policy is that many students can rise to the challenge of a demanding college-level AP course in high school. AP courses need not be just for the smartest and most able students in the school. In marathon races, there is only one winner, but most of the runners finish the race and get as much out of it as the winner does. AP Calculus courses can be similar. The teacher, by following the *AP Calculus Course Description*, sets the pace and can often get remarkable performances from students who hadn't been effectively challenged in previous mathematics courses.
>
> — John Mahoney, Benjamin Banneker Academic High School, Washington, D.C.

Course Overview

Here it is important to communicate your high expectations to your students. It is easy to justify those high expectations: AP Calculus is a college-level course, with a demanding topic outline. Students need to be dedicated and conscientious in order to succeed in such a challenging course. Prepare them from the outset to work hard so that there will be no surprises. I tell my students to expect a *minimum* of 30–40 minutes of homework *every night*, including the night after a test.

You could borrow here from the philosophy and goals of AP Calculus found in the current Course Description.

Course Planner

Thinking about the course in terms of covering a few major categories of topics can make the task seem more manageable. These major categories in Calculus AB are:

- limits and continuity
- the idea and definition of the derivative, calculating derivatives, derivative "theory"
- applications of the derivative
- the idea and definition of the integral, calculating integrals, integral "theory"
- applications of the integral
- differential equations

In addition to a few minor topics like integration by parts, partial fractions, the logistic model, Euler's method, and L'Hospital's Rule, Calculus BC teachers must include the following material:

- parametric, polar, and vector functions
- series

Although each teacher's situation is different, for a traditional schedule most AB teachers hope to finish with differential calculus at the halfway point in their schedule. Whatever you decide, the most important thing to keep in mind when mapping out a timetable for your class is the critical need for ample review time prior to the AP Exams in May.

Obviously, for a BC class the schedule needs to be somewhat accelerated, since a month or more should be dedicated to coverage of series. Additional BC-only topics also require more time (although none as much as series). My own rule of thumb for BC is to have the integral defined and the Fundamental Theorem covered at about the halfway point.

> Since I know there will be an ongoing review throughout the year, I do not necessarily stay with a topic until the students have mastered it. I tend not to be concerned about their lack of mastery and move on after I feel the topic has been sufficiently covered and the students have had some practice. There will be plenty of opportunities for the students to receive comments tailored to their particular needs when I grade their free-response questions.
>
> —Steve Olson, Hingham High School,
> Hingham, Massachusetts

Teaching Strategies

Students learn best by doing problems and working activities. Incorporate these strategies into your planning.

There is an enormous supply of past AP Exam questions, both multiple-choice and free-response. Give these to students regularly for homework, group work in class, quizzes, and tests. Look carefully at these questions before you use them. You will find that there are qualitative differences in questions from more recent exams. The *AP Calculus Course Description* underwent a major change in 1998, and exams since then reflect those changes. Nonetheless, there are still excellent questions from earlier exams that you can use. All the free-response questions since 1998, with solutions and scoring guidelines, are available for free download at AP Central. Questions and solutions from exams dating back to 1969 can be purchased through AP Central. This resource is a gold mine!

You will also want to use calculator-based activities in your class. This will serve three main purposes. First, students need to be adept users of technology, minimally competent in the four required calculator uses on the exam, which are enumerated in the Course Description:

- Plot the graph of a function within an arbitrary viewing window
- Find the zeros of functions (solve equations numerically)
- Numerically calculate the derivative of a function
- Numerically calculate the value of a definite integral

Calculator-based activities should make students more proficient and confident users of technology.

Second, calculator activities can help promote an atmosphere of exploration and discovery. Students tend to take ownership of the mathematical objects they work with on a handheld grapher.

Third, such activities encourage students to write about their understanding and interpretation of mathematical results. Writing is increasingly becoming a part of students' work on the AP Exams themselves. For many students, this type of writing experience is unusual in a math class. Encourage them to strive for clarity. For example, it is in this context that I tell my students to avoid the use of pronouns in their writing. Saying, "The function is increasing for the values of x where the derivative is greater than zero" is obviously better than saying, "It's increasing where it's positive." Moreover, such clarity of expression is expected of students on the AP Exams.

The four required calculator capabilities listed above should be seen as a set of minimal competencies. I make a point to cover a little more with my own students. For example, I want to be sure they can define and graph a function using the built-in numerical derivative tool in their calculator, and find zeros both on the graph screen and using the calculator's built-in solver. I also expect them to be able to store a value they have found (often a zero) into a built-in calculator variable and later retrieve that value for use in a subsequent calculation. For instance, they may need to solve for a zero of a function that ends up being used as a limit of integration. It's better to store that value away in a variable and then use that variable in calculating the integral.

Students should also be comfortable using the calculator to store functions using the "Y= menu." For example, a typical free-response question about area or volume may require the student to evaluate definite integrals using the calculator. By storing functions in the calculator as Y1(X) or Y2(X), students only need to enter the functions carefully one time. They do not need to reenter them each time that they have to use numerical integration on the calculator. This is particularly helpful when the integrand involves the sum or difference of two functions. This saves the students time and decreases the chances of making entry mistakes (particularly with parentheses, exponents, or radicals) under the pressure of the exam.

> Embedding AP Exam questions into chapter tests gave these tests a spiraling character that they had always lacked. The added comprehensiveness was a challenge to most of my students, and many of them found themselves moving their study approach away from last-minute cramming of only the current chapter. This may have been the most important byproduct of incorporating AP Exam questions into chapter tests. This inclusion also forced me to have separate calculator and noncalculator sections on many of my chapter tests.
>
> —Michael Grasse, Elk Grove High School,
> Elk Grove Village, Illinois
>
> (Michael notes that since typical AP Exam questions cover several topics, he has to wait until after he's covered applications of the derivative before his approach can work.)

Evaluation

To make use of challenging questions on your tests and quizzes, you should consider a grading policy that does not rely on a strict percentage-based system, such as 90–100 = A, 80–89 = B. Earning even 80 percent of the available points on the AP Exam is just about a sure-fire guarantee of getting the highest grade of 5. So, if you are going to make use of questions from past exams, and perhaps some of your own challenging questions, you should not be expecting students to get 90 percent of the points in order to get an A. In my own class, I'm usually pleased if the median score on a test is 75 percent or so, and the median usually falls in the middle of the B's. Occasionally, though, the median score may fall to 50 percent or below. Whatever grading scale you choose, it should be fair, communicated to the students, and flexible enough that you can safely ask tough questions without compromising your stated standards.

Writing Component

Consider making journal writing a part of the course expectations. If you look closely at AP Exam questions since 1998, you'll see many instances in which students are asked to interpret a result, explain the meaning of some value in a particular context, or give a reason for a particular answer. Students need guidance and practice in writing about mathematics, so I have my students keep a calculus journal. I require them to write about their understanding of concepts and to respond to questions I pose in class or in homework assignments. A sample of journal questions appears in appendix 1.

- Consider the following when you develop a course timeline:
  - If you were not able to gather your class before summer vacation, you may have to include a review week on algebra, functions, function notation, and trigonometry.
  - Teacher resource books of textbooks often have timelines—refer to these.
  - Include quiz and test dates.
  - Include writing assignments.
- During preplanning:
  - Speak with your department head or administration officer to determine when you are to be scheduled for help/review sessions (mornings or afternoons).
  - Get together with your school's media specialist, who will help you set aside math books for your calculus students at a reserve bookshelf in the library. These reserved books should include:
    - A math history book or encyclopedia; and
    - Readable calculus books such as
      - Dick, Thomas P., and Charles M. Patton. *Calculus of a Single Variable*. Boston: PWS Publishing Company, 1994.
      - Anton, Howard, Irl Bivens, and Stephen Davis. *Calculus: Early Transcendentals.* 7th ed. New York: John Wiley & Sons, 1999.
      - Hughes-Hallet, Deborah, et al. *Calculus—Single Variable.* 3rd ed. New York: John Wiley & Sons, 2002. (Ed. note: The 4th edition was published in 2005.)
- On the first day of class:
  - Distribute the course timeline to your students so the AP Calculus dates are first to make it onto their calendars—these dates get filled quickly with college visitations and family vacations.
  - Give your students the directions for the AP Exam they are taking the following spring. You want your students to know what will be expected with their answers. With this in mind, the students will be practicing all year on justifications and verifications.
  - Provide your students with the times you are available for help/review sessions.

—María Pérez Randle, Bishop Kenny High School,
Jacksonville, Florida

Six Sample Syllabi

The following syllabi include four from secondary school teachers of AP Calculus and two from college instructors of calculus courses. Be sure to read them all, even those that you don't think apply to your situation, because they all include teaching tips, resources, and suggestions for student activities that you may be able to use in your own classroom.

Important note: The AP Course Audit

The syllabi included in this Teachers Guide were developed prior to the initiation of the AP Course Audit and the identification of the current AP Calculus Curricular Requirements. These syllabi contain rich resources and will be useful in generating ideas for your AP course. In addition to providing detailed course planners, the syllabi contain descriptions of classroom activities and assignments, along with helpful teaching strategies. However, they should not necessarily be used in their entirety as models that would be authorized under the guidelines of the AP Course Audit. To view the current AP Curricular Requirements and examples of syllabi that have been developed since the launch of the AP Course Audit and therefore meet all of the AP Calculus Curricular Requirements, please see AP Central.

http://apcentral.collegeboard.com/courseaudit/resources

Syllabus 1 (Calculus AB)

Ann Davidian
General Douglas MacArthur High School
Levittown, New York

School Profile

School Location and Environment: General Douglas MacArthur High School is one of two high schools in the Levittown School District, a primarily residential community in central Nassau County on Long Island, approximately 10 miles east of New York City.

Grades: 9–12

Type: Public high school

Total Enrollment: Approximately 1,300 students

Ethnic Diversity: 4.8 percent Hispanic/Latino; 4 percent Asian or Pacific Islander; 0.8 percent African American

College Record: Approximately 54 percent attend four-year colleges, and 35 percent attend two-year colleges.

Personal Philosophy

Teaching AP Calculus is one of the greatest joys of my career. I have the opportunity to work with the best mathematics students and lead them to discover the beauty of calculus. As one of my students once said, “Everyone should take a calculus course. It uses all of the math that you ever learned and puts it all together.” Calculus gives students the opportunity to utilize their knowledge of algebra, geometry, and trigonometry in one course. The “Rule of Four” (representing functions verbally, graphically, algebraically, and numerically) is an essential component of our course.

I believe that students learn best when they are involved and active in the lesson. As Eric Butterworth said, “I tell you and you forget. I show you and you remember. I involve you and you understand.” Thus, I try to teach calculus using exploration and discovery as often as possible. Today’s technology opens up a new world of exploration for students. I believe that, with proper guidance, students can discover many of the important concepts of calculus. We work as a team to explore and master these concepts.

Class Profile

We generally run one section of AP Calculus AB each year. (We also run a section of AP Calculus BC.) Class periods are 47 minutes, and we meet five days a week for one period.

Course Overview

My main objective in teaching AP Calculus is to enable students to appreciate the beauty of calculus and receive a strong foundation that will give them the tools to succeed in future mathematics courses. Students know that they will work harder than ever, and our expectation is that this hard work will enable them to succeed in the course. We work together to help students discover the joys of calculus.

Course Planner

The primary textbook I use is Deborah Hughes-Hallett's *Calculus—Single Variable*, 3rd ed. (Numerous ancillary materials are available, including an instructor's manual, solutions manual, test bank, ConcepTests, and a teacher's guide. There is also an instructor's resource CD-ROM.)

School starts after Labor Day, so we have about 150 school days before the AP Exam. The following breakdown presents the approximate time we allow for each chapter of our text. There is some degree of flexibility in the timing, and labs are interspersed throughout the chapters, which adds to the time allotted for each chapter. I try to finish the material by the middle of April to allow time for review.

A Library of Functions (Chapter 1)

Students complete this review of precalculus materials over the summer.

Key Concept: The Derivative (Chapter 2)

3 weeks

1. How Do We Measure Speed?
2. Limits
3. The Derivative at a Point
4. The Derivative Function
5. Interpretations of the Derivative
6. The Second Derivative
7. Continuity and Differentiability

CBL Ball Toss Lab (see Student Activity 1, below)

Graphing the Derivative of a Function (see Student Activity 2)

Shortcuts to Differentiation (Chapter 3)

5 weeks

1. Powers and Polynomials
2. The Exponential Function
3. The Product and Quotient Rules

4. The Chain Rule

5. The Trigonometric Functions

6. Applications of the Chain Rule and Related Rates

7. Implicit Functions

8. Linear Approximation and the Derivative

9. Using Local Linearity to Find Limits

Calculus in the Year 2000: New Ways of Teaching the Derivative and the Definite Integral, a three-hour video by Steve Olson (available at the College Board Store). Steve Olson gives a wonderful explanation on finding the derivative of an exponential function.

"Related Rates" from *Calculus in Motion* (see Teacher Resources, below): Five classic related rates problems are presented in an interactive format.

Tootsie Roll Pops Lab (see Student Activity 3)

"Investigating the Accuracy of the Tangent Line Approximation" (from the ancillary materials that accompany *Calculus—Single Variable*)

Using the Derivative (Chapter 4)
4 weeks

1. Using First and Second Derivatives

2. Families of Curves

3. Optimization and Modeling

4. Theorems About Continuous and Differentiable Functions

Optimization Project (see Student Activity 4)

Mathematical Association of America Calculus Films video, *Theorem of the Mean Policeman*

Key Concept: The Definite Integral (Chapter 5)
3 weeks

1. How Do We Measure Distance Traveled?

2. The Definite Integral

3. Interpretations of the Definite Integral

4. Theorems About Definite Integrals

Using Calculus to Determine Distance Driven (see Student Activity 5)

Midterm Exam

Constructing Antiderivatives (Chapter 6)
3 weeks

1. Antiderivatives Graphically and Numerically
2. Constructing Antiderivatives Analytically
3. Differential Equations
4. Second Fundamental Theorem of Calculus
5. The Equations of Motion

Graphing the Antiderivative of a Function (similar to the activity on Graphing the Derivative of a Function from chapter 2)

Integration (Chapter 7)
2 weeks

Integration by Substitution

1. Approximating Definite Integrals

Exploring Approximation Techniques Using a Calculator Program: I use a program written by one of my students. Similar programs are available on the Web, or see Sam Gough et al., *Work Smarter Not Harder—Calculus Labs for TI-82 and TI-83*.

Differential Equations (Chapter 11)
4 weeks

1. What Is a Differential Equation?
2. Slope Fields
3. Separation of Variables
4. Growth and Decay
5. Applications and Modeling

Drawing Slope Fields (see Student Activity 6)

Using the Definite Integral (Chapter 8)
3 weeks

1. Areas and Volumes
2. Applications to Geometry

Play-Doh Lab (see Student Activity 7)

Winplot Program (see Student Activity 7)

Volumes of Solids of Revolution—The Disk Method and Exploring Volume by Cross Sections (see Student Activity 7)

Teaching Strategies

I feel that the most important aspect of my calculus course is that students come to class each day prepared to work. To demonstrate this commitment to hard work, students are required to complete a summer assignment before entering my class. Since the first chapter of our calculus text is a review of our precalculus text, students are able to work through this chapter on their own. I give them a packet to complete and submit to me in August. (I am available for extra help and to answer questions.) This enables me to grade the assignment before classes begin and drop students who have not completed the assignment. (Students and parents sign a contract stating that the summer assignment will be completed by the due date, or students will be dropped from the course.) Two weeks after the start of the school year, I test students on chapter 1.

On the first day of school, I begin with chapter 2 (Key Concept: The Derivative). This sets the tone for the year. As discussed in the Student Activities section below, I start with a Calculator-Based Laboratory (CBL) ball toss experiment. Students explore the concept of an average rate of change and discover the concept of instantaneous rate of change. Thereafter, student exploration and discovery continue to be an important aspect of the remaining topics.

Throughout the course, students work together on a regular basis, both formally and informally. At times, I set up groups to work on a particular activity, but students do not need to be told to work together. Our classroom has tables instead of desks to make it more conducive to group work. When students are working on a problem, they will often work alone initially but then turn to their partners to collaborate on their mathematics.

In discovering new concepts, the class works as a whole. It is not necessary for students to raise their hands. If they have a thought to share, they are welcome to make a contribution. If they are so inclined, students will go up to the board to illustrate a point. At times, I am able to step back and just listen to the interaction among my students as they explore a topic.

Technology can be used to help make calculus concepts come alive. Students are issued TI-89 calculators. Our classroom contains 10 computers and a SMART Board. We use *TI-Navigator*, *TI InterActive!*, and CBLs. Technology enables students to "see" what is being discussed. Topics are presented using the "Rule of Four"—graphically, numerically, algebraically, and verbally. Through this multifaceted approach, students gain an in-depth understanding of the material.

Our textbook and its ancillaries have a fabulous collection of true-false and multiple-choice questions that probe students' understanding of some of the finer points of calculus. There are also several books referenced below that contain wonderful multiple-choice questions. Starting in September, students are given several true-false or multiple-choice questions each evening for homework. All of these assignments include questions on topics previously covered. Through the use of *TI-Navigator*, we can determine where students are having difficulty and then focus on those topics.

Each evening, students are also assigned in-depth questions that are discussed in class the following day. Some of these questions are on the new material; others provide students with a review of concepts previously learned. These questions also come from the books discussed above as well as the resources listed below.

Lab Component

We do not have a formal lab component in our calculus class. However, I do devote class time to lab experiments. Except for two of the labs that are performed completely at home, all labs are conducted in school during a class period and then finished at home. I strive to provide students with as much hands-on work as possible. Students work together in groups of two. They submit one report for each group and receive a group grade on the report.

Several wonderful resources for calculus labs are *A Watched Cup Never Cools: Lab Activities for Calculus and Precalculus* by Ellen Kamischke; *Work Smarter Not Harder—Calculus Labs for TI-82 and TI-83* by Sam Gough et al.; and the *AP Teacher's Guide to Accompany* "Calculus—Single Variable" (Deborah Hughes-Hallett et al., 3rd ed.) by Benita Albert et al. (see Teacher Resources below).

Student Evaluation

I rarely give full-period tests in my calculus class. Since our periods are only 47 minutes long, I hate to give up a full class period for a test. As a result, I quiz students more often and also have students work on in-class labs, take-home assignments, and projects. At the end of the quarter, I divide the sum of the points the student earned by the total number of points available for the quarter.

Most of my quizzes are cumulative, providing students with a constant review of previous topics. During the course of the year, I give several noncalculator tests to help prepare students for the noncalculator section of the AP Exam. The instructor's manual of our textbook has some excellent "gateway tests" that I use to test students' skill in computing derivatives and antiderivatives. One section of our midterm is also noncalculator.

Starting in October, I assign six AP free-response questions for students to work on for two weeks. Students can work on these questions with one other person, and they can come to me for extra help. At the end of the two weeks, I randomly select one of these questions for a quiz. Students are graded as they would be graded on an AP Exam. (Free-response questions and scoring guidelines are available on AP Central.)

When the second semester begins in February, we work on multiple-choice sections of AP Calculus Released Exams. Students have two weeks to complete a multiple-choice section. They can work on these questions with one other student and come to me for extra help. At the end of the two weeks, students hand in their work for grading. They are also quizzed on selected questions from the work they submit. The multiple-choice work alternates with the free-response work until the AP Exam administration.

In March, students take the multiple-choice section of the 1997 AP Exam under testing conditions by using the APCD® (see Teacher Resources). The CD has a timer so that students can learn to pace themselves. It grades the students and identifies areas that need improvement. The CD is interactive, enabling students to see where their errors were on individual questions. In April, students take the multiple-choice section of the 1998 exam using the APCD. This enables students to see where they have improved and what areas still need improvement.

Teacher Resources

Primary Textbook

Hughes-Hallett, Deborah, et al. *Calculus—Single Variable.* 3rd ed. New York: Wiley & Sons, 2002. (Ed. note: The 4th edition was published in 2005.)

References

- Adams, Colin, Joel Hass, and Abigail Thompson. *How to Ace Calculus: The Streetwise Guide*. New York: W.H. Freeman and Company, 1998.

- Albert, Benita, et al. *AP Teacher's Guide to Accompany* "Calculus—Single Variable" (Hughes-Hallett et al., 3rd ed.). New York: John Wiley & Sons, 2002.

- Best, George, and J. Richard Lux. *Preparing for the AP Calculus (AB) Exam*. Andover, Mass.: Venture Publishing, 2004.

- Foerster, Paul A. *Calculus Explorations.* Emeryville, Calif.: Key Curriculum Press, 1997.

- Kamischke, Ellen. *A Watched Cup Never Cools: Lab Activities for Calculus and Precalculus.* Emeryville, Calif.: Key Curriculum Press, 1999.

- *Multiple-Choice and Free-Response Questions in Preparation for the AP Calculus (AB) Examination*. 8th ed. Brooklyn, N.Y.: D&S Marketing Systems, n.d.

- Lifshitz, Maxine. *Amsco's AP Calculus AB/BC: Preparing for the Advanced Placement Exams.* New York: Amsco School Publications, 2004.

- MAA Notes. *Resources for Calculus Collection*. Washington, D.C.: MAA, 1993.

  Volume 1: *Learning by Discovery: A Lab Manual for Calculus*, edited by Anita E. Solow. MAA Notes Number 27.

  Volume 2: *Calculus Problems for a New Century*, edited by Robert Fraga. MAA Notes Number 28.

  Volume 3: *Applications of Calculus*, edited by Philip Straffin. MAA Notes Number 29.

  Volume 4: *Problems for Student Investigation,* edited by Michael B. Jackson and John R. Ramsay. MAA Notes Number 30.

  Volume 5: *Readings for Calculus*, edited by Underwood Dudley. MAA Notes Number 31.

- Pilzer, Scott, et al. *ConcepTests to Accompany* "Calculus—Single Variable" (Hughes-Hallett et al., 3rd ed.). New York: John Wiley & Sons, 2003.

Technology

- Alvirne High School, Hudson, N.H. AP Calculus Web site. homepages.seresc.net/%7Esray/alvirne.html.
  This is a rich resource for AP Calculus teachers and students.

- Gough, Sam, et al. *Work Smarter Not Harder—Calculus Labs for the TI-82 and TI-83* (also available for the TI-83 Plus) and *Work Smarter Not Harder—Calculus Labs for the TI-89*. Andover, Mass.: Venture Publishing, n.d.
  www.vent-pub.com
  Each book contains a disk with wonderful calculator programs.

- Husch, Lawrence S. Visual Calculus Web site. Mathematics Department, University of Tennessee, Knoxville.
  archives.math.utk.edu/visual.calculus/
  Visit this site for a collection of excellent modules that can be used by teachers and students.

- Parris, Richard. *Winplot*. Phillips Exeter Academy.
  math.exeter.edu/rparris/winplot.html
  This is a free general-purpose plotting utility to draw and animate curves and surfaces. It is fabulous for enabling students to see and manipulate solids of revolution.

- Texas Instruments Inc.
  education.ti.com
  Any of the links on this site will lead you to wonderful calculus activities and resources.

- Texas Instruments Inc. *TI InterActive!* CD-ROM.
  The program contains a word processor with an integrated mathematics system, TI graphing calculator functionality, and an integrated Web browser.

- The College Board's AP Central Web site
  apcentral.collegeboard.com
  This site contains everything you need to know about AP Calculus. It contains a wealth of information.

- College Board AP Calculus Electronic Discussion Group
  Go to the AP Central Home Page and click on the link to Electronic Discussion Groups.
  I highly recommend joining this EDG. It is an amazing source of teaching ideas and strategies, many of which I have incorporated into my course.

- College Board. *APCD: Calculus AB.* College Entrance Examination Board, 2000.
  This interactive CD-ROM includes the 1997 and 1998 Released Exams, which can be taken as either a review or a test simulation. The CD also contains study-skill suggestions and test-taking strategies. For sale at store.collegeboard.com.

- Weeks, Audrey. *Calculus in Motion.* CD-ROM. Burbank, Calif.: Calculus in Motion, 2005.
  This CD has great calculus animations for *Geometer's Sketchpad* v4. www.calculusinmotion.com

Videos

- MAA Calculus Films in VHS Format (3-tape series). Mathematical Association of America. These films are no longer sold by MAA, but many calculus teachers have copies.

- *Calculus in the Year 2000: New Ways of Teaching the Derivative and the Definite Integral*, VHS, 3 hours, by Steve Olson. For sale at store.collegeboard.com.

Professional Development

Before I taught AP Calculus for the first time, I attended a weeklong AP Summer Institute. I have attended several other Summer Institutes since then. The information I have received has been invaluable. I would advise anyone teaching AP Calculus to attend such a workshop.

Student Activities

1. On the first day of school, we begin the study of derivatives with a CBL experiment. Students toss a ball in the air and examine the height versus time graph generated. They fit the data with a quadratic equation for the position function to determine how high the ball went and how long it was in the air. Students compute the average velocity over a time interval. They are then asked to determine the velocity of the ball at exactly 0.06 seconds after the ball was tossed and explain how their answer was obtained. Finally, they zoom in on the graph of the position function near $t = 0.06$ until the graph looks like a line. Students find the slope of the line and compare it to their estimation of the instantaneous velocity at $t = 0.06$.

2. When students are first learning about derivatives, I sketch the graph of an unknown function on the board. Each student comes up to the board and plots a point that would lie on the graph of the derivative of the function. We watch as the graph of the derivative unfolds. For example, I often graph a sine function on the board. At the maximum value of this function, the derivative would be zero. Students are anxious to plot these points. At the points of inflection, the derivative would be at a maximum or minimum. These are also easy points to plot. Gradually, students fill in the remaining values, and we can see that the derivative of a sine function is a cosine function.

3. During our study of related rates, students suck on Tootsie Roll Pops to determine the rate of change of their radius; then they calculate the rate of change of the Pops' volume. Students measure the initial radius of a Pop with dental floss. They then suck on the Pop for 30 seconds, record its radius, suck for another 30 seconds, etc. They model the rate of change of the radius with some function of time. Students then use this rate of change to estimate the rate of change of the volume of the Pop when its radius is three-fourths of its original radius. This lab, "How Many Licks?" can be found in Ellen Kamischke's book, *A Watched Cup Never Cools*.

4. The study of optimization can be made more meaningful to students by asking them to design an optimum can. Students obtain a can of soda, soup, tuna, etc. They measure the height and diameter of the can and determine its volume. Then they find the radius and height of the most cost-effective can that will hold the same volume and write an explanation of the mathematics involved in making their determination. Students then construct the most cost-effective can, bring their original can and constructed can to school, and make a presentation to the class. (I obtained the idea for this activity, as well as the next one, from Christine Healy, a teacher at Bethpage High School in Bethpage, New York.)

5. After learning how to approximate a definite integral, students use these techniques to calculate the distance covered during a 20-minute drive with a friend or parent. Before beginning the drive, students record the car's odometer reading. Using the speedometer, they record the car's speed at one-minute intervals, noting any traffic conditions. At the end of the drive, they check the odometer reading again. Students then graph speed versus time and use integration techniques to approximate the distance traveled over the 20-minute interval. They compare this distance with the actual mileage determined by the odometer. Students are often amazed at the closeness of their approximation to the odometer reading.

6. As an introduction to slope fields, I use an activity from the *AP Teacher's Guide to Accompany* "Calculus—Single Variable." Using the graphing calculator screen with the grid turned on, I project a 3 × 3 grid onto the board and assign each student several coordinate points in the region ((1,1), (1,2), etc.).

   For a given differential equation, each student computes the slope at his or her coordinate position and then goes to the board to draw a short line segment with the calculated slope and the coordinate point as the midpoint of the segment. For example, if $dy/dx = y$, the student with coordinates (1,1) would go to the board and at the point (1,1) draw a little line segment with a slope of 1. The student with coordinates (1,2) would go to the board and at the point (1,2) draw a little line segment with a slope of 2. (It is important that the second student draw a line segment whose slope is steeper than the slope of the first student's line segment.) Continuing in this fashion, the class would complete the slope field. At this point, all sorts of discussions can ensue.

7. One of the most difficult concepts for me as a student of calculus was finding the volume of solids. If students cannot visualize the solids, they have a more difficult time understanding how to compute the volume. To enable students to "see" the solids of revolution, I have purchased several "open-up" party decorations from a party supply store. Instead of just discussing how a line revolving around an axis forms a cone, students see the cone generated. (I found an open-up ice cream cone in one of the party stores.)

   I bring cans of Play-Doh into school and ask students to construct solids whose bases are bounded by two curves and whose cross sections are squares or equilateral triangles, etc. For example, students are given the graph of a circle and asked to construct a solid such that each cross section perpendicular to the base is an equilateral triangle. Students build the solids using Play-Doh and then use plastic knives or dental floss to cut through the solid and obtain the required cross sections.

   Students also use the *Winplot* program on the computer and see the solids come alive.

   We finish off this topic with two activities from *Work Smarter Not Harder*, a book of labs accompanied by a disk of calculator programs. Students can download a program onto their calculators that will enable them to enter a function, graph the function, and rotate the function about a line. The calculator will then display a cross section of the solid generated. Another program enables students to enter a function and display a cross section that is a square, isosceles right triangle, etc. By the time the students have completed these activities, they are quite comfortable with the topic.

Syllabus 2 (Calculus AB)

Carol H. Miller
Glenbrook North High School
Northbrook, Illinois

School Profile

School Location and Environment: Northbrook is a suburban community located about 10 miles north of Chicago, Illinois. The district is well supported financially. Approximately 75 percent of the faculty have master's degrees. The district has been recognized nationally in both academic and extracurricular activities.

Grades: 9–12

Type: Public high school

Total Enrollment: Approximately 2,075 students (2003-04 school year)

Ethnic Diversity: As of the 2002-03 school year:
14.6 percent Asian and Pacific Islander
1.1 percent Hispanic/Latino
0.3 percent African American

College Record: Approximately 96 percent of students in the class of 2003 enrolled in college.

Personal Philosophy

I love teaching, and I love mathematics. I believe that mathematics is the best content through which to help young people to analyze and solve problems. I approach students of AP Calculus with the philosophy that we are in this together. Since we do not control the date that the AP Exam is given or the way in which the questions are asked, we must understand the material and trust our problem-solving skills. I tell the students that it will be easy for me to teach them the content of calculus, but that seeing good problems will teach them to solve problems.

Class Profile

The school day consists of four 90-minute class periods. In general, classes meet every other day. However, block 1 meets every day for 40 minutes between 6:55 a.m. and 7:35 a.m. (The only classes offered in block 1 are early-bird gym and jazz band). AP science classes meet every day for 90 minutes, and AP Calculus classes meet one day for 90 minutes and the next day for 45 minutes. After the AP Exam, calculus students are only required to attend class every other day for 90 minutes.

The average class size is 25. We teach either four sections of Calculus AB and one of Calculus BC or three of AB and two of BC, depending on enrollment. Juniors who take Calculus AB can take our "Calculus C" course the first semester of their senior year to finish the Calculus BC topics. During the second semester, these students can take multivariable calculus. Juniors who take Calculus BC take multivariable calculus and linear algebra their senior year. A few sophomores take Calculus BC. These students take multivariable calculus and linear algebra their junior year and do independent study with a member of our faculty as seniors. We also offer two or three sections of AP Statistics.

Course Overview

We cover everything in the Calculus AB topic outline as it appears in the *AP Calculus Course Description*, including integration by parts. The primary textbook is *Calculus from Graphical, Numerical, and Symbolic Points of View* (2nd ed.) by Arnold Ostebee and Paul Zorn. The two stated objectives of this course are that the students do well on the AP Exam and in the subsequent course, whether that is taken at Glenbrook North or in college. Consequently, there is an attempt to balance understanding, skills, and use of technology.

Course Planner

Below is the sequence of our AP Calculus AB course.

First Semester AP Calculus AB

Section Numbers	Topics	Timeline
A10–A20	Real Numbers and the Coordinate Plane	2 days
A21–A26	Lines and Linear Functions	1 day
A28–A34	Polynomials and Rational Functions	1 day
A35–A41	Algebra of Exponentials and Logarithms	1 day
1.1	Functions, Calculus Style	2 days
1.2	Graphs	2 days
A42–A46	Trigonometric Functions	2 days
1.3	A Field Guide to Elementary Functions	2 days
1.4	Amount Functions and Rate Functions: The Idea of the Derivative	1.5 days
1.5	Estimating Derivatives: A Closer look	2.5 days
1.6	The Geometry of Derivatives	1.5 days
1.7	The Geometry of Higher-Order Derivatives	1.5 days
2.3	Limits	4 days
2.1	Defining the Derivative	3 days
2.2	Derivatives of Power Functions and Polynomials	2 days
2.4	Using Derivative and Antiderivative Formulas	3 days
2.6	Derivatives of Exponential and Logarithmic Functions	1 day
2.7	Derivatives of Trigonometric Functions Modeling Oscillation	1 day
3.1	Algebraic Combinations: The Product and Quotient Rules	3.5 days
3.2	Composition and the Chain Rule	2.5 days
3.3	Implicit Functions and Implicit Differentiation	4 days
3.4	Inverse Functions and Their Derivatives, Inverse Trigonometric Functions	2.5 days
3.5	Miscellaneous Derivatives and Antiderivatives	2 days
4.3	Optimization	3 days
4.5	Related Rates	3 days
2.4	Using Derivative and Antiderivative Formulas (problems p. 120)	1 day
2.5	Differential Equations, Modeling Motion (problems pp. 129-31)	1 day
2.6	Derivatives of Exponential and Logarithmic Functions, Modeling Growth (problems pp. 138-39)	1 day
2.7	Derivatives of Trigonometric Functions: Modeling Oscillation (problems pp. 147-48)	1 day
4.1	Slope Fields, More Differential Equation Models	2.5 days
4.2	More on Limits: Limits Involving Infinity and L'Hospital's Rule	1.5 days

Second Semester AP Calculus AB

Section Numbers	Topics	Timeline
4.6	Newton's Method	2 days
4.7	Building Polynomials to Order, Taylor Polynomials (Linear Only)	2 days
4.8	Why Continuity Matters	1 day
4.9	Why Differentiability Matters, the Mean Value Theorem	1 day
5.1	Areas and Integrals	2 days
5.2	The Area Function	2 days
5.6	Approximating Sums: The Integral as a Limit	3 days
5.7	Working with Sums	3 days
5.3	The Fundamental Theorem of Calculus	2 days
5.4	Finding Antiderivatives, the Method of Substitution	4 days
7.1	Measurement and the Definite Integral, Arc Length	2 days
7.2	Finding Volumes by Integration	5 days
7.4	Separating Variables: Solving Differential Equations Symbolically	3 days

After the AP Exam

At present, we cover topics from chapter 8 of our primary textbook; these topics are not part of the Calculus AB syllabus. A test is given on this material. In the past, we taught a unit introducing students to a computer software program such as *Mathematica* or *Derive* to do calculus. However, this was discontinued after the introduction of the TI-92.

Section Numbers	Topics	Timeline
8.1	Integration by Parts	2 days
8.2	Partial Fractions	2 days
8.3	Trigonometric Antiderivatives	2 days
8.4	Miscellaneous Antiderivatives	1 day

Teaching Strategies

For about 50 percent of our students, Calculus AB is the first course that they take from our honors track. For these students, the expectations are considerably higher than they had been up to this point. A complete tentative schedule, showing clearly the target day of the AP Exam, is given to students on day one. The teacher tries to be seen as a coach, with the student and coach working together toward a common goal of doing well on the AP Exam.

In the Calculus AB course, teachers spend a significant amount of time reviewing and improving skills in algebra, geometry, trigonometry, and problem solving. In addition to using the appendix of the primary textbook, teachers use worksheets extensively.

In 1984-85, we began giving a mock AP Exam. Students are scheduled for an in-school field trip, during which we administer the mock exam. We try to simulate the actual AP Exam administration as closely as we can.

Technology and Computer Software

Teachers use TI-83 and TI-89 graphing calculators for presentations. Almost all students use one of these two calculators.

One of our Calculus AB teachers uses a number of PowerPoint presentations, which he has produced to aid in teaching many calculus concepts, such as the definition of the derivative. At the present time, none of our teachers are using computer software.

Student Evaluation

Quarter grades are computed using homework, writing, quizzes, and tests as individual categories. Each quarter grade represents 40 percent of the semester grade. The final exam represents the remaining 20 percent of the grade. Homework is awarded a score based on each individual teacher's preference. Quizzes vary depending on the teacher, but tests tend to be the same for everyone who teaches a particular course. On about half of the tests, students are allowed to use a calculator. As early as possible in the course, teachers try to incorporate multiple-choice practice problems. The semester final and mock AP Exams use multiple-choice questions that follow the format of the AP Exam. Questions from previous AP Exams strongly influence assessment during the year.

Teacher Resources

Primary Textbook

Ostebee, Arnold, and Paul Zorn. *Calculus from Graphical, Numerical, and Symbolic Points of View*. 2nd ed. Boston: Houghton Mifflin, 2002.

Reference

- Best, George, Stephen Carter, and Douglas Crabtree. *Calculus Concepts and Calculators*. 2nd ed. State College, Pa.: Venture Publishing, n.d.
- Hughes-Hallett, Deborah, et al. *Calculus—Single Variable*. 3rd ed. New York: John Wiley & Sons, 2002. [Ed. note: The 4th edition was published in 2005.]
- MAA Notes. *Resources for Calculus Collection*. Vol. 2: *Calculus Problems for a New Century,* edited by Robert Fraga. Washington, D.C.: MAA, 1993.
- *Multiple-Choice and Free-Response Questions in Preparation for the AP Calculus (AB) Examination*. 8th ed. Brooklyn, N.Y.: D&S Marketing Systems, n.d.
- Stewart, James. *Calculus*. 5th ed. Pacific Grove, Calif.: Brooks/Cole Publishing Company, 2003.

Technology Resources

Fischbeck, Sally E. *The TI-89, A Graphing Calculator with Computer Algebra, Tips for TI-83 Users*. Texas Instruments. education.ti.com/us/product/tech/89/down/tips.html

In all of our calculus sections, graphing calculators play a major role in both teaching and learning. Calculator papers written by Sally E. Fischbeck have strongly influenced our use of the graphing calculator.

Students are required to have a calculator. The TI-83 Plus is recommended for Calculus AB and the TI-89 for Calculus BC.

Review Material

- AP Calculus Released Exams (see chapter 5, Resources for Teachers).
- Free-response questions posted annually on AP Central.
- Calculus problems from the film *Calculus in the Year 2000: New Ways of Teaching the Derivative and the Definite Integral*, VHS, 3 hours, by Steve Olson. For sale at store.collegeboard.com.

Student Activities

The following is a computer lab activity that I originally developed after attending the 1993 CalcNet Workshop. It was intended to introduce my students to the computer software *Derive* and reinforce or introduce some calculus ideas as well as to serve as a group activity and writing assignment. As the course moved on to graphing calculators, the activity was revised for calculators. Three students were selected for each group; this was done so that they could all use the same calculator and, preferably, share both the TI-83 and TI-89. At present, all of our students use either the TI-83 or TI-89.

The main goal of this activity is for students to take an in-depth look at continuity. The particular areas of focus are (1) endpoint activity, (2) continuity over various intervals, and (3) the Extreme Value Theorem.

The students in each group work together and present their conclusions in a single paper that is graded based upon the correctness of the mathematics and the quality of the presentation. The paper should conclude with remarks on what the students believe the purpose of the activity was.

1. For the following function, answer questions (i) through (vi) with careful, thoughtful, and complete answers.

$$f(x)=\begin{cases} x^2-1 & \text{for } -1\le x<0 \\ 2x & \text{for } 0\le x<1 \\ 1 & \text{for } x=1 \\ -2x+4 & \text{for } 1<x<2 \\ 0 & \text{for } 2<x\le 3 \end{cases}$$

   (i) Does $f(-1)$ exist? Does $\lim_{x\to 1^+} f(x)$ exist? Does $\lim_{x\to 1^+} f(x)=f(-1)$? Is *f* continuous at $x=-1$?

   (ii) Does $f(1)$ exist? Does $\lim_{x\to 1} f(x)$ exist? Does $\lim_{x\to 1} f(x)=f(1)$? Is *f* continuous at $x=1$?

   (iii) Is *f* defined at $x=2$? Is *f* continuous at $x=2$?

   (iv) At what values is *f* continuous?

   (v) What is the value of $\lim_{x\to 2} f(x)$? What value should be assigned to $f(2)$ to make *f* continuous at $x=2$?

   (vi) To what new value should $f(1)$ be changed to make *f* continuous at $x=1$?

2. Write down precisely what conditions must hold for a function $y=g(x)$ to be continuous on (i) $(-\infty,\infty)$ (ii) $[-2,3]$ (iii) $[7,\infty)$ and (iv) $(-5,4]$.

3. For the greatest integer function $y = \text{int}(x)$, tell whether each of the following statements is true or false, and give reasons based on definitions or theorems from our textbook.
   (i) The function int is defined on $(-\infty, \infty)$.
   (ii) The function int is continuous on $(-\infty, \infty)$.
   (iii) Right-hand and left-hand limits exist for int at all real numbers $x = c$.
   (iv) The function int has two-sided limits at each integer x in its domain.
   (v) For every integer n, int is continuous on the interval $(n, n+1)$.
   (vi) For every integer n, int is continuous on the interval $[n, n+1)$.
   (vii) For every integer n, int is continuous on the interval $(n, n+1]$.

4. Write down a formula for a function $y = f(x)$ together with an interval $[a, b]$ that will match each of the following four situations.
   (i) f has both a maximum and a minimum on the interior of $[a, b]$.
   (ii) f has a maximum at one endpoint and a minimum at the other endpoint of $[a, b]$.
   (iii) f has a maximum at an interior point and a minimum at an endpoint of $[a, b]$.
   (iv) f has a maximum at an endpoint and a minimum at an interior point of $[a, b]$.

5. On the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, the cosine takes on a maximum value of 1 once and a minimum value of 0 twice, and the sine takes on a maximum value of 1 and a minimum value of -1. Do these two examples contradict the Extreme Value Theorem? Why or why not?

6. The function $f(x) = x$ has neither a largest nor a smallest value on the interval $(0, 1)$. Does this contradict the Extreme Value Theorem? Explain.

7. Even a single point of discontinuity can keep a function from having either a maximum or minimum on a closed interval. The function

$$f(x) = \begin{cases} x+1 & \text{for } -1 \le x < 0 \\ 0 & \text{for } x = 0 \\ x-1 & \text{for } 0 < x \le 1 \end{cases}$$

   is continuous at every point of the interval $[-1, 1]$ except at $x = 0$, yet its graph over the interval had neither a highest nor a lowest point. Does this contradict the Extreme Value Theorem? Why or why not?

8. The greatest integer function on $(1.5, 4.3)$ takes on both a maximum and a minimum value. What are these values? Does this example contradict the Extreme Value Theorem? Why or why not?

Please summarize what you believe that I wanted you to gain from this activity.

Syllabus 3 (Calculus BC)

Kathleen Goto
Iolani School
Honolulu, Hawaii

School Profile

School Location and Environment: Iolani School is located in Honolulu, a cosmopolitan, multiethnic city of 800,000 people. The University of Hawaii and the East-West Center are located within a mile of the Iolani campus.

Grades: K–12

Type: Independent, college-preparatory school

Total Enrollment: Approximately 1,792 students in K–12 and 1,277 students in grades 7–12

Ethnic Diversity: 70 percent Asian
7 percent Pacific Islander

College Record: Approximately 99 percent attend four-year colleges, and 1 percent attend two-year colleges.

Personal Philosophy

The students in AP Calculus BC are well prepared to handle the quick pace and amount of work required. They are accustomed to academic success and will work hard to remain successful. Although they usually measure their achievements by the grades they earn, I want them to see our course as more than a series of units to master.

Number crunching and symbol manipulation are only small parts of learning calculus. I want my students to feel they understand each concept we cover. One of our major goals is for the students to learn how to use precise language to describe these concepts and the relationships between ideas.

I subscribe to the notion that any student who is willing to work hard can succeed in calculus.

Class Profile

We offer classes in both Calculus AB (four sections, about 65–75 students) and Calculus BC (three sections, about 40–50 students). The usual schedule calls for three 42-minute classes and two 45-minute classes each week.

Course Overview

This is first-year calculus of a single variable: differential and integral calculus, and infinite sequences and series.

Course Planner

Primary textbook

Finney, Ross L., et al. *Calculus: Graphical, Numerical, Algebraic.* Menlo Park, Calif.: Scott Foresman/ Addison-Wesley, 1999.

The chapter numbers follow the textbook. Note that we work on chapter 10 before chapter 9.

Chapter 1: Prerequisites for Calculus (7 days)

Elementary functions:

Linear, power, exponential/logarithmic, trigonometric/inverse trigonometric

Parametric equations

Getting familiar with the graphing calculator

Chapter 2: Limits and Continuity (10 days)

Limits:

Limit at a point, limit at infinity, infinite limits
Properties of limits

Continuity

Tangent line to a curve

Slope of a curve at a point

Chapter 3: Derivatives (15 days)

Definition of f'

Derivative at a point

Relating the graphs of f and f'

When does $f'(a)$ fail to exist?

Rules for differentiation:

Sum, product, quotient

Chain Rule

Implicit differentiation

Derivatives of trigonometric, inverse trig, exponential and logarithmic functions

Chapter 4: Applications of Derivatives (17 days)

Mean Value Theorem

Using the derivative to find:

- Critical point(s) and extreme values
- When the function is increasing or decreasing
- Point(s) of inflection
- When the function is concave up or concave down

Optimization problems

Using the tangent line to approximate function values

Newton's method

Differentials and change

Related rates

Chapter 5: The Definite Integral (14 days)

RAM (Rectangle Approximation Method)

Riemann sums

Finding an antiderivative

Using a definite integral to find area, volume, average value of a function

Fundamental Theorem of Calculus

Approximating the definite integral:

- Trapezoidal Rule, Simpson's Rule, Error

Chapter 6: Differential Equations and Mathematical Modeling (15 days)

Slope fields

Antiderivatives and the indefinite integral

Techniques of integration:

- Substitution, integration by parts, trig substitution, partial fractions

Separable differential equations

Euler's method

Exponential growth and decay

Logistic growth

Chapter 7: Applications of Definite Integrals (12 days)

Using the definite integral to discuss:

Net change—motion on a line, consumption over time

Area, volume, length of a curve, surface area of a solid of revolution

Work, fluid force

Chapter 8: L'Hospital's Rule, Improper Integrals, Partial Fractions (13 days)

Indeterminate forms $\left(\frac{0}{0}, \frac{\infty}{\infty}, \infty-\infty, 1^{\infty}, 0^{0}, \infty^{0}\right)$ and L'Hospital's Rule

Relative rates of growth

Improper integrals (partial fractions and trig substitutions—done with chapter 6)

Chapter 10: Parametric, Vector, and Polar Functions (14 days)

Parametric functions:

Derivative at a point

$\frac{d^2y}{dx^2}$

Length of a curve, surface area of a solid of revolution

Vectors:

Angle between vectors

Scalar product

Using vectors to describe motion in the plane

Polar coordinates and pole graphs:

Slope, horizontal and vertical tangent lines

Area, length of a curve

Chapter 9: Infinite Series (15 days)

Geometric series

Power series:

Term-by-term differentiation and integration to find power series of new functions

Taylor's series/Maclaurin series

Lagrange form of the remainder

Tests for convergence/divergence:

nth term test

Direct Comparison

Ratio Test

Integral Test

Limit Comparison Test

Alternating Series Test (Leibniz's Theorem)

Radius and Interval of convergence

Review for AP Exam (15 days)

In addition to providing the students with some AP scoring tips, we review concepts and discuss the solutions to both free-response and multiple-choice questions.

Students start by working on an old multiple-choice test on the first weekend. They say this test reminds them of many things they have stored away and "forgotten." Over the following few weeks, they work on another multiple-choice test at home and one in class under testing conditions.

We go through a few free-response questions together in class and discuss the scoring guidelines so students understand the need for "complete" solutions. Over the next two weeks, students work on free-response sections from three previous AP Calculus Exams (all six questions in each exam) at home. They usually have two nights or a weekend to do each test. They are encouraged to talk to each other but are supposed to write their own solutions. They also take a couple of free-response "minitests" in class. I choose four of the six questions (two from the calculator section and two from the noncalculator section), and they work on these under AP Exam-like conditions—that is, 20 minutes with the calculator for the first two questions, and the remainder of the 42-minute period without the calculator to finish the test.

Finally, I use four days to test one last time: multiple-choice without calculator, multiple-choice with a calculator, free-response with a calculator, and free-response without a calculator. For this last test I make mock exam booklets with the spaces for the work and the questions printed on appropriate mock exam paper.

While this is going on, I am grading furiously every night at home. At school, we are discussing the students' work in order to point out common errors and to suggest alternate solution methods.

Teaching Strategies

My goal for each class period is to develop an interesting discussion. I tend to get into the lecturer mode too easily, so I always welcome questions or comments. In fact, some of the best classes occur when a student asks "Why?" or "Why not?" and the class proposes answers that need to be examined.

Students discuss homework questions in small groups (two to four members) for a few minutes, and unresolved questions are saved for discussion with the entire class. Individual students serve several days or several weeks as the "homework boss." In that role, the student leads the homework review for the entire class and solicits volunteers who share their work. The "homework boss" also ensures that everyone else is following along.

I preach a positive attitude and encourage hard work, a lot of practice, and organized thinking and speaking.

Student Evaluation

Student grades are based on homework, quizzes, and tests. Homework is assigned daily and collected often (about four out of five days). Assignments are designed to take about one hour and include not only problems from the current discussion, but almost always some problems from the last few days of class discussion as well. Students meet in small groups of two to four for a few minutes at the beginning of the class period to discuss immediate questions from the assignment. Questions that are not resolved are passed on for discussion with the entire class. Homework that is collected is given a mark of 1–5 points (this mark is based on completeness more than on correctness). If a student has missed the point completely, I will return the paper and ask for it to be redone (with no grade penalty).

Quizzes are unannounced and cover small pieces of material, usually work from the last two or three days. Quizzes are worth 10 points.

Tests cover larger amounts of material and are almost always cumulative. In-class tests are divided into calculator/noncalculator sections similar to the free-response section of the AP Exam. There is no other set format for the tests. I occasionally include some multiple-choice questions, but most of the questions require students to show the work that leads to their answers or are short-answer-type questions.

There is one take-home test each of the first three quarters. Students are encouraged to talk about their ideas with their classmates but are told they should write their own solutions to turn in. Each test is worth 45–60 points. Tests are not scheduled at the conclusion of chapters or units, but rather when they fit into the schedule. The tests tend to include material that is covered up to a few days before each test is given.

Quarter averages are figured using these guidelines:

20 percent homework—two homework assignments are forgiven

30 percent quiz average—two lowest quiz scores are dropped

50 percent test average

Quarter grades are assigned as follows:

90–100 = A

80–89 = B

70–79 = C

60–69 = D

below 60 = failing

Teacher Resources

- Anton, Howard, Irl Bivens, and Stephen Davis. *Calculus: Early Transcendentals*. 7th ed. New York: John Wiley & Sons, 2003.

- Bradley, Gerald L., and Karl J. Smith. *Single Variable Calculus*. 2nd ed. Upper Saddle River, N.J.: Prentice-Hall, 1998.
  I especially like the historical notes that are spaced throughout the book.

- Foerster, Paul A. *Calculus: Concepts and Applications*. Emeryville, Calif.: Key Curriculum Press, 2004.
  This is a wonderful source of interesting problems for discussion.

- Foerster, Paul A. *Calculus: Concepts and Applications, Instructor's Resource Book*. Emeryville, Calif.: Key Curriculum Press, 2004.
  The explorations often help me think of a good way to introduce a new topic or to review a topic.

- Hughes-Hallett, Deborah, et al. *Calculus—Single Variable*. 3rd ed. New York: John Wiley & Sons, 2002. [Ed. note: The 4th edition was published in 2005.]
  So many interesting problems!

- MAA Notes. *Resources for Calculus Collection*. Washington, D.C.: MAA, 1993.

  - Volume 1: *Learning By Discovery: A Lab Manual for Calculus*, edited by Anita Solow. MAA Notes Number 27.
  - Volume 2: *Calculus Problems for a New Century*, edited by Robert Fraga. MAA Notes Number 28.
  - Volume 3: *Applications of Calculus,* edited by Philip Straffin. MAA Notes Number 29.
  - Volume 4: *Problems for Student Investigation*, edited by Michael B. Jackson and John R. Ramsay. MAA Notes Number 30.
  - Volume 5: *Readings for Calculus*, edited by Underwood Dudley. MAA Notes Number 31.

- Ostebee, Arnold, and Paul Zorn. *Calculus from Graphical, Numerical, and Symbolic Points of View*. 2nd ed. Boston: Houghton Mifflin, 2002.

- Stewart, James. *Calculus*. 5th ed. Pacific Grove, Calif.: Brooks/Cole Publishing Company, 2003.

- Weeks, Audrey. *Calculus in Motion*. Burbank, Calif.: Calculus in Motion, 2005. (calculusinmotion.com).
  This software has wonderful animations and visuals of things "changing."

Web Resources

- AP Central (apcentral.collegeboard.com)
  This is the College Board site, and it has lots of helpful resources. This year I found some nice slope field worksheets ready to use. Free-response questions, scoring guidelines, and scoring commentaries are available on AP Central.

- AP Calculus Electronic Discussion Group (EDG)
  The volume of mail that goes through this list is astounding. You can read other teachers'

questions and responses, or you can post your own questions—you'll be sure to receive several good responses!

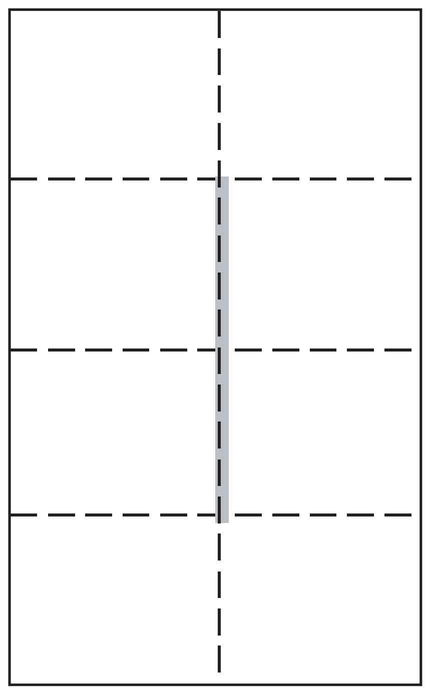
Student Activities

1. Calculus Minibook

This activity can be used at any point in the course. Have the students make a little booklet. The writing assignment can be creative or serious. I have assigned "The Story of Derivatives" or the "The Story of Limits" and have been impressed with the creative ways students show what they know.

Follow the directions shown below to create the booklet.

Use one 8 1/2 × 14 sheet of paper. Fold along the broken lines:



Cut along the section marked by this line: ————————————

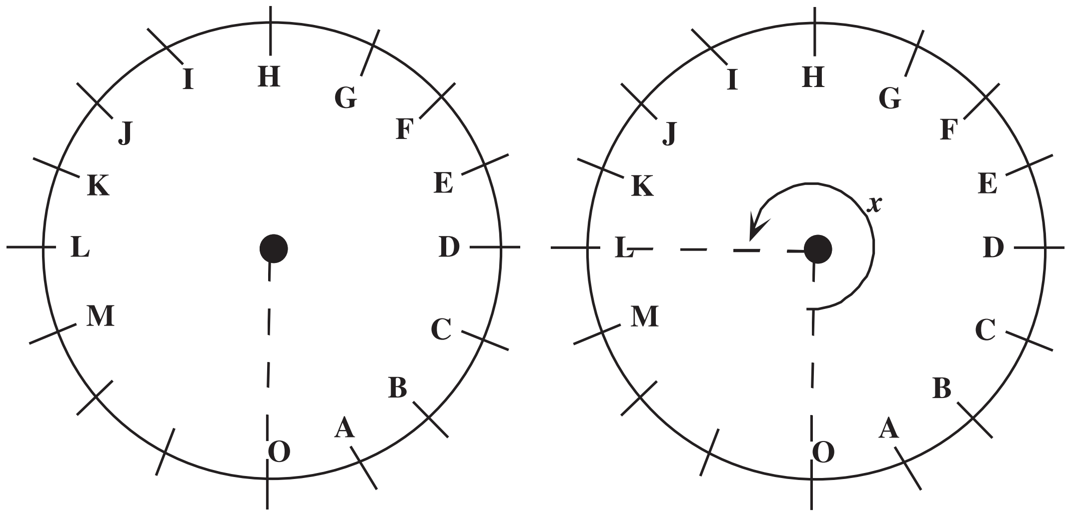
Fold the sheet in half vertically, and fold into a "book."

Write your story!

2. M&M Cone Lab

This activity is based on Exploration 1 in Finney et al., *Calculus*, p. 210.

1. Cut out circle (this circle has a radius of 8 cm.).



2. Cut along radius from O to center of circle.

3. Create various sizes of cones from the sectors of the circle.

   We'll call these cones M (when O slides over to meet M), L, K, etc.
   (Note cone **L** is made from a 270° sector of a circle, or $\boldsymbol{x} = 1.5\pi$ radians.)

4. Fill each cone with M&M's and complete this table.

Cone	Number of M&M's to Fill	Central Angle of Sector
M		
L		
K		
J		
I		
H		
G		
F		
E		
D		
C		

5. We want to know the central angle (call it x) that gives the cone with maximum volume.

   I copy this circle onto paper that is a little heavier. Each group of four students has one cone and a cup of M&M's. After the table is complete, they set up a plot and look for the answer to the question.

   Finally, we work the problem analytically to see which value of x will give a cone of maximum volume.

Syllabus 4 (Calculus BC)

Nancy Stephenson
William P. Clements High School
Sugar Land, Texas

School Profile

School Location and Environment: William P. Clements High School is a suburban school in the Fort Bend Independent School District, southwest of Houston. Most of the students who attend Clements come from white-collar, middle- to high-income families with parents in professional careers. A large number of students are classified as gifted and talented, and the students are very competitive academically. Many of the students take five or six AP classes during their senior year, especially in the areas of mathematics and science.

Grades: 9–12

Total Enrollment: Approximately 2,400 students

Ethnic Diversity: Approximately 38 percent Asian
5 percent Hispanic/Latino
3 percent African American

College Record: Approximately 85 percent attend four-year colleges, and 10 percent attend two-year colleges.

Personal Philosophy

I really enjoy teaching AP Calculus. My students are very bright and inquisitive, and they are enthusiastic about learning. Calculus pulls together many of the concepts the students have studied in previous courses and helps them see the relevance of the material they were taught prior to calculus. I believe that AP Calculus BC gives the students a strong foundation for the math and science courses they will take in college, and I feel privileged to have the opportunity to help them lay this foundation. Students frequently come back to tell me that they felt very well prepared for their college math courses. I think that this is due in large part to the challenging AP Calculus curriculum and the effort made by students to be successful on the AP Exam.

Class Profile

Usually there are five sections of AP Calculus AB and three sections of AP Calculus BC each year. The average size of each class is 30 students. Clements is on a seven-period day, with classes meeting each day for 50 minutes.

Course Overview

My primary textbook is *Calculus with Analytic Geometry* (5th edition), by Ron Larson, Robert P. Hostetler, and Bruce H. Edwards. Following is a brief list of the course objectives:

Limits and Continuity. Limits are introduced, including the idea of "zooming in" on a particular point both graphically and by using a table of values. The students also learn to find limits by using algebraic techniques. The properties of limits, right- and left-hand limits, and asymptotic behavior of a function are discussed, as well as the meaning of continuity and the graphical properties of a continuous function.

The Derivative and Applications of Differentiation. Students are introduced to the idea of a local linearity as a way to determine whether a function is differentiable at a particular point. They learn how to find the derivative by using the definitions of the derivative and proving that a function is differentiable. Students learn to analyze rates of change by using a graph and a table of values; to find the derivative of a function analytically by using the rules of differentiation; and to evaluate a derivative at a particular point by using their graphing calculators. Students learn how to interpret the meaning of a derivative and describe it verbally. They also learn how to do implicit differentiation and how to apply their knowledge to related rates problems. They perform a discovery activity to help them understand the Mean Value Theorem, and they find absolute extrema. Students also engage in a discovery activity to help them learn the First and Second Derivative Tests, and they apply their knowledge to optimization problems and to problems in which the graphs of f' and f'' are given. They also apply their knowledge of derivatives to solve problems involving rectilinear motion.

The Integral and Applications of Integration. Students are introduced to the idea of antidifferentiation and indefinite integrals. They study area and the definition of a definite integral, as well as Riemann sums, including left, right, and midpoint sums. The Fundamental Theorem of Calculus is introduced through a discovery activity, and students learn how to apply the Fundamental Theorem to evaluate definite integrals, both analytically and by using their graphing calculators. They also learn how to use Riemann sums and the Trapezoidal Rule to estimate the value of a definite integral, particularly in situations in which graphical or tabular information is given. Students also learn integration techniques, including integration by parts and integration by partial fractions. Students apply their knowledge of integration to solve problems involving accumulation, area, volume, average value, arc length, rectilinear motion, and logistic growth.

Transcendental Functions. Students learn the definition of the natural logarithmic function, as well as its properties, and differentiation and integration involving e^u, $\ln u$, a^u, and $\log_a u$, and the inverse trigonometric functions. They learn how to solve separable differential equations and apply their knowledge of differential equations to draw a slope field for a particular differential equation and to sketch a solution curve on the slope field. They also learn to estimate a solution to a differential equation by using Euler's method.

Indeterminate Forms and Improper Integrals. Students apply their knowledge of L'Hospital's Rule to evaluate limits involving indeterminate forms. They also learn the techniques needed to evaluate improper integrals.

Infinite Series. Students study infinite sequences series and learn tests for determining their convergence or divergence. They study alternating series and absolute and conditional convergence. They learn to find a Taylor polynomial for a given function and how to use it to approximate values, as well as how to estimate the error involved in their approximation. Students learn to find a power series for a given function both by using Taylor's Theorem and by manipulation of a known series. They also learn to find the radius and interval of convergence of a given series.

Parametric and Polar Equations. Students learn to graph parametric equations and to find the derivative and integral of a parametric equation, as well as how to apply their knowledge to vectors. They learn to find the derivative and integral of a polar equation and how to apply their knowledge to problems involving area.

Course Planner

At the end of the school year, I give students planning to study Calculus BC when school reconvenes a summer packet with the types of problems they studied in precalculus. This packet is due on the first day of school. By doing this I avoid beginning the course with a review of prerequisites and am able to start on chapter 1 of the textbook after a day of review and test on the summer packet.

First Semester

Chapter 1: Limits and Their Properties (10 days—one test)

Lab on Limits
An Introduction to Limits
Properties of Limits
Techniques for Evaluating Limits
Continuity and One-Sided Limits
Infinite Limits

Chapter 2: Differentiation (19 days—two tests)

Zooming-in Activity and Local Linearity
The Derivative and the Tangent Line Problem
The Meaning of the Derivative
Basic Differentiation Rules and Rates of Change
The Product and Quotient Rules of Higher-Order Derivatives
The Chain Rule
Implicit Differentiation
Related Rates
Position, Velocity, Acceleration, and Rectilinear Motion

Chapter 3: Applications of Differentiation (19 days—two tests)

Extrema on an Interval
Discovery Lesson on the Mean Value Theorem
Rolle's Theorem and the Mean Value Theorem
Lab on the First Derivative Test
Increasing and Decreasing Functions and the First Derivative Test
Lab on Concavity and Points of Inflection
Concavity and the Second Derivative Test
Limits at Infinity
A Summary of Curve Sketching
Relating the Graphs of f, f', and f''
Optimization
Tangent Line Approximations

Chapter 4: Integration (15 days—two tests)

Antiderivatives and Indefinite Integration
Area
Meaning of the Definite Integral
Riemann Sums, including Left, Right, and Midpoint Sums
Discovery Lesson on the First Fundamental Theorem of Calculus
The First Fundamental Theorem of Calculus and the Mean Value Theorem for Integrals
Integration by Substitution
Discovery Lesson on the Second Fundamental Theorem of Calculus
The Second Fundamental Theorem of Calculus and Functions Defined by Integrals

Chapter 5: Logarithmic, Exponential, and Other Transcendental Functions (16 days—two tests)

The Natural Logarithmic Function and Differentiation
The Natural Logarithmic Function and Integration
Inverse Functions
Exponential Functions: Differentiation and Integration
Bases Other Than *e* and Applications
Differential Equations
Applications of Differential Equations
Slope Fields
Euler's Method

First Semester Exam (two review days)

Second Semester
Chapter 5: Logarithmic, Exponential, and Other Transcendental Functions (4 days—one test)
- Inverse Trig Functions and Differentiation
- Inverse Trig Functions and Integration

Chapter 6: Applications of Integration (10 days—one test)
- The Integral as an Accumulator of Rates of Change
- Area of a Region Between Two Curves
- Volume of a Solid with Known Cross Sections
- Volume of Solids of Revolution
- Arc Length

Chapter 7: Integration Techniques, L'Hospital's Rule, and Improper Integrals (17 days—two tests)
- Review of Basic Integration Rules
- Integration by Parts
- Trigonometric Integrals
- Partial Fractions
- Logistic Growth
- Discovery Lab on L'Hospital's Rule
- Relative Rates of Growth
- Discovery Lab on Improper Integrals
- Improper Integrals

Chapter 8: Infinite Series (17 days—two tests)
- Lab on Sequences
- Convergence and Divergence of Sequences
- Series and Convergence
- The Integral Test and *p*-series
- Comparisons of Series
- Alternating Series and the Alternating Series Remainder
- The Ratio and Root Tests
- Taylor Polynomials and Approximations
- Power Series and Radius and Interval of Convergence
- Taylor and Maclaurin Series
- Taylor's Theorem with the Lagrange Form of the Remainder

Chapter 10: Plane Curves, Parametric Equations, and Polar Curves (12 days—one test)
- Plane Curves and Parametric Equations
- Parametric Equations and Calculus
- Parametric Equations and Vectors—Motion Along a Curve
- Polar Coordinates and Polar Graphs
- Area in Polar Coordinates

AP Exam Review (minimum of 15 days)
AP Exam

Chapter 16: Differential Equations (6 days—one test)
- Definitions and Basic Concepts of Differential Equations
- First Order Linear Differential Equations
- Second Order Homogeneous Linear Equations

Chapter 5: Hyperbolic Functions (4 days—one test)
- Hyperbolic Functions and Applications

Second Semester Exam (two review days)

Teaching Strategies

Learning by Discovery. I like to introduce each unit with a discovery lesson. I think that exploration and discovery are great ways for students to learn because they have more ownership in the material being covered than they have from a traditional lecture approach. The discovery lessons are done in groups of two or three students.

Graphing Calculator. Many of the discovery lessons rely heavily on the use of the graphing calculator. The calculator helps students develop a visual understanding of the material that they would not otherwise have. My students use the graphing calculator almost every day in class and also on homework. However, many homework problems and about half of the problems on quizzes and tests are done without the use of the graphing calculator. Since the AP Exam is half calculator and half noncalculator, I feel that it is important for students to have practice working problems both ways. We spend time in class discussions talking about the types of questions that they must know how to work *with* their calculators and the types of questions that they must know how to work *without* their calculators. We also discuss the techniques needed to use the calculator most efficiently (storing functions in the $y =$ screen, storing values that will be used later in the problem, etc.).

Rule of Four. I give my students many opportunities to work problems presented in a variety of ways: graphical, numerical, analytical, and verbal. Most of the problems in my primary textbook are written with an analytical representation, so I frequently supplement these problems with problems giving a graph or tabular data. I also often ask students for verbal explanations to give them the opportunity to communicate their reasoning in words.

Justification of Answers. I ask my students to justify their answers on homework, quizzes, and tests, and I prefer that they write the justifications in sentences. We talk a lot about the amount of work they need to show and the correct way to justify their work on various types of problems. (The "Commentary on the Instructions for the Free-Response Section of the AP Calculus Exams" on AP Central is very helpful in showing examples of correct justifications.)

Homework. My students have homework each night, usually requiring about 30 to 45 minutes of their time. In order to cover all of the Calculus BC topics, we move very quickly through each chapter; it is very important that students do their homework each night so that they gain the maximum benefit from the homework discussion that occurs the next day in class.

Assessment. Our school district requires a minimum of three major grades and six daily grades for every six-week period. The major average and the daily average each count as 50 percent of a student's six-week grade. I generally use tests as major grades. I use quizzes, "Problems of the Week" (see AP Review below), and homework as daily grades, with an occasional project counting as a daily grade.

AP Review. At the beginning of the third six-week period, I start giving the students a "Problem of the Week," which is a free-response question from a previous AP Exam. I grade the problem on a nine-point scale, using the scoring guideline that was used at the AP Calculus Reading. I try to pick questions from topics that the students learned several weeks (or months) earlier so that they are reviewing as they work the problems. For example, as they are learning applications of integration, I might give them a related rates question or a problem in which they are given a graph of the derivative and asked questions about it. On each major test, I also include a problem that is similar to the "Problems of the Week" that they have worked on recently. My students feel that the "Problems" provide them with constant review for the free-response section of the AP Exam, so they do not forget the topics they learned earlier in the year. In February, I begin giving a page of three or four multiple-choice questions as part of the homework each night. The multiple-choice questions that I use are taken from AP Released Exams and AP Calculus review books. Just as with the "Problems of the Week," I try to choose questions from topics that students learned earlier in the year.

I try to allot a minimum of three weeks before the AP Exam to devote to review. During this three-week period, students work on the sample questions in the *AP Calculus Course Description* and on multiple-choice and free-response questions from Released Exams. Some of these are assigned for homework, while others are given as a quiz or test. The calculus teachers in my district also administer a practice AP Exam on a Saturday morning about a week and a half before the actual AP Exam is given. For the practice exam, we use the multiple-choice section from the most recent Released Exam and the previous year's free-response questions. I grade the practice exams, and we spend the next few days discussing the problems. The practice exam lets students see which topics they need to review. It also gives them an idea of how exhausting the "real" exam will be, so that they realize they need to get a good night's sleep before exam day and eat a good breakfast before taking the exam.

Teacher Resources

Primary Textbook

Larson, Ron, Robert P. Hostetler, and Bruce H. Edwards. *Calculus with Analytic Geometry.* 5th ed. Boston: Houghton Mifflin, 1994.

Supplementary Texts

I supplement quite a bit with materials from other calculus textbooks, AP Released Exams, and free-response questions from AP Central. My resources include the following textbooks:

- Anton, Howard. *Calculus—A New Horizon.* 6th ed. New York: John Wiley & Sons, 1998.
- Best, George, Stephen Carter, and Douglas Crabtree. *Concepts and Calculators in Calculus.* 2nd ed. Andover, Mass.: Venture Publishing. n.d.
- Dick, Thomas P., and Charles M. Patton. *Calculus of a Single Variable.* Boston: PWS Publishing Company, 1994.
- Finney, Ross, et al. *Calculus—Graphical, Numerical, Algebraic.* 2nd ed. Menlo Park, Calif.: Scott Foresman/Addison-Wesley, 1999.
- Foerster, Paul A. *Calculus—Concepts and Applications.* Emeryville, Calif.: Key Curriculum Press, 2004.
- Hughes-Hallett, Deborah et al. *Calculus—Single Variable.* 3rd ed. New York: John Wiley & Sons, 2002. [Ed. note: The 4th edition was published in 2005.]
- Ostebee, Arnold, and Paul Zorn. *Calculus from Graphical, Numerical, and Symbolic Points of View.* 2nd ed. Boston: Houghton Mifflin, 2002.
- Stewart, James. *Calculus: Concepts and Contexts.* 3rd ed. Pacific Grove, Calif.: Brooks/Cole Publishing Co., 2005.

More Resources

- D&S Marketing Systems. www: dsmarketing.com
  - *Multiple-Choice and Free-Response Questions in Preparation for the AP Calculus (AB) Examination*
  - *Multiple-Choice and Free-Response Questions in Preparation for the AP Calculus (BC) Examination*
- Edwards, Bruce H., Ron Larson, and Robert P. Hostetler. *Themes for Advanced Placement Calculus.* 7th ed. Boston: Houghton Mifflin, 2002.
- Foerster, Paul A. *Instructor's Resource Book*. Emeryville, Calif.: Key Curriculum Press, 2004. This publication accompanies his calculus textbook.
- Hockett, Shirley O., and David Bock. *How to Prepare for the AP Calculus Advanced Placement Exam*. 7th ed. Hauppauge, N.Y.: Barron's Educational Series, 2002.
- Mathematical Association of America (www.maa.org)
  MAA Notes. *Resources for Calculus Collection*. Washington, D.C.: MAA, 1993.
  - Volume 1: *Learning By Discovery: A Lab for Calculus,* edited by Anita Solow. MAA Notes Number 27.
  - Volume 2: *Calculus Problems for a New Century,* edited by Robert Fraga. MAA Notes Number 28.
  - Volume 3: *Applications of Calculus,* edited by Philip Straffin. MAA Notes Number 29.
  - Volume 4: *Problems for Student Investigation,* edited by Michael B. Jackson and John R. Ramsay. MAA Notes Number 30.
  - Volume 5: *Readings for Calculus,* edited by Underwood Dudley. MAA Notes Number 31.
- Venture Publications. www.vent-pub.com
  - *AP Calculus with the TI-83*
  - *AP Calculus with the TI-89 Graphing Calculator*
  - *Preparing for the Calculus AB Exam*
  - *Preparing for the Calculus BC Exam*

Student Activities

Below are two of the discovery labs that I use with my students.

Discovering Relationships Lab

For each of the functions given, you will calculate its first derivative. By observing the graphs of the function and its derivative, you will discover a relationship between the function and its derivative.

Given $f(x)=x^2-6x-8$. Graph $f(x)$ and $f'(x)$ in a friendly window. Sketch below.

1. Where do the relative maximum and minimum values of $f(x)$ occur? ____________
2. For which values of x is $f'(x)$ equal to zero or undefined? ____________
3. For which values of x is $f(x)$ increasing? ____________
4. For what intervals is $f'(x)$ positive? ____________
5. For which values of x is $f(x)$ decreasing? ____________
6. For what intervals is $f'(x)$ negative? ____________

Given $f(x)=(x-1)^{2/3}$. Graph $f(x)$ and $f'(x)$ in a friendly window. Sketch below.

7. Where do the relative maximum and minimum values of $f(x)$ occur? ____________
8. For which values of x is $f'(x)$ equal to zero or undefined? ____________
9. For which values of x is $f(x)$ increasing? ____________
10. For what intervals is $f'(x)$ positive? ____________
11. For which values of x is $f(x)$ decreasing? ____________
12. For what intervals is $f'(x)$ negative? ____________

Given $f(x)=|x-1|$. Graph $f(x)$ and $f'(x)$ in a friendly window. Sketch below.

13. Where do the relative maximum and minimum values of $f(x)$ occur? ____________
14. For which values of x is $f'(x)$ equal to zero or undefined? ____________
15. For which values of x is $f(x)$ increasing? ____________
16. For what intervals is $f'(x)$ positive? ____________
17. For which values of x is $f(x)$ decreasing? ____________
18. For what intervals is $f'(x)$ negative? ____________

After you have finished your investigation, complete these statements:

The relative maximum and minimum values of f occurred when ______________________________

____

The function f is increasing when ____

____

The function f is decreasing when ____

____

First Derivative Test

The relative maximum of a function $f(x)$ occurs when ______________________________

____

The relative minimum of a function $f(x)$ occurs when ______________________________

____

Lab on Indeterminate Forms

In this section we will be evaluating limits that may give results that may surprise you. For the following limit problems, make a guess as to what you think the limit will be and then evaluate the given expression for the values given and see if you think your guess was correct.

1. $\lim_{x\to 0}\left(\frac{e^{3x}-1}{x}\right)=?$ Guess: ________

   To see if your guess is correct, evaluate the following:

   $\left.\frac{e^{3x}-1}{x}\right|_{x=0.1}=$ ________

   $\left.\frac{e^{3x}-1}{x}\right|_{x=0.01}=$ ________

   $\left.\frac{e^{3x}-1}{x}\right|_{x=0.001}=$ ________

   Now what do you think the original limit is? ________

2. $\lim_{x\to 1^+}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right)=?$ Guess: ________

   To see if your guess is correct, evaluate the following:

   $\left.\left(\frac{1}{\ln x}-\frac{1}{x-1}\right)\right|_{x=1.1}=$ ________

   $\left.\left(\frac{1}{\ln x}-\frac{1}{x-1}\right)\right|_{x=1.01}=$ ________

   $\left.\left(\frac{1}{\ln x}-\frac{1}{x-1}\right)\right|_{x=1.001}=$ ________

   Now what do you think the original limit is? ________

3. $\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^x=?$ Guess: ________

   To see if your guess is correct, evaluate the following:

   $\left.\left(1+\frac{2}{x}\right)^x\right|_{x=100}=$ ________

   $\left.\left(1+\frac{2}{x}\right)^x\right|_{x=1000}=$ ________

   $\left.\left(1+\frac{2}{x}\right)^x\right|_{x=10,000}=$ ________

   Now what do you think the original limit is? ________

Syllabus 5 (Calculus I)

Benjamin G. Klein
Davidson College
Davidson, North Carolina

College Profile

School Location and Environment: Davidson College is a nationally recognized, highly selective independent liberal arts college located 20 miles north of Charlotte, North Carolina, in the town of Davidson. Founded in 1837 by Presbyterians, today Davidson enrolls approximately 1,600 men and women.

Some 3,900 students applied for admission to the class of 2007. Of these, 31.8 percent were accepted; and of those accepted, 39.4 percent enrolled. The mean SAT® scores for the enrolled students were 671 (verbal) and 677 (mathematics). Students in the class of 2007 come from 36 different states and 10 foreign countries: 12.2 percent are "domestic students of color," and 51.6 percent of the class are males.

The college's academic requirements are fairly traditional. A so-called core requirement ensures that students take courses in a variety of different departments. Although no one course is required, all students must take at least one course in mathematics. For the majority of students, this course is Calculus I, either via AP credit or taken at Davidson.

The college offers majors in 20 different departments, one of which is mathematics. Davidson also offers so-called concentrations, which students pursue on top of their declared major. Two of these concentrations—applied mathematics and computer science—are particularly attractive to students who are interested in mathematics. The latter concentration would be the equivalent of a minor in computer science at many colleges.

The number of majors in mathematics at Davidson fluctuates quite a bit, from fewer than 10 to more than 20. The average is about 15, which represents about 4 percent of each graduating class. This is a larger number than one would find at most liberal arts colleges like Davidson.

In the fall semester, enrollment in Calculus I and II sections is capped at 26 and 25, respectively, and classes typically fill. Enrollments in spring semester calculus classes tend to be somewhat smaller. Regular members of the Davidson faculty teach all the mathematics courses, and every member of the Department of Mathematics holds a doctorate in mathematics.

AP Policy at Davidson

Davidson College is an enthusiastic supporter of the Advanced Placement Program, and every relevant department gives AP credit for certain of its courses based on student performance on an appropriate AP Exam. All but one of these departments requires a grade of 4 on the AP Exam before credit for a Davidson course is awarded. The one exception is the Department of Mathematics. It is important to note that students who enroll at Davidson with more than four AP credits can apply only four of those credits toward Davidson's 32-course graduation requirement. A grade of 4 or 5 on the AP Statistics Exam earns an unspecified course credit at Davidson—that is, a credit that counts toward graduation but does not satisfy any other requirement.

Davidson College awards credit for Calculus I to students with a grade of 3 or better on either the Calculus AB or Calculus BC Exam. A student who gets a 4 or a 5 on the BC Exam earns two course credits, one for Calculus I, and a second credit that can be used to help satisfy the core requirement in natural science and mathematics but does not correspond to any Davidson course.

The issue of placement in Calculus I versus Calculus II is very different from the issue of credit. We encourage students with an AP grade of 3 to enroll in Calculus II at Davidson, even if these students do not take the corresponding AP credit. In fact, even students with a grade of 2 who feel that their AP Exam grade does not accurately reflect their knowledge of calculus are encouraged to consider enrolling in Calculus II.

Since Calculus II at Davidson deals primarily with multivariable calculus, even students with grades of 4 or 5 on the BC Exam (and hence two course credits in mathematics) should start with Calculus II if they wish to continue their study of calculus. Students who have had some multivariable calculus should consult with a member of the Department of Mathematics to decide which Davidson course best fits their needs.

Personal Philosophy and Teaching Strategy

As described in the Class Profile section below, Davidson College offers special sections of Calculus I for students with no previous exposure to calculus. In the remaining sections, we assume that students have seen some calculus previously. The approach I take when teaching Calculus I depends on which type of section I am teaching.

We cover the same material in every Calculus I section. (See the Course Overview below.) This is essential because students from any Calculus I section can enter any Calculus II section. The difference in approach arises in the way we present the material and the expectations we have of the students.

In both versions of Calculus I, I use the same basic instructional method: a lecture format in which I actively solicit questions and comments from students. Given the amount of material that we cover and the limited time available, I do not do any group work or individual investigations during class. I encourage and, when possible, require these activities outside of class since I feel they are valuable.

I firmly believe that calculus, among other things, involves developing a set of skills, and that students learn skills through practice. Thus, I give and expect students to do a lot of homework and turn it in for grading. Since I believe that homework is important, I devote at least a few minutes at the beginning of each class to my comments and students' questions about the homework that was due that day.

Since I am committed to the intelligent use of technology as a piece of learning and doing mathematics, I also require active use of graphing calculators in both versions of Calculus I. I hope that my students will learn not only how to use their calculators to solve calculus problems, but equally important, *when* to use them. To this end, I have both calculator-active and noncalculator sections in all of my in-class tests.

When teaching a section of Calculus I for students who have not seen any calculus previously, I try to keep several things in mind:

- These students will, for the most part, not be the most talented mathematically or they would have taken calculus in high school. Therefore, I have to be careful not to go too fast or to make unreasonable assumptions about their precalculus knowledge and skills.

- Most of these students are in their first or second semester at Davidson and are still adjusting to the social and academic differences between high school and college. In particular, most of them are not ready to assume primary responsibility for their own learning. Therefore, for example, they need daily homework assignments with the incentive of credit toward their final grades.

- For many of the students in the course, Calculus I will be the last mathematics course they take. Less than half of them will go to on to take Calculus II. Therefore, I have to be sure that we do some inherently interesting and clearly worthwhile things in Calculus I, since saying "You'll see why this is important when you get to Calculus II" is not much motivation to a student who plans to throw in the mathematical towel after Calculus I.

- The students, my departmental colleagues, and Davidson College all expect me to offer a course that is rigorous and challenging and thereby consistent with the high standards of the institution. Therefore, I need to set expectations that I know some students will not be able to meet unless they work very hard and come to me for extra help. Not all students will do so. Consequently, almost certainly some students will do poorly in the course.

When I teach Calculus I to students who have had previous exposure to calculus, all of the above items, except for the first, apply. When teaching these students, however, I assume that they have both talent and an interest in mathematics, knowing full well, of course, that this assumption is not true for some of the students. I also assume that they will know how to do at least simple symbolic manipulations for differentiation and perhaps also for integration. Thus, I spend much less time discussing and giving routine examples of differentiation formulas. Instead, I focus on more challenging examples and applications and, in some cases, on the underlying theory. I also take this approach in assigning homework and in designing tests. I assign very few routine problems in the differentiation sections and ask higher-level questions on tests and problem sets.

Philosophy of the Department of Mathematics

The approach to teaching Calculus I by other members of my department is similar to mine. We all believe that students learn mathematics by doing mathematics, so all Calculus I students are required to turn in daily homework assignments and also work on a variety of special assignments. Some instructors give more tests than I do and also use other types of tests.

We all believe that Calculus I students should be able to work both routine and nonroutine problems that use calculus-based techniques and have an appreciation for the underlying analytical theory. We therefore emphasize problem solving but do not neglect topics like mean value theorems for derivatives and for integrals.

We all believe that students should learn how to use appropriate technology as they learn calculus. The number of students taking Calculus I in the fall makes it impossible to require students to use *Mathematica* (the campus standard computer algebra system). Instead, we require students to use a graphing calculator and give them assignments that ensure that they do so. In the interest of uniformity, we require students to use a TI-89 graphing calculator. Some students are able to get by with a less powerful machine, but the vast majority of students, when informed of the advantages offered by the TI-89, decide to get one of their own.

Finally, we also believe that students need the opportunity to work with instructors in informal settings outside of the classroom, and hence we encourage students to take advantage of our generous schedule of office hours.

Class Profile

As noted above, Davidson College operates on a traditional semester calendar. Monday-Wednesday-Friday courses have three 50-minute meetings per week, and Tuesday-Thursday courses have two 75-minute meetings per week. Calculus I is most often offered in the Monday-Wednesday-Friday format, but there are typically one or two Tuesday-Thursday sections in the fall semester.

The vast majority of students in Calculus I are first-year students, and the course is designed with first-year students in mind. Many of the students who enroll in Calculus I have taken calculus previously, often in AP Calculus courses. Some of these students will actually have earned AP credit for Calculus I but will forfeit this credit and take Calculus I at Davidson. In an attempt to better serve students with no previous exposure to calculus, each fall the Department of Mathematics offers two sections of Calculus I for students with "no previous experience" in calculus, the so-called X sections. Students who have taken a course in high school that contains any calculus at all are not eligible to enroll in either of these specially designated sections at Davidson.

Calculus instructors in the non-X sections assume that their students have seen some calculus before and feel free to cover the simpler aspects of the course a little more rapidly. Of course, the amount of prior exposure to calculus varies widely among the students in these courses. Some will have seen only a little differential calculus, while others will have taken a strong AP Calculus course and perhaps have earned a creditable grade on the AP Exam.

Course Prerequisites

Davidson College requires three units of secondary mathematics for admission and strongly encourages students to take more mathematics in secondary school. The understanding is that the last of the three units would be a precalculus course of some sort. Thus, any student enrolled at Davidson is eligible to enroll in Calculus I. There is no placement test for enrollment in Calculus I; students who are not certain that they are ready for Calculus I are encouraged to consult with a member of the Department of Mathematics to determine their readiness for the course.

Note that Davidson does not offer a precalculus course. Therefore, a student who wants to start calculus at Davidson and does not have an appropriate background in mathematics must take a precalculus course at another institution.

Course Overview

The only textbook used in the course is *Calculus: Early Transcendentals* (7th ed.) by Howard Anton, Irl Bivens, and Stephen Davis. We use this book for Calculus II and part of our third semester calculus course as well. In Calculus I, we cover essentially all of chapters 1 through 6 and selected topics from chapter 7; thus, we cover all of the standard topics in differential calculus for polynomial, logarithmic, exponential, trigonometric, and inverse trigonometric functions. We also define the definite (Riemann) integral, discuss and apply the two Fundamental Theorems of Calculus, and treat a selection of applications of the definite integral including area and volume.

Our Calculus I course covers less material than a Calculus AB course. Specifically, we do less with applications of the integral and do not cover separable differentiable equations. We do, however, say a little about slope fields.

Course Planner

The following table presents the sections in the textbook covered for each week of the course and a description of the topics covered.

Week Number	Sections Covered	Topics Discussed
1	2.1–2	Limits—informal
2	2.3–4	Limits—formal
3	2.5–6 and 3.1	Continuity, rates of change
4	3.2–4	Definition of derivative and derivative formulae
5	3.5–7	Chain rule, implicit differentiation, related rates
6	3.8 and 4.1	Differentials, inverse functions
7	4.2–4	Exponential, log, and inverse trig functions
8	4.5 and 5.1	L'Hospital's Rule, derivatives and graphing
9	5.2–3	First and Second derivative tests
10	5.4–6	Rectilinear motion, max-min problems
11	5.7–8 and 6.1–2	Newton's method, Mean Value Theorem, the area function, slope fields
12	6.3–4	U-substitutions, the definition of the definite integral—introduction
13	6.5	Definition and properties of the definite integral
14	6.6	Fundamental Theorems of Calculus, Thanksgiving
15	6.7–8 and 7.1	Applications of the definite integral

Student Evaluation

I give three in-class, full-period tests and a three-hour final examination. Each in-class test has both a calculator and a noncalculator section. The in-class tests constitute 42 percent of the final course grade, and the final exam constitutes 28 percent of the final course grade.

I assign nine so-called pledged problems during the semester. I try to avoid giving a pledged problem in a week when one of the in-class tests is scheduled. Examples are given in the Student Activities section below. Students typically have a week to work on each of these problems, and the pledged problems constitute 20 percent of the final grade for the course. I allow, and indeed encourage, students to work with a partner on some of these assignments. These problems are meant to stretch students. Although they cannot get help from one another unless they have a partner, they can get (limited) help from me.

The remaining 10 percent of the final grade for the course comes from the daily homework assignments (typically about 35). I allow students to get as much help as they need from whomever they choose as they work on these assignments; daily homework does not count heavily toward the final grade, even though, fully and carefully done, it represents a good bit of work.

Teacher Resources

Anton, Howard, Irl Bivens, and Stephen Davis. *Calculus: Early Transcendentals*. 7th ed. New York: John Wiley & Sons, 2003.

Other than this book and its accompanying answer book, I do not use any other resources when I teach Calculus I. I generate all of the handouts, special problem assignments, etc., myself.

Student Activities

Below are two of the "pledged problems" that I assigned in a section of Calculus I that I recently taught. It was a section for students who had not had calculus before. The problems give the flavor of what I expect from these students.

Third pledged problem

1. A function, *f*, is defined on the interval [-4,4] in such a way that its graph consists of four line segments. The first has endpoints (-4,6) and (-2,2), the second has endpoints (-2,2) and (0,0), the third has endpoints (0,0) and (2,2), and the fourth has endpoints (2,2) and (4,6).
   a. Sketch the graph of the function *f*.
   b. The function is not differentiable at $x = 2$. Explain why not. Your explanation can be graphical and/or algebraic in nature. [The function also fails to be differentiable at $x = -2$ and $x = 0$ for essentially the same reason.]
   c. Find $f'(x)$ for each value of x for which f is differentiable.
2. Consider the function $g(x)=\begin{cases} -x^2 & \text{if } x<0 \\ x^2 & \text{if } x\geq 0 \end{cases}$. Is this function differentiable at $x = 0$? Be sure to provide a graphical and/or algebraic justification for your answer.
3. Consider the graph of $y=x^2$. Let T_1 be the line which is tangent to the graph at the point (a,a^2), and let T_2 be the line which is tangent to the graph at the point (b,b^2), where $a < b$. Determine the coordinates of the point of intersection of T_1 and T_2. (Each coordinate will depend on both a and b.) [NOTE: If you cannot solve the problem at the given level of generality, you can take a five-point penalty and solve it with $a = -2$ and $b = 4$. However, before you opt for the penalty and the simpler version of the problem, be sure that you know how to use your calculator to solve two equations in two unknowns. It can solve the equations you need to solve in the general case.]

Fourth pledged problem

1. The equation $xy+y^2=14$ defines y as a differentiable function of x at the point $x = 5$, $y = 2$.
   a. Use implicit differentiation to find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at the point $x = 5$, $y = 2$.
   b. Solve the given equation for y in terms of x using the quadratic formula, your TI-89, or some other method. Be sure, though, to pick the right sign for the square root in the solution you choose, i.e., the sign that gives $y = 2$ when $x = 5$. By solving for y, you are determining y as an explicit function of x.
   c. Check your answer to part a) by finding the first and second derivatives of the explicit function of x obtained in part b) and evaluating these derivatives at $x = 5$.
2. As a mothball evaporates, it maintains a spherical shape with a shrinking radius. [Recall that the volume and surface area of a sphere with radius r are given by $V=\frac{4\pi r^3}{3}$ and $A=4\pi r^2$, respectively.]
   a. Suppose that at a particular instant the length of the radius of a mothball is 3/2 = 1.5 inches and this length changes at a rate of -1/4 = -0.25 inch per day. At what rate does the mothball's volume change at this instant?
   b. Suppose now that the volume of another evaporating mothball changes at a rate that is proportional, with a fixed constant of proportionality, to the surface area of the mothball. (i) Show that the radius of the mothball changes at a <u>constant</u> rate. (ii) Find the constant rate at which the radius changes if the volume of the mothball changes at a rate of -1/8 = -0.125 cubic inches per day at the instant when the radius of the mothball is 3/4 = 0.75 inches.

Syllabus 6 (Calculus I/Calculus II)

James Peters
Weber State University
Ogden, Utah

University Profile

School Location and Environment: Weber State University is a four-year institution of higher education located in Ogden, Utah. WSU offers over 200 certificate and degree programs—the largest and most comprehensive undergraduate program offering in the state of Utah. Although the student body of over 18,000 is drawn predominantly from the Wasatch Front, it includes students from 48 states and 46 foreign countries. Founded in 1889, Weber State University has grown to include nearly 50 departments and programs in seven colleges. A leader in online instruction, WSU offers 10 online degree programs that serve over 6,000 students from all over the world.

Personal Philosophy and Philosophy of the Department

To provide students with a solid foundation in the fundamental notions of calculus, I believe that conceptual understanding, as well as a mathematically rigorous treatment of the subject, is central to the course. Because homework is so important to student success, I do spend a sizable proportion of time going over homework problems that students suggest; however, most of the class time is spent presenting new material. Whatever I am doing, I always try to involve as many students as possible by asking for and encouraging as much input as is practicable. Of course, all groups are different, and while sometimes I need to rein in certain students, other times I need to coax them to participate. In general, though, I believe that the more I can get students to do their own independent thinking during class, the more likely they will continue to think for themselves outside of class.

I also believe that whether we like it or not, students are coming to us already exposed to current technologies. We would be remiss, therefore, if we did not attempt to instruct them in the proper and improper uses of these technologies. At WSU, there are two aspects to any discussion of technology. Almost all students taking calculus already have a graphing calculator. Thus, in my sections of Calculus I, I require students to have a graphing calculator, and I strive to constantly demonstrate its proper use. On the other hand, because we want students in courses beyond calculus to have a working knowledge of a computer algebra system (CAS), we recently instituted a *Mathematica* laboratory course (Math 1100) as a corequisite for Calculus I. Thus, as often as I am able, I demonstrate the usefulness of *Mathematica*. As a basis for all this attention to technology, I have handouts that illustrate the basic operations of the various graphing calculators, and students are expected to be able to use them effectively.

Class Profile

Both Calculus I and Calculus II are offered each semester in the Department of Mathematics. In a typical year, we enroll approximately 350 students in Calculus I and 225 students in Calculus II. The average class size is approximately 25. The semester is 15 weeks in length, and each class meets the equivalent of four times a week for a 50-minute period. Each class is worth four semester credit hours.

Math 1100 is a one-credit hour course that consists of several projects covering an entire spectrum of topics that students work through and complete independently. Thus, after students complete Calculus I, we assume that they are familiar enough with *Mathematica* that we can routinely make assignments that involve its use.

Course Prerequisites

Students arrive in a calculus course via many avenues. Some have completed the prerequisite courses of college algebra and trigonometry, and some have taken precalculus. Others might have placed into Calculus I by means of a score of 75 or more on a COMPASS placement test, or a grade of 3 or more on the AP Calculus AB Exam. Some students go straight to Calculus II by means of a grade of 4 or 5 on the AP Calculus AB Exam or a grade of 3 or 4 on the AP Calculus BC Exam.

Course Overview

The main objective of the course is to provide students with a solid foundation in the fundamental notions of calculus. A secondary objective is to provide guidance in the appropriate use of technology in the pursuit of the main objective.

Course Planner

For the calculus sequence at WSU, we currently use *Calculus* (5th ed.) by James Stewart as our primary text. Each section in the course planner is covered in one or two class periods. Chapters refer to the primary text.

Calculus I

Chapter 2

1. The Tangent and Velocity Problems
2. The Limit of a Function
3. Calculating Limits Using the Limit Laws
4. The Precise Definition of a Limit
5. Continuity
6. Tangents, Velocities, and Other Rates of Change

Chapter 3

1. Derivatives
2. The Derivative as a Function
3. Differentiation Formulas

5. Derivatives of Trigonometric Functions
6. The Chain Rule
7. Implicit Differentiation
8. Higher Derivatives

9. Related Rates
10. Linear Approximations and Differentials

Chapter 4

1. Maximum and Minimum Values
2. The Mean Value Theorem
3. How Derivatives Affect the Shape of a Graph
4. Limits at Infinity; Horizontal Asymptotes
5. Summary of Curve Sketching
6. Graphing with Calculus and Calculators
7. Optimization Problems
9. Newton's Method
10. Antiderivatives

Chapter 5

1. Areas and Distances
2. The Definite Integral
3. The Fundamental Theorem of Calculus
4. Indefinite Integrals and the Total Change Theorem
5. The Substitution Rule

Chapter 6

1. Areas between Curves
2. Volumes
3. Volumes by Cylindrical Shells
4. Work
5. Average Value of a Function

Calculus II

Chapter 7

1. Inverse Functions
2. The Natural Logarithmic Function
3. The Natural Exponential Function
4. General Logarithmic and Exponential Functions
5. Inverse Trigonometric Functions
7. Indeterminate Forms and L'Hospital's Rule

Chapter 8

1. Integration by Parts
2. Trigonometric Integrals
3. Trigonometric Substitution
4. Integration of Rational Functions by Partial Fractions
5. Strategy for Integration
6. Integration Using Tables and Computer Algebra Systems
7. Approximate Integration
8. Improper Integrals

Chapter 9

1. Arc Length
2. Area of a Surface of Revolution
3. Applications to Physics and Engineering

Chapter 11

1. Curves Defined by Parametric Equations
2. Tangents and Areas
3. Arc Length and Surface Area
4. Polar Coordinates
5. Areas and Lengths in Polar Coordinates
6. Conic Sections

Chapter 12

1. Sequences
2. Series
3. The Integral Test and Estimation of Sums
4. The Comparison Tests
5. Alternating Series
6. Absolute Series
7. Strategy for Testing Series
8. Power Series
9. Representations of Functions as Power Series
10. Taylor and Maclaurin Series
11. The Binomial Series
12. Applications of Taylor Polynomials

Teaching Strategies

In the interest of the most efficient and effective use of class time, I work all the problems that are worked at the board. That being said, however, I also make a point of rarely writing anything down that is not first suggested by a student. Thus the problem-solving portion of my class often seems a bit unruly. Within reason, I tend to like it that way and strive for the most "ordered chaos" that I can achieve with an eye toward constructive, "on-task" discussions. During the presentation of new material, the class is certainly much less unruly, but I still encourage students to lead me in the right direction.

I try to use technology, especially the graphing calculator, as often as possible. I find that I can do virtually everything that I need to illustrate both the graphing calculator and the CAS technologies with my old TI-92 and overhead view screen. When I use the CAS capabilities of the TI-92, I usually write the equivalent *Mathematica* syntax on the board. Thus, I actually demonstrate with *Mathematica* relatively rarely.

Lab Component

As I have mentioned, at WSU we have a separate *Mathematica* laboratory course that is a corequisite for Calculus I. During this lab, the students independently work through and complete several projects covering an entire spectrum of topics.

Student Evaluation

Approximately every four weeks, we have an in-class, hour-long exam consisting of roughly eight free-response type questions on the main ideas of the material that has been covered most recently. These exams constitute 50 percent of the student's final grade. Each week without an exam, we have another assignment. Usually this is a 20-minute quiz consisting of two routine homework problems; sometimes it is a hand-in set of homework problems, and other times a technology assignment involving a graphing calculator or CAS. All of these assignments together comprise 25 percent of the student's final grade. At the end of the course, there is a comprehensive final exam consisting of roughly 14 free-response type problems. This final exam counts for the remaining 25 percent of the student's final grade. The student's point totals are then compared to a standard 90 percent = A, 80 percent = B, 70 percent = C, 60 percent = D grading scale to determine the final grade.

Teacher Resources

The only resources I typically use for my calculus course are the aforementioned textbook (see below) and accompanying student solutions manual; the *AP Calculus Teacher's Guide*; the handouts that I have prepared covering the basic operation of each available model of graphing calculator; and the handout that I have prepared covering the basic operation of the *Mathematica* CAS.

Stewart, James. *Calculus.* 5th ed. Pacific Grove, Calif.: Brooks Cole Publishing Company, 2003.

Chapter 4
The AP Exams in Calculus AB and Calculus BC

Exam Format

Like all AP courses, AP Calculus prepares students for a national assessment administered in May each year. The AP Calculus courses are meant to be equivalent to college-level calculus courses. Students can earn college credit or placement in an advanced class by scoring sufficiently well on the AP Exam.

Students in calculus have the option of taking the Calculus AB or the Calculus BC Exam. In keeping with the characterization of Calculus BC as an extension rather than an enhancement of AB, students who take the BC exam receive an AB subscore grade that measures their performance on AB topics alone (approximately 60 percent of the exam). This subscore grade can be helpful to colleges in making decisions about credit, placement, or both. It is recommended that colleges and universities apply the same AP policy to the AB subscore grade as they do to the Calculus AB Exam grade.

The AP Calculus Exams consist of two main sections: 45 multiple-choice questions in Section I and 6 free-response questions in Section II. Sections I and II are equally weighted when calculating a student's composite score, which is then translated into an AP grade of 1–5.

Both of these sections are further divided into parts on which calculators are or are not allowed. Part A of the multiple-choice section consists of 28 questions in 55 minutes, and calculators are not allowed. For Part B of the multiple-choice section, which consists of 17 questions in 50 minutes, a graphing calculator is required for some questions. All 45 multiple-choice questions are equally weighted, and a deduction is taken for incorrect answers. Part A of the free-response section consists of 3 questions in 45 minutes, and a graphing calculator is required for parts of questions. Part B of the free-response section consists of 3 questions in 45 minutes, and a calculator is not allowed. During the administration time for Part B, students may also work on the questions in Part A without the use of a calculator.

Important Exam Instructions

The timing and mechanics of the exams are complicated and should be communicated to students well before the administration. An incorrect answer on a multiple-choice question results in a deduction of one-fourth of a point. As a consequence, if a student can eliminate one or more of the choices as definitely incorrect, it is to the student's advantage to guess from among the remaining choices.

The instructions that students are to follow when presenting their work on the free-response section appear in the *AP Calculus Course Description*, as well as on AP Central. A more extensive commentary from the Development Committee expanding on those instructions is available at AP Central. It contains useful information that teachers should communicate to students and review just before the exam. A good portion of this commentary addresses the presentation of student work. The goal is to get students to present their work in a way that another person can easily understand what they are trying to say. Writing about mathematics has received increased emphasis in recent years, as technology has removed some of the computational drudgery. Looking at a few parts of Section II free-response questions that have appeared on recent AP Calculus Exams, the importance of communication becomes clear:

- "Justify that your answer is an absolute minimum." (1998 Calculus AB, question 2, part (b))
- "Using correct units, explain the meaning of this integral." (1998 Calculus AB, question 3, part (d))
- "Using correct units, explain the meaning of your answer in terms of water flow." (1999 Calculus AB/BC, question 3, part (a))
- "Use the slope field for the given differential equation to explain why a solution could not have the graph shown below." (2000 Calculus BC, question 6, part (b))
- ". . . explain the meaning of $H(17)$ and $H'(17)$ in the context of the amusement park." (2002 Calculus AB/BC, question 2, part (c))
- ". . . explain the meaning of $\int_0^b R(t)dt$ in terms of fuel consumption for the plane." (2003 Calculus AB, question 3, part (d))
- "Explain the meaning of $\frac{1}{b}\int_0^b R(t)dt$ in terms of fuel consumption for the plane." (2003 Calculus AB, question 3, part (d))

Beginning with the 2005 exams, sign charts by themselves are not accepted as a sufficient response when a free-response question asks for a justification for the existence of either a local or an absolute extremum of a function at a particular point in its domain. For more information, see "On the Role of Sign Charts in AP Calculus Exams" on the Calculus AB or Calculus BC home pages on AP Central.

Preparing Students

Adequate preparation for the AP Exams consists primarily of delivering a solid course during the school year. However, successful AP teachers also recognize the importance of dedicating a significant amount of time before the exam for review and exam preparation. Teachers develop their own strategies for this part of the course, but setting aside a substantial amount of time is crucial. My own target is a minimum of three to four weeks of review.

> The best tip I can give for an AP class is to have an ongoing review of the major topics in the course. The way that I accomplish this is by using "old" AP free-response questions that are available on AP Central. In this way, my students write solutions to questions from previous AP Exams throughout the year. These questions are suited to this purpose because they incorporate ideas from various parts of the course. The only thing you need to verify is that the material has been covered in your class by the time you ask the students to solve the problem. The specifics of how you do this will vary from course to course; however, in my class the students hand in solutions to two problems each week from October until April. I score the papers as they would be scored at the AP Reading and have them count for 10 percent of the term grade (the scoring guidelines for all the free-response questions in recent years are available on AP Central).
>
> —Steve Olson, Hingham High School,
> Hingham, Massachusetts

The tone of my own classes during this review time is decidedly positive and encouraging. During the year, I try to build a relationship of collaboration with my students, and we all face the same exam at the end of the course. Since none of us knows what questions will appear on the exam, the teacher becomes an ally of the students, and the feeling is one of teamwork directed toward a common goal. At the outset of the review period, I make a point to deliver a pep talk, conveying my belief that every student is fully capable of performing well on the exam. Since review is so critical for success, I urge them all to work hard and stay focused during that time.

During the review period, most of the students' time is devoted to working on problems from past AP Calculus Exams (both free-response and multiple-choice questions). Problems are assigned as homework, as in-class group work, and on tests and quizzes. A vast supply of good questions is available (see chapter 5 for details). The 1998 AP Calculus Exams can still be purchased in their entirety, and the *2003 AP Calculus AB and Calculus BC Released Exams* book was published in 2005. In addition, complete exams were also released in 1969, 1973, 1985, 1988, 1993, and 1997. The College Board offers the *AP Calculus Multiple-Choice Question Collection 1969–1998* for purchase at store.collegeboard.com.

> Students realize that the stakes are high on any AP Exam. Anxiety can be minimized for most students by getting a couple of entire exams under their belts before sitting for the real exam and getting grades on at least one practice exam that are reflective of their ability. The 1998 and 2003 Released Exams include worksheets that help students compute their AP grade for the exam using the 1 to 5 scale. This helps students get a feel for the kind of raw score that must be attained to get a passing AP grade.
>
> The 1998 AP Calculus Exams each take 180 minutes of testing time, and the 2003 exams were 195 minutes long; these times are impossible to duplicate in most school situations. Fortunately, the exams can be split into 45-minute segments that can be done separately: noncalculator multiple-choice questions, calculator multiple-choice questions, free-response questions 1–3, and free-response questions 4–6. During the end-of-the-year review, I expect my students to finish a segment of an exam each night for homework, which frees up class time for explaining multiple-choice questions, grading free-response questions using the scoring guidelines, and talking about strategy. I also ask students to do each exam segment in one 45-minute sitting with no distractions. Unfortunately this method does not allow students to experience the fatigue that goes along with a three-hour test.
>
> No three words have ever struck as much terror in the hearts of AP Calculus students as "justify your answer." Some students assume that the justification is "in their work somewhere" and happily go on to the next problem. Other students feel the need to write lengthy paragraphs that amount to explaining basic calculus concepts to the Exam Reader. I have found that having students grade their own free-response questions using the actual scoring guidelines (included in the Released Exams and also found on AP Central) is an effective way for students to learn how to justify their answers completely and efficiently. Having students get familiar with some of the scoring guidelines also helps illustrate two other main principles of AP scoring: that "bald answers" (answers without any accompanying work or setup) typically get no points, and that it is quite possible to win points on one part of a free-response question even though mistakes are made on an earlier part of the same question.
>
> —Michael Grasse, Elk Grove High School,
> Elk Grove Village, Illinois

My own plan for review usually includes a topic-targeted run through all of the exam topics. That is, we devote a day or two to each of the major content areas: limits and continuity, idea and definition of derivative, techniques and applications of the derivative, idea and definition of the integral, evaluating integrals and applications of the integral, and differential equations. For BC students, we devote time to series and parametric, polar, and vector functions as well. I don't try to reteach the material at this time but rather to give a broad perspective, pointing out the highlights and likely types of exam questions. This run-through of the content lasts at least 10 days (or more for BC). I distribute many, many questions, both multiple choice and free response, that fall within each content area. We spend the remaining review time before the exam working on problems that are not targeted at specific content. I also plan one major review test (usually administered over two or more days).

A topic that appears frequently on the free-response section of the Calculus BC Exam is "Motion Along a Curve." Teachers frequently ask to what extent vectors need to be taught for students to do well on the motion questions. An extensive unit on vector analysis is not necessary, but the concept that a particle is moving both in the *x*-direction and *y*-direction as it moves along the curve needs to be understood. I usually find that if you teach motion along a curve parallel to teaching motion along a line, the students find the topic easy to understand and do quite well on this topic on the AP Exam.

To start, I build on the students' knowledge of parametric equations by taking a set of parametric equations and making the $x(t)$ and the $y(t)$ the two components of a vector, usually written as $s = \langle x(t), y(t) \rangle$ where *s* is the position of the particle on the curve at any time "*t*". We then define the absolute value (magnitude) of a vector using the Pythagorean Theorem as $\|s\| = \sqrt{(x(t))^2 + (y(t))^2}$.

Now I have the students remember their work with particle motion along a line and recall that we found the velocity, acceleration, speed, and distance traveled. We now need to parallel their earlier work while investigating motion along the curve. On a line, the velocity was $x'(t)$, and on a curve we have the velocity vector $\langle x'(t), y'(t) \rangle$ The acceleration along a line was $x''(t)$, and the acceleration vector is $\langle x''(t), y''(t) \rangle$. The speed along a line was the absolute value of the velocity $|v(t)|$, and along a curve the absolute value of the velocity vector $\|v(t)\| = \sqrt{(x'(t))^2 + (y'(t))^2}$. Finally, the distance a particle travels along a line is the integral of the speed $\int_a^b |v(t)|\, dt$, and the distance traveled along a curve is also the integral of the speed $\int_a^b \|v(t)\|\, dt = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$, which is the same as the formula for arclength in parametric equations.

We conclude this unit by working several AP free-response questions on this topic, thus showing students that they have everything they need to succeed on these questions on the AP Exam.

—Tom Becvar, St. Louis University High School, Missouri

Some teachers administer practice exams outside of actual class time. Nancy Stephenson of William P. Clements High School in Sugar Land, Texas, joins with other calculus teachers in her district to offer a practice exam on a Saturday morning shortly before the actual AP Exam is administered (see the AP Review section of Nancy's syllabus in chapter 3).

With the variety of schedules in the lives of my students, I have to have a variety of ways to prepare them for the exam. Of course, the first way is to occasionally include old AP questions in my tests throughout the year so that students get used to the language, format, and scoring. However, as we near the exam, we begin to prepare about a month in advance while still finishing any relevant material that may need to get done. We alternately use old multiple-choice sections and free-response sections as take-home quizzes or tests. In late April we also hold a practice exam at a local school in the area and invite all students. Our local AP Calculus teachers' association puts this on every year and hundreds of students attend. At the start of every year, the teachers agree not to use the latest released multiple-choice and free-response questions in class so that they can be used for the practice exam, ensuring that they will be fresh for students. After the practice exam, we go over the key with the students and let them score their own exams so they have a better idea of where they stand. In the final two weeks before the exam, I hold about three to four evening sessions for the students who want more practice (serving pizza greatly increases attendance). At these sessions, we fine-tune our technology (calculator) skills, review justification responses, and work on what students identify as their weakest content areas. Results are always good after I implement these several techniques for preparation.

—Joe Milliet, St. Mark's School of Texas, Dallas

You will find more tips that apply to the actual exam at AP Central. One of the most important is making sure students are well rested and well fed on the day of the exam. Last-minute, late-night cramming can be counterproductive.

Important Reminders from the Development Committee and Chief Reader

Here are some things to remember as you prepare students for the AP Calculus Exams. Some of these points have been mentioned before, but they are things that cannot be overemphasized.

- Remind students of the importance of showing their work. Answers without supporting work may not receive credit.
- Students should have their calculators in **radian mode** before they begin the exam.
- The instructions for the free-response section are important to share with students prior to the exam date. On Part A, students may use their calculators to (1) graph a function; (2) numerically solve an equation; (3) numerically compute the value of a derivative at a point; and (4) numerically calculate the value of a definite integral. A student can freely use a calculator for any of these purposes without showing any intermediate work, as long as the student clearly indicates what the calculator is used for (often referred to as the “setup”). This means (a) labeling the function and the scaling for a graph sketched from the calculator; (b) stating what equation the calculator solver is used for; (c) stating the function and the point at which the numerical derivative was calculated; and (d) stating the definite integral that has been calculated. Note, however, that while a student can sketch a graph from one obtained on a calculator, the graph should not be used as the basis of a mathematical justification.
- The AP Calculus Exams have moved away from rote manipulation toward problems that probe understanding of fundamental concepts. The exam is less predictable than it used to be.
- It is important for students to practice mathematical writing skills to help communicate their reasoning and explanations to the Readers who will be scoring their answers.
- Students should have experience throughout the year with justifying conclusions using mathematical (calculus) arguments, such as use of the First Derivative Test or Second Derivative Test.
- Emphasize an approach to calculus that includes functions represented in graphical, tabular, and analytic form, and the connections between these representations.
- Students should try **each part** of **each** free-response problem. If students feel that they can’t “get into” part (a) of a problem, tell them to go on to part (b) and try to start from there.

Share these tips with students

Use the Calculator Effectively on the AP Calculus Exams

- Set the calculator to radian mode.
- On free-response problems, report decimal approximations to three decimal places after the decimal point.
- Be proficient with the four expected calculator capabilities:
  - Graphing a function
    Use information in the question to help determine the viewing window.
  - Finding zeros of functions (solving equations numerically)
    Be comfortable with the graphical solver (using intersect, root/zero commands) and equation solver capabilities.
  - Numerically calculating the derivative of a function at a point
  - Numerically calculating the value of a definite integral
- On the calculator portions of the exam, use the calculator to evaluate definite integrals and numerical derivatives. Show the "setup" in free-response problems.
- Watch parentheses carefully when entering functions.
- Store functions in the "Y=" menu to avoid entering functions multiple times.
- If a point of intersection or zero is needed for a calculation (derivative or integral), store that value into a variable. Use the variable in the subsequent calculation.
- Tracing along a graph to find a point of intersection or zero might not produce the required accuracy.
- Calculations from the graph screen (derivatives or integral values) might not produce the required accuracy.
- A justification requires a mathematical (noncalculator) argument.
- Use standard mathematical notation, not calculator syntax, on the exam.

After the Exam

Every May, a discussion develops on the calculus electronic discussion group concerning what to do with students after the AP Exam and before the end of school. You can search through the archives of the EDG to look at the entire discussion, or you can visit AP Central for a subset of these ideas. Click on the Calculus AB or Calculus BC home page and look under "Teaching Resource Materials."

On the first day I see my students after the exam, the most important question I ask them is whether there was anything on the exam for which they felt unprepared. I want to know where the preparation may have been inadequate so I can address the issue with future classes. There are limitations, though, on the types of discussions you can legally have with your students (see below).

Multiple-Choice Questions

AP teachers should take the opportunity to discuss the exam with their students, but the focus should be on a general discussion of topics on the exam rather than on individual questions. Begin any postexam discussion by reminding students of their agreement not to disclose specific multiple-choice questions. Students sign a statement to said effect on their multiple-choice answer sheets before they take an AP Exam.

It is perfectly appropriate to ask students how well the course prepared them for topics *X, Y, or Z* on the exam, and to discuss areas where students felt especially prepared or unprepared. Avoid asking questions, however, that would lead students to discuss actual exam questions.

The AP Program reuses a set of multiple-choice "equating" questions each year. These come from a previous AP Exam—not necessarily from the most recent one. (For more detail about the process, see the "Exams" section on AP Central.) If exam questions are disclosed, the AP Program can no longer use those questions in the future. This creates a potential threat to the quality of the scoring process and the fairness and reliability of AP grades.

Free-Response Questions

The free-response questions can be discussed in their entirety once they are posted on the AP Central Web site 48 hours after each administration. If, however, your students take the AP Exam during a late-testing administration (usually the week after the regular two-week AP Exam period), those free-response questions remain secure and are not disclosed; you are not permitted to discuss those specific questions with students.

After the exam, some teachers show their students the film *Stand and Deliver,* which chronicles the success of a group of AP Calculus students from a Los Angeles high school. It's available in VHS and DVD formats.

The best activity that I have come up with after the AP Exam is having students study a new topic. I have students work in groups of two or three. They pick a topic from a list of projects that includes (but is not limited to) integration by parts, integration by partial fractions, integration by trig substitution, integrals with powers of trig functions, arc length of a curve, surface area of a solid of revolution, force and work, partial differentiation, multiple integrals, polar coordinates, Taylor series, Euler's method, improper integrals, and hyperbolic functions. I teach an AB course, so these topics give the students a feel for some of the BC topics or some of the other topics that they will encounter in a second or third calculus course in college. Students have several days in class to study and discuss the topic with their group. They are required to write a report on the topic and include an introduction, several examples, and an assignment of at least three problems. They may use our text and any other books that I have in my calculus library in the classroom. Groups then make oral presentations in which they teach their topic to the rest of the class and give the assignment that they have constructed. A portion of their grade for the project is how successful they have been at teaching the class. The project is graded on the written report, the oral presentation, the class responses to the assignment, and effort during the work sessions.

If time is short after the AP Exam, I have variations on this project that do not include teaching it to the class. One variation is to have each student take a quiz on his/her topic after the report is completed. This ensures that all students have done their part in the group work and usually eliminates any slackers.

Another group project is one I found in the May 2002 *Mathematics Teacher* called "Design a Window." It is a nice group activity, requiring three or four class periods, which starts out easy and ends up requiring some good thinking skills. It is a more relaxing, but still academic, project if you only have a few days following the AP Exam.

For many years I have worked with the AP Physics teacher on a field trip to Cedar Point (an amusement park). Many amusement parks have similar days, when students are encouraged to come and analyze the rides at the park. At our school, we have turned this day into a "Physics and Calculus Day." I have written problems requiring students to determine the length of a hill on a coaster (arc length), the cost of building a hill on a coaster (area and/or volume), the number of people in a queue or in the park (see free-response question AB/BC 2 from the 2002 Calculus Exams), the area or volume of a pond at the park, etc. This can be a very fruitful exercise, but it requires preparation to determine the problems you'd like to elaborate for your students.

Whatever activity you plan for the time following the AP Exam, I recommend that you make it academic. It should be a little fun, a change of pace from the regular routine, and not quite as demanding as all the work leading up to the exam. The projects that I plan are always a portion of the students' grades for the last marking period.

—Martha Montgomery, Fremont Ross High School,
Fremont, Ohio

College Board Questions and Answers About the AP Exam

- **Who is eligible to take the AP Exam?**

  All students are eligible to take the AP Exam. The College Board strongly discourages the use of practices or policies that restrict students from participating in the AP Program or taking the AP Exam. See the Equity and Access section in at the beginning of this Teacher's Guide.

- **What is the role of the AP Coordinator?**

  Each participating school designates an AP Coordinator who takes primary responsibility for organizing and administering that school's AP program. The AP Coordinator may be a full- or part-time administrator or counselor, or a faculty member who is not teaching an AP course. AP Coordinators manage the receipt, distribution, administration, and return of AP Exam materials.

- **How do I work with this person in my new role as an AP teacher?**

  AP teachers and the AP Coordinator work closely together throughout the academic year. Early in the spring, AP teachers consult with the Coordinator to help determine the correct number and type of exams that need to be ordered. During the exam administration weeks, Coordinators may designate AP teachers to serve as proctors for exams in a subject area other than the one they teach.

  Coordinators are the bridge between AP teachers, students, and administrators, and the AP Program. Questions about exam fees, dates and deadlines, and exam-specific policies such as the calculator policy should be directed to the AP Coordinator.

- **How is the AP Exam administered? How should an AP teacher work with the school's AP Coordinator?**

  AP Exams are administered worldwide in May. To avoid any conflict of interest, the actual administration of the exams is performed by the Coordinator and designated proctors. However, in the months leading up to the exams, AP teachers can assist Coordinators with tasks such as the collection of exam fees from students. Teachers can help their students by familiarizing them with the format and timing of the exam.

- **What should I expect after the exam (e.g., grades, reports, transcripts, etc.)?**

  AP grades are reported to students, their schools, and their designated colleges in July. Each school automatically receives an AP Grade Report for each student, a cumulative roster of all students, rosters of all students by exam, an AP Scholar roster for any qualifying students, and a *AP Instructional Planning Report*. (Note: Data for students testing late with an alternate form of the exam are not included in this report.) For a fee, schools may also request their students' free-response booklets.

  Schools receive the *AP Instructional Planning Report* for each of their AP classes in September. The report compares your students' performance on specific topics in the AP Exam to the performance of students worldwide on those same topics, helping you target areas for increased attention and focus in the curriculum. To get the most out of the report, please read the interpretive information on the document. It explains how the data, when used correctly, can provide valuable information for instructional and curricular assessment as well as for planning and development. Contact your school's AP Coordinator for this report.

- **How can I find AP credit policy information?**

  Go to apcentral.collegeboard.com/colleges to search for colleges' AP policies.

Chapter 5
Resources for Teachers

How to Address Limited Resources

The number one outside resource for all teachers is AP Central. Every AP Calculus teacher should register in order to take full advantage of the wealth of support materials available there. (It's free!) Here's a list of just some of the important information you will find there:

- The complete *AP Calculus Course Description*
- Free-response questions and scoring guidelines, dating from 1998, available for free download
- A link to the College Board Store, where you can order AP Released Exams (complete with multiple-choice items), collections of released multiple-choice and free-response questions with solutions, the AP Calculus CD, and other resources
- Reviews of textbooks, Web sites, and other materials
- A link to the AP Calculus electronic discussion group, an online community of teachers interested in AP Calculus
- Section II (free-response) instructions for students, with a commentary on those instructions from the AP Calculus Development Committee

Local circumstances may force a particular school or district to face challenges that are not present in other localities. Here are some strategies that may help you address those challenges:

- Collaborate with a local college or university: Find a professor who teaches calculus and try to form a cooperative relationship. Those institutions want your AP students, so they have an interest in supporting you.
- Form a local support group: If there are several schools within a reasonable distance from yours, contact the calculus teachers in those schools. Arrange to meet a few times a year to discuss teaching calculus. Pool resources with nearby schools in circumstances similar to yours. There may be other schools in your area that you can share materials with: tests, quizzes, activities, lesson plans, textbooks, review books, and the like.
- Look into College Board programs that offer assistance: In chapter 3, for example, you'll find information about the AP Fellows Program.
- Develop a plan to ensure that all students have access to a graphing calculator: See John Mahoney's suggestions in the box on page 91.

- A vast supply of sample multiple-choice and free-response questions is available for free or low cost on AP Central, but consider writing your own test questions as well. See Joe's Milliet's advice on page 93.

> Schools have used a variety of approaches to ensure that AP Calculus students have access to graphing calculators while they take the course. Some have purchased a set of calculators and loan these to students in the same way students are loaned textbooks at the beginning of the year. Students who borrow the calculators are responsible for replacing the batteries when necessary and, of course, for returning the calculators at the end of the year. Other schools have adopted a buy-back scheme: Students can buy a calculator from the school at the beginning of the year (registering the serial number) and sell it back to the school at the end of the year for $15 less than the purchase price. Both of these approaches emphasize the importance of students having access to a graphing calculator both at school and at home.
>
> — John Mahoney, Benjamin Banneker Academic High School, Washington, D.C.

Resources (Annotated Bibliography)

Calculus is a mature and stable AP subject. Consequently, the supply of ancillary teaching materials is ample. For new teachers, the sheer volume of resources can be overwhelming. AP Central maintains a substantial collection of reviews of resources. This collection is available from the Teachers' Resources tab on the main AP Central page and is searchable by subject, type of resource, and keyword. Even browsing that collection, though, can be daunting. There are over 100 reviews for Calculus AB alone. To get you started, we list and comment on a few favorite resources here. Notice, too, that each of the sample syllabi in chapter 3 has a list of resources used by that contributor. (Note: Our listing of resources is not meant as an endorsement on the part of the College Board of any particular author or publication.)

College Board Publications

AP Released Exam books in each AP subject are published every four or five years, on a staggered schedule. Each book contains a complete copy of a particular exam, including the multiple-choice questions and answers, a description of the process of scoring the free-response questions, examples of students' actual responses, scoring guidelines, and commentary that explains why the responses received the scores they did. The most recent AP Released Exams (2003 for Calculus) also contain a diagnostic guide to help students and teachers evaluate areas of strength and weakness. The 1998 and 2003 AP Calculus Released Exams are available for purchase at the College Board Store (store.collegeboard.com).

In addition to the syllabi contained in this Teacher's Guide, **Sample Syllabi** for each AP subject are available on AP Central. High school teachers from public or private schools wrote most of these syllabi, although because AP courses cover material usually taught at the college level, some syllabi from college professors are also included.

A number of other products for AP Calculus can be purchased at the College Board Store, including free-response questions from exams dating between 1969 and 1997 and multiple-choice questions from several released exams. Free downloads of the free-response questions from exams since 1998 are available by clicking on "The Exams" tab on the AP Central Home Page.

Basic Calculus Textbooks

- Hughes-Hallet, Deborah, et al. *Calculus—Single Variable*. 3rd ed. New York: John Wiley & Sons, 2002. [Ed. note: The 4th edition was published in 2005.]

Known in calculus circles as the "Harvard" book (due to the institutional affiliation of several members of the author team), this textbook has been at the forefront of the modern calculus reform movement. Highly regarded for its excellent problems, the book is certainly true to the "Lean and Lively" slogan popularized by a Mathematical Association of America publication from the early days of calculus reform. Every AP Calculus teacher should read the material in this book on the derivative and the integral. I'm particularly fond of the "Check your Understanding" questions that appear at the end of every chapter. Savvy veterans have noted how closely many problems in this book mirror AP problems.

- Ostebee, Arnold, and Paul Zorn. *Calculus from Graphical, Numerical, and Symbolic Points of View*. 2nd ed. Boston: Houghton Mifflin, 2002.

Another innovative and refreshing calculus reform textbook, the Ostebee and Zorn (OZ) book puts its commitment to the multirepresentational approach upfront in its title. As in the Harvard text, the problems in OZ are creative and invigorating. Notably, both OZ and the Harvard text have included more "standard" types of exercise problems in later editions. The text is highly readable and maintains a conversational tone.

- Foerster, Paul A. *Calculus: Concepts and Applications*. Emeryville, Calif.: Key Curriculum Press, 2004.

Paul Foerster, the author of this text, teaches mathematics at Alamo Heights High School in San Antonio, Texas. Foerster has a long and storied history in AP Calculus and is highly respected in the AP Calculus community. This book is notable for its quick introduction to the concepts of calculus (there's no review chapter covering precalculus topics at the outset), rich applications, and fidelity to the verbal component of the "Rule of Four." Foerster weaves journal-keeping directly into the text. Chapters begin with engaging "Exploratory Problem Sets" that serve to connect students to major blocks of material from the get-go.

- Finney, Ross L., et al. *Calculus: Graphical, Numerical, Algebraic*. 2nd ed. Menlo Park, Calif.: Scott Foresman/Addison-Wesley, 1999.

Now a distant offspring of its famous ancestor (the "Thomas" book), this is a popular choice for the primary text in AP Calculus. Like OZ, it announces its multirepresentational approach in the title. Dan Kennedy, the most recent addition to the author team, is a teacher of AP Calculus at the Baylor School in Chattanooga, Tennessee, and served as chair of the AP Calculus Development Committee in the early 1990s as it began to embrace graphing calculator technology. The chapter on infinite series is marvelous, and the explorations written into the text encourage students to work together.

- Stewart, James. *Calculus*. 5th ed. Pacific Grove, Calif.: Brooks/Cole Publishing Company, 2003.

Although this text is more like a traditional calculus book than many others, it is nonetheless informed by the use of technology and sensitive to the changes in the AP Calculus program. Examples are plentiful, and there is no shortage of theory and proof. As the author states in the preface, "The book contains elements of reform, but within the context of a traditional curriculum." A "leaner" version is titled *Calculus—Concepts and Contexts* (2nd ed.).

- Larson, Ron, Robert P. Hostetler, and Bruce H. Edwards. *Calculus*. 8th ed. Boston: Houghton Mifflin, 2006.

The new edition of the popular AP calculus textbook emphasizes the importance of traditional calculus skills and includes new technology and calculus reform ideas.

Exam Review Guides

Many exam preparation books are available for AP Calculus. Most are reviewed at AP Central, and you should read the thorough reviews there. Here, we list just a few of them. Of course, the very best sources of review questions are the Released Exams, especially the ones from recent years, and the actual AP Exam free-response questions, which are posted on AP Central following each administration.

- Hockett, Shirley O., and David Bock. *How to Prepare for the AP Calculus Advanced Placement Examination*. 7th ed. Hauppauge, N.Y.: Barron's Educational Series, 2001.

- *Multiple-Choice & Free-Response Questions in Preparation for the AP Calculus (AB) Examination.* 8th ed. Brooklyn: D&S Marketing Systems, n.d.
  There is also a version for the BC exam.

- Best, George, and J. Richard Lux. *Preparing for the Calculus (AB) Exam*. Andover, Mass.: Venture Publishing, 2004.
  There is also a version for the BC exam.

- Howell, Mark, and Martha Montgomery. *Be Prepared for the AP Calculus Exam*. Andover, Mass.: Skylight Publishing, 2005.

> I still write the majority of my own test, quiz, and handout questions. It forces me to think about the level of difficulty, to deeply consider what I am trying to test with each question, and to think about what I value in assessing a score. Writing multiple-choice questions forces me to think about common student errors, both mechanical and conceptual, while writing free-response questions makes me think about how students express themselves and justify their work. I never get as much feedback from questions that I "borrow" as I do from questions that I write myself. If you are new to teaching AP Calculus or feel that this is something you want to try without affecting scores too much at first, I suggest you initially assign your own questions as extra credit or extra practice problems. You will gain confidence quickly and soon find the knack for writing questions without too much extra time or difficulty.
>
> — Joe Milliet, St. Mark's School of Texas, Dallas

Laboratory Resources (Manuals, etc.)

Most modern textbooks come with ancillary materials that generally include instructor's guides, solutions manuals, test banks, and explorations. Sometimes these come in separate publications; other times they are combined in one. The best place to get information about these materials is from the publisher's Web site. Even if you don't adopt the parent material as your primary text, the explorations often stand well on their own. There are other independent publications that contain activities or explorations but are not tied to a particular text. Here are a few such publications.

- Foerster, Paul A. *Instructor's Resource Book*. Emeryville, Calif.: Key Curriculum Press, 2004.
  Accompanies the Foerster *Calculus* text and has a lot of nice explorations.

- Albert, Benita, et al. *AP Teacher's Guide to Accompany* "Calculus-Single Variable" (Hughes-Hallett et al., 3rd ed.). New York: John Wiley & Sons, 2002.
  This guide has sample exam questions, explorations, and tips.

- Antinone, Linda, et al. *Calculus Activities*. Dallas: Texas Instruments, 2004.
  This collection of activities is tied to the TI-83/84 family of calculators. TI also publishes activities for the TI-86 and TI-89.

Other Teaching Resources

- MAA Notes. *Resources for Calculus Collection*. Washington, D.C.: Mathematical Association of America, 1993.

  Volume 1: *Learning by Discovery: A Lab Manual for Calculus*, edited by Anita Solow. MAA Notes Number 27.

  Volume 2: *Calculus Problems for a New Century*, edited by Robert Fraga. MAA Notes Number 28.

  Volume 3: *Applications of Calculus*, edited by Philip Straffin. MAA Notes Number 29.

  Volume 4: *Problems for Student Investigation*, edited by Michael B. Jackson and John R. Ramsay. MAA Notes Number 30.

  Volume 5: *Readings for Calculus*, edited by Underwood Dudley. MAA Notes Number 31.

- McMullin, Lin. *Teaching AP Calculus*. Brooklyn: D&S Marketing Systems, 2002.
  As the title suggests, this is a handy guide for teaching AP Calculus. It consists primarily of a run-through of course content, with teaching tips, suggestions for using technology, and lists of exam questions on each topic.

- Foster Manufacturing Company, Plano, Texas
  Phone/Fax: 972 424-3644
  E-mail: fmco@flash.net
  Foster makes physical wood models illustrating volumes of solids of revolution and volumes of solids with known cross sections.

Multimedia

- APCDs are available for AP Calculus AB, among other subjects. Each CD has actual AP Exams, interactive tutorials, exam descriptions, answers to frequently asked questions, study-skill suggestions, and test-taking strategies. The teacher version of each CD, which can be licensed for up to 50 workstations, enables you to monitor student progress and provide individual feedback. Included is a *Teacher's Manual* that gives full explanations along with suggestions for utilizing the APCD in the classroom. The APCD for Calculus AB includes the 1997 and 1998 Released Exam questions. More information, including worksheets to accompany the APCD for Calculus AB, is available on AP Central. You can purchase the APCDs at the College Board Store.

- *Calculus in the Year 2000: New Ways of Teaching the Derivative and the Definite Integral*, VHS, presented by Steve Olson. 3 hours.
  Available at the College Board Store.

- Instructional videos on using graphing calculators (TI-8X series), presented by Sally Fischbeck. Venture Publishing, n.d.
  www.vent-pub.com/pages/books/videos.html

- *Stand and Deliver*, VHS/DVD, directed by Ramòn Menéndez (1988). 104 minutes. Warner Studios, 2004.

Web Sites

- AP Calculus Web Guide
  apcentral.collegeboard.com/apc/members/courses/teachers_corner/27435.html

- National Council of Teachers of Mathematics
  www.nctm.org

- Mathematical Association of America
  maa.org
  Calculus resources: www.maa.org/pubs/books/calculus.html

- North Carolina Association of Advanced Placement Mathematics Teachers
  www.ncaapmt.org/calculus
  The association publishes an excellent newsletter twice a year. Membership is only $5 a year ($10 for non-U.S. addresses).

- Calculator company Web sites

  - Casio
    www.casio.com/index.cfm?fuseaction=Products.Catalog&catalog=Calculators
  - Hewlett-Packard
    www.hp.com/calculators/
  - Sharp
    www.sharpusa.com/products/FunctionLanding/0,1050,4,00.html
  - Texas Instruments
    education.ti.com/

- The Math Forum
  mathforum.org/

- MERLOT: Multimedia Educational Resource for Learning and Online Teaching
  www.merlot.org
  This is a clearinghouse of Web sites covering many subjects, including calculus. One significant advantage of the site is that all of the sites it references have been peer-reviewed. Thus, you are likely to find high-quality activities using MERLOT as a launching pad.

Software

- Weeks, Audrey. *Calculus in Motion*. Burbank, Calif.: Calculus in Motion, 2005. calculusinmotion.com.
  Dynamic calculus animations to use with the *Geometer's Sketchpad*.

- Graphing software—used for inserting in your tests and handouts.
  Winplot: math.exeter.edu/rparris/winplot.html
  Graphmatica: www8.pair.com/ksoft/
  GrafEq: www.peda.com/grafeq/
  Mathplotter: mathplotter.lawrenceville.org/mathplotter/index.htm

- *MathType*
  mathtype.com.
  This is the professional version of *Equation Editor*. It is useful for creating handouts and tests.

- *Derive*
  education.ti.com/us/product/software/derive/features/features.html

- *Mathematica*
  www.wolfram.com/products/mathematica/index.html

- *Maple*
  www.maplesoft.com

- *TI InterActive!*
  education.ti.com/us/product/software/tii/features/features.html

- *TI Connect*
  education.ti.com/us/product/accessory/connectivity/features/software.html

Professional Development

In the following section, the College Board outlines its professional development opportunities in support of AP educators.

The teachers, administrators, and AP Coordinators involved in the AP Program compose a dedicated, engaged, vibrant community of educational professionals. Welcome!

We invite you to become an active participant in the community. The College Board offers a variety of professional development opportunities designed to educate, support, and invigorate both new and experienced AP teachers and educational professionals. These year-round offerings range from half-day workshops to intensive weeklong summer institutes, from the AP Annual Conference to AP Central, and from participation in an AP Reading to Development Committee membership.

Workshops and Summer Institutes

At the heart of the College Board's professional development offerings are workshops and summer institutes. Participating in an AP workshop is generally one of the first steps to becoming a successful AP teacher. Workshops range in length from half-day to weeklong events and are focused on all 37 AP courses and a range of supplemental topics. Workshop consultants are innovative, successful, and experienced AP teachers; teachers trained in developmental skills and strategies; college faculty members; and other qualified educational professionals who have been trained and endorsed by the College Board. For new and experienced teachers, these course-specific training opportunities encompass all aspects of AP course content, organization, evaluation, and methodology. For administrators, counselors, and AP Coordinators, workshops address critical issues faced in introducing, developing, supporting, and expanding AP programs in secondary schools. They also serve as a forum for exchanging ideas about AP.

While the AP Program does not have a set of formal requirements that teachers must satisfy prior to teaching an AP course, the College Board suggests that AP teachers have considerable experience and an advanced degree in the discipline before undertaking an AP course.

AP Summer Institutes provide teachers with in-depth training in AP courses and teaching strategies. Participants engage in at least 30 hours of training led by College Board-endorsed consultants and receive printed materials, including excerpts from AP Course Descriptions, AP Exam information, and other course-specific teaching resources. Many locations offer guest speakers, field trips, and other hands-on activities. Each institute is managed individually by staff at the sponsoring institution under the guidelines provided by the College Board.

Participants in College Board professional development workshops and summer institutes are eligible for continuing education units (CEUs). The College Board is authorized by the International Association for Continuing Education and Training (IACET) to offer CEUs. IACET is an internationally recognized organization that provides standards and authorization for continuing education and training.

Workshop and institute offerings for the AP Calculus teacher (or potential teacher) range from introductory to topic-specific events and include offerings tailored to teachers in the middle and early high school years. To learn more about scheduled workshops and summer institutes near you, visit the Institutes & Workshops area on AP Central: apcentral.collegeboard.com/events.

Online Events

The College Board offers a wide variety of online events, which are presented by College Board-endorsed consultants and recognized subject-matter experts to participants via a Web-based, real-time interface. Online events range from one hour to several days and are interactive, allowing for exchanges between the presenter and participants and between participants. Like face-to-face workshops, online events vary in focus from introductory themes to specific topics, and many offer CEUs for participants. For a complete list of upcoming and archived online events, visit apcentral.collegeboard.com/onlineevents.

Archives of many past online events are also available for free or for a small fee. Archived events can be viewed on your computer at your convenience.

AP Central

AP Central is the College Board's online home for AP professionals. The site offers a wealth of resources, including Course Descriptions, sample syllabi, exam questions, a vast database of teaching resource reviews, lesson plans, course-specific feature articles, and much more. Bookmark the information on AP Central about AP Calculus: apcentral.collegeboard.com/calculusab or apcentral.collegeboard.com/calculusbc.

AP Program information is also available on the site, including exam calendars, fee and fee reduction policies, student performance data, participation forms, research reports, college and university AP grade acceptance policies, and more.

AP professionals are encouraged to contribute to the resources on AP Central by submitting articles or lesson plans for publication and by adding comments to Teacher's Resources reviews.

Electronic Discussion Groups

The AP electronic discussion groups (EDGs) were created to provide a moderated forum for the exchange of ideas, insights, and practices among AP teachers, AP Coordinators, consultants, AP Exam Readers, administrators, and college faculty. EDGs are Web-based threaded discussion groups focused on specific AP courses or roles, giving participants the ability to post and respond to questions online to be viewed by other members of the EDG. To join an EDG, visit apcentral.collegeboard.com/community/edg.

AP Annual Conference

The AP Annual Conference (APAC) is a gathering of the AP community, including teachers, secondary school administrators, and college faculty. The APAC is the only national conference that focuses on providing complete strategies for middle and high school teachers and administrators involved in the AP Program. The 2007 conference will be held July 11 to 15 in Las Vegas, Nevada. Conference events include presentations by each course's Development Committee, course- and topic-specific sessions, guest speakers, and pre- and postconference workshops for new and experienced teachers. To learn more about this year's event, please visit www.collegeboard.com/apac.

AP professionals are encouraged to lead workshops and presentations at the conference. Proposals are due in the fall of each year prior to the event (visit AP Central for specific deadlines and requirements).

Professional Opportunities

College Board Consultants and Contributors

Experienced AP teachers and educational professionals share their techniques, best practices, materials, and expertise with other educators by serving as College Board consultants and contributors. They may lead workshops and summer institutes, sharing their proven techniques and best practices with new and experienced AP teachers, AP Coordinators, and administrators. They may also contribute to AP course and exam development (writing exam questions or serving on a Development Committee) or evaluate AP Exams at the annual AP Reading. Consultants and contributors may be teachers, postsecondary faculty, counselors, administrators, and retired educators. They receive an honorarium for their work and are reimbursed for expenses.

To learn more about becoming a workshop consultant, visit apcentral.collegeboard.com/consultant.

AP Exam Readers

High school and college faculty members from around the world gather in the United States each June to evaluate and score the free-response sections of the AP Exams at the annual AP Reading. AP Exam Readers are led by a Chief Reader, a college professor who has the responsibility of ensuring that students receive grades that accurately reflect college-level achievement. Readers describe the experience as providing unparalleled insight into the exam evaluation process and as an opportunity for intensive collegial exchange between high school and college faculty. (More than 8,500 Readers participated in the 2006 Reading.) High school Readers receive certificates awarding professional development hours and CEUs for their participation in the AP Reading. To apply to become an AP Reader, go to apcentral.collegeboard.com/readers.

Development Committee Members

The dedicated members of each course's Development Committee play a critical role in the preparation of the Course Description and exam. They represent a diverse spectrum of knowledge and points of view in their fields and, as a group, are the authority when it comes to making subject-matter decisions in the exam-construction process. The AP Development Committees represent a unique collaboration between high school and college educators.

AP Grants

The College Board offers a suite of competitive grants that provide financial and technical assistance to schools and teachers interested in expanding access to AP. The suite consists of three grant programs: College Board AP Fellows, College Board Pre-AP Fellows, and the AP Start-Up Grant, totaling over $600,000 in annual support for professional development and classroom resources. The programs provide stipends for teachers and schools that want to start an AP program or expand their current program. Schools and teachers that serve minority and/or low income students who have been traditionally underrepresented in AP courses are given preference. To learn more, visit apcentral.collegeboard.com/apgrants.

Our Commitment to Professional Development

The College Board is committed to supporting and educating AP teachers, AP Coordinators, and administrators. We encourage you to attend professional development events and workshops to expand your knowledge of and familiarity with the AP course(s) you teach or that your school offers, and then to share that knowledge with other members of the AP community. In addition, we recommend that you join professional associations, attend meetings, and read journals to help support your involvement in the community of educational professionals in your discipline. By working with other educational professionals, you will strengthen that community and increase the variety of teaching resources you use.

Your work in the classroom and your contributions to professional development help the AP Program continue to grow, providing students worldwide with the opportunity to engage in college-level learning while still in high school.

Appendix 1
Journal Questions

1. Explain what it means to say $\lim_{x\to\infty}\frac{1}{x}=0$.

The next two questions are meant to initiate a discussion of why "closer and closer" language doesn't work:

2. Explain what it means to say $\lim_{x\to\infty}\frac{\sin x}{x}=0$.

(Note: Language such as "The values of $\frac{\sin x}{x}$ get closer and closer to 0 as x gets larger and larger" is not quite correct. Look at a table of values for large values of x and you see that $\frac{\sin x}{x}$ spends about half the time actually getting farther and farther away from 0! What is important is that the values of $\frac{\sin x}{x}$ can be made arbitrarily close to 0 by making x sufficiently large.)

3. Explain what it means to say $\lim_{x\to 0}\frac{\sin 3x}{x}=3$.

The Intermediate Value Theorem (IVT) says:

> If f is continuous on the closed interval $[a, b]$ and K is any real number between $f(a)$ and $f(b)$, then there is at least one number c in the open interval (a, b) such that $f(c)=K$.

Let $a = 1$, $b = 5$, $f(1) = 2$, $f(5) = 4$, and $K = 3$. Draw the graph of a function that meets each of the specified criteria, or explain why it is impossible to do so.

4. A function that fails to meet the hypothesis (the "If" part) of the IVT, and does not satisfy the conclusion (the "then" part).

5. A function that fails to meet the hypothesis of the IVT, but does satisfy the conclusion.

6. A function that meets the hypothesis of the IVT, but does not satisfy the conclusion.

7. A function that meets the hypothesis of the IVT, and does satisfy the conclusion.

The Extreme Value Theorem (EVT) says (in the maximum case):

> If f is continuous on the closed interval $[a, b]$, then there exists at least one number c in $[a, b]$ such that $f(x)\le f(c)$ for all x in $[a, b]$.

Let $a = 1$ and $b = 5$. Draw the graph of a function that meets each of the specified criteria, or explain why it is impossible to do so.

8. A function that fails to meet the hypothesis (the "If" part) of the EVT, and does not satisfy the conclusion (the "then" part).

9. A function that fails to meet the hypothesis of the EVT, but does satisfy the conclusion.

10. A function that meets the hypothesis of the EVT, but does not satisfy the conclusion.

11. A function that meets the hypothesis of the EVT, and does satisfy the conclusion.

The Mean Value Theorem (MVT) says:

> If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there is at least one number c in the open interval (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

Let $a = 1$ and $b = 5$. Draw the graph of a function that meets each of the specified criteria, or explain why it is impossible to do so.

12. A function that fails to meet the hypothesis of the MVT, and does not satisfy the conclusion.

13. A function that fails to meet the hypothesis of the MVT, but does satisfy the conclusion.

14. A function that meets the hypothesis of the MVT, but does not satisfy the conclusion.

15. A function that meets the hypothesis of the MVT, and does satisfy the conclusion.

16. Draw the graph of a function that has exactly one removable discontinuity. Give a symbolic definition for such a function, i.e., $f(x) = \dots$

17. Draw the graph of a function that has exactly one essential discontinuity. Give a symbolic definition for such a function, i.e., $g(x) = \dots$

18. Draw the graph of a function that has an infinite number of essential discontinuities. Give a symbolic definition for such a function, i.e., $h(x) = \dots$

19. Draw the graph of a function that has an infinite number of removable discontinuities. Extra credit: Give a symbolic definition for such a function, i.e., $r(x) = \dots$

20. Explain the difference between average rate of change and instantaneous rate of change.

21. How is this difference manifested in the definition of the derivative of a function f at a point?

22. If you zoom in with equal scaling at a point on the graph of a function where it is differentiable, you will eventually see a line. Explain what this line has to do with the value of the derivative of the function at the point.

23. Explain, in plain English, what the Fundamental Theorem of Calculus says.

24. What's the most important idea you've seen in calculus so far, and why?

25. Explain how the idea of limit is used to define a definite integral.

26. How is the process of finding the equation of a line with slope m and passing through the point (a, b) in algebra like solving a differential equation in calculus?

27. Translate this Algebra 1 problem into a differential equation problem, and solve it: Write the equation of a line with slope -3 and x-intercept 7.

28. Explain the difference between the general solution of a differential equation and the particular solution that satisfies an initial condition.

29. Show that the function $y(x)=2+\int_1^x \cos\left(t^3\right)dt$ solves the differential equation $\frac{dy}{dx}=\cos\left(x^3\right)$ with $y(1)=2$.

30. What kinds of questions does the derivative let you answer that you could not answer with your precalculus knowledge?

31. What kinds of questions does the integral let you answer that you could not answer with your precalculus knowledge?

Appendix 2
Compendium of Terms

The *AP Calculus Course Description* does not attempt to include every term that a student ought to encounter while studying calculus. It is reasonable to assume that students who have taken a calculus course will be familiar with the vocabulary of the subject, and this compendium is an attempt to establish what that vocabulary is—without implying that a calculus course consists merely of defining it.

absolute convergence (BC)	Cartesian coordinate system
absolute maximum	Chain Rule
absolute minimum	circle
absolute value	circular functions
acceleration	closed interval $[a, b]$
acceleration vector (BC)	coefficient
algebraic function	Comparison Test (BC)
alternating series (BC)	complex number
Alternating Series Test (BC)	components of a vector (BC)
amplitude	composition $f \circ g$
antiderivative	concave down
antidifferentiation	concave up
approximation	conditional convergence (BC)
arccosine function	conic section
arclength (BC)	constant function
arcsine function	constant of integration
arctangent function	continuity at a point
asymptote	continuity on an interval
average rate of change	continuous function
average value	convergent improper integral (BC)
axis of rotation	convergent sequence (BC)
axis of symmetry	convergent series (BC)
base (exponential and logarithmic)	coordinate axes
bounded	cosecant function
bounded above	cosine function
bounded below	cotangent function
Cardioid (BC)	critical point

critical value	geometric sequence (BC)
cross-sectional area	geometric series (BC)
decay models	graph
decreasing function	growth models
decreasing on an interval	growth rate
definite integral	half-life
delta notation (D)	harmonic series (BC)
derivative	hyperbola
difference quotient	imaginary number
differentiability	implicit differentiation
differential	improper integral (BC)
differential equation	increasing function
differentiation	increasing on an interval
discontinuity	increment
disk method	indefinite integral
distance (from velocity)	indeterminate form (BC)
distance formula	infinite limit
divergent improper integral (BC)	inflection point
divergent sequence (BC)	initial condition
divergent series (BC)	initial value problem
domain	inscribed rectangle
dummy variable of integration	instantaneous rate of change
$\frac{dy}{dx}$ (Leibniz notation)	instantaneous velocity
e	integer
ellipse	integrable function
end behavior	integrand
endpoint extremum	integration
Euler's method (BC)	integration by partial fractions (BC)
even function	integration by parts (BC)
exponent laws	integration by substitution
exponential function	Intermediate Value Theorem
exponential growth and decay	interval
extremum	interval of convergence (BC)
factorial	inverse function f^{-1}
First Derivative Test	irrational number
frequency of a periodic function	Lagrange Error Bound (BC)
function	Law of Cosines
Fundamental Theorem of Calculus	Law of Sines

left-hand limit	period
left-hand sum	periodic function
Leibniz, Gottfried	perpendicular curves
L'Hospital's Rule (BC)	piecewise-defined function
limit	polar coordinates (BC)
limit at infinity	polynomial
limit of integration	position function
linear function	position vector (BC)
local extrema	power series
local linearity	prime notation $(f'(x))$
local linearization	Product Rule
logarithm laws	proportionality
logistic equation (BC)	p-series (BC)
logistic growth (BC)	quadrant
lower bound	quadratic formula
Maclaurin series (BC)	Quotient Rule
maximum	radian
mean value	radius of a circle
Mean Value Theorem	radius of convergence (BC)
midpoint formula	range
minimum	rate of change
monotonic	Ratio Test (BC)
motion	rational function
natural logarithm	real number
Newton, Isaac	rectangular coordinates
nonremovable discontinuity	region (in a plane)
normal line	related rates
numerical derivative	relative error
numerical integration	relative maximum
odd function	relative minimum
one-to-one function	removable discontinuity
open interval (a, b)	Riemann sum
optimization	right-hand limit
order of a derivative	right-hand sum
origin	root of an equation
parametric equations (BC)	roundoff error
partial fractions (BC)	scalar (BC)
partial sum of a series (BC)	secant function
partition of an interval	secant line
percentage error	second derivative

Second Derivative Test	term of a sequence or series (BC)
separable differential equation	transcendental function
sequence (BC)	Trapezoidal Rule
series (BC)	trigonometric function
set	truncation error for power series (BC)
sigma notation	unit circle
sine function	unit vector (BC)
slope	upper bound
slope field	u-substitution
solid (in 3-space)	vector (BC)
solid of revolution	velocity
speed	velocity vector (BC)
sphere	vertex
subset	viewing window
symmetry	volume by slicing
tangent function	x-axis
tangent line	x-intercept
tangent vector (BC)	y-axis
Taylor polynomial (BC)	y-intercept
Taylor series (BC)	zero of a function