Question 1

Let R be the region in the first and second quadrants bounded above by the graph of $y = \dfrac{20}{1 + x^2}$ and below by the horizontal line $y = 2$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the x-axis.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles. Find the volume of this solid.

$\dfrac{20}{1 + x^2} = 2$ when $x = \pm 3$

(a) Area $= \displaystyle\int_{-3}^{3} \left(\frac{20}{1 + x^2} - 2 \right) dx = 37.961$ or 37.962

(b) Volume $= \pi \displaystyle\int_{-3}^{3} \left(\left(\frac{20}{1 + x^2} \right)^2 - 2^2 \right) dx = 1871.190$

(c) Volume $= \dfrac{\pi}{2} \displaystyle\int_{-3}^{3} \left(\frac{1}{2} \left(\frac{20}{1 + x^2} - 2 \right) \right)^2 dx$

$\qquad\quad = \dfrac{\pi}{8} \displaystyle\int_{-3}^{3} \left(\frac{20}{1 + x^2} - 2 \right)^2 dx = 174.268$

1 : correct limits in an integral in (a), (b), or (c)

(a) $2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) $3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
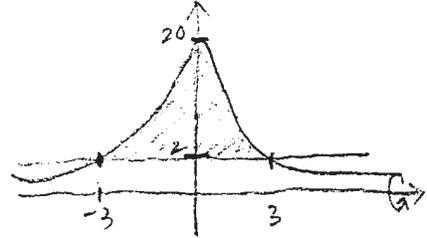
CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

IA₁

Work for problem 1(a)

$$\text{Area} = \int_{-3}^{3} \left(\frac{20}{1+x^2} - 2 \right) dx$$

$$= 37.9618$$

-4-

Do not write beyond this border.

Work for problem 1(b) $R = \frac{20}{1+x^2}$ $r = 2$

$$V = \pi \int_{-3}^{3} \left[\left(\frac{20}{1+x^2}\right)^2 - 4 \right] dx$$

$$= 1871.1901$$

Work for problem 1(c) $D = \frac{20}{1+x^2} - 2$

$$r = \frac{\frac{20}{1+x^2} - 2}{2} = \frac{10}{1+x^2} - 1$$

$$V = \int_{-3}^{3} \frac{\pi \left(\frac{10}{1+x^2} - 1\right)^2}{2} dx = \frac{\pi}{2} \int_{-3}^{3} \left(\frac{10}{1+x^2} - 1\right)^2 dx$$

$$= 174.2685$$

CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

1B

Work for problem 1(a)

$(0, 20)$

$(3, 2)$

$y = 2$

$(0, 2)$

$y = \dfrac{20}{1 + x^2}$

$1 + x^2 = \dfrac{20}{y}$

$$A_R = 2 \int_0^3 \left[\left(\dfrac{20}{1 + x^2} \right) - 2 \right] \, dx$$
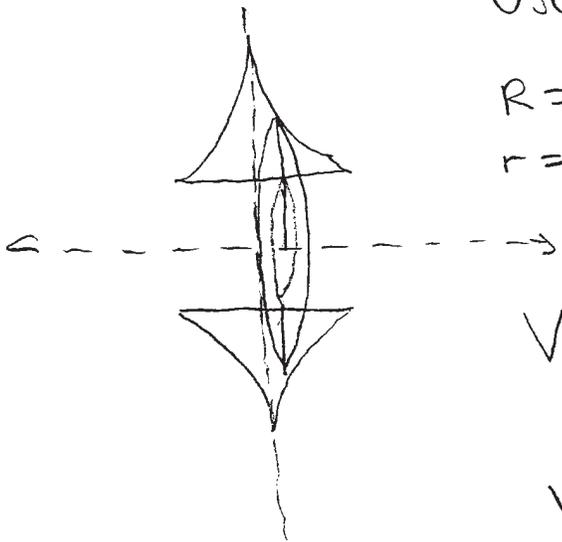
$$A_R = 37.962$$

Do not write beyond this border.

-4-

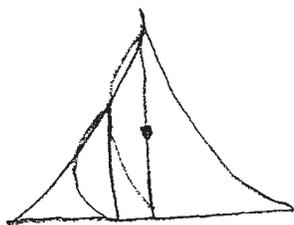Work for problem 1(b)

Use washers method.

$R = \dfrac{20}{1+x^2}$

$r = 2$

$$V = 2\left[\pi \int_0^3 \left(\left(\dfrac{20}{1+x^2}\right)^2 - 2^2\right) dx\right]$$

$V = 1871.190$

Work for problem 1(c)

$r = \dfrac{\left(\dfrac{20}{1+x^2}\right)}{2} = \dfrac{10}{1+x^2}$

$$V = 2\left[\dfrac{1}{2}\pi \int_0^3 \left(\dfrac{10}{1+x^2}\right)^2\right]$$

$V = 243.324$

GO ON TO THE NEXT PAGE.

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

IC₁

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

R shaded region bounded by $y = \dfrac{20}{1+x^2}$ & $y = 2$

Intersect at $(-3, 2)$ & $(3, 2)$

A of $R =$

$$\int_{-3}^{3} \left(\frac{20}{1+x^2} - 2 \right) dx$$

Area of shaded region $R = \boxed{37.962 \text{ units}^2}$

-4-

Work for problem 1(b)

~~scribbled out~~

$$V = \pi \int_a^b \left(R(x) - r(x) \right)^2 dx$$

$$V = \pi \int_{-3}^{3} \left(\frac{20}{1+x^2} - 2 \right)^2 dx$$

$$\boxed{V = 1394.148 \text{ units}^3}$$

Work for problem 1(c)

A of semicircle $\frac{1}{2} \pi r^2$

$$V = \int_{-3}^{3} \frac{1}{2} \pi r^2 \quad \text{where } r = \frac{20}{1+x^2}$$

$$V = \int_{-3}^{3} \frac{1}{2} \pi \left(\frac{20}{1+x^2} \right)^2$$

$$\boxed{V = 973.294 \text{ unit}^3}$$

-5-

Question 1

Overview

This problem presented students with a region bounded above by the graph of a function and below by a horizontal line. Because no picture was provided, students were expected to graph the function on their calculators or use their knowledge of rational functions to sketch the graph, and then identify the appropriate region from their graph. The points of intersection of the graph and the horizontal line could be found either algebraically or with the calculator. Students needed to find, in part (a), the area of the region; in part (b), the volume of the solid generated when the region was rotated about the *x*-axis; and in part (c), the volume of the solid above the region for which the cross sections perpendicular to the *x*-axis were semicircles.

Sample: 1A
Score: 9

The student earned all 9 points.

Sample: 1B
Score: 6

The student earned 6 points: the region point, 2 points in part (a), 3 points in part (b), and no points in part (c). In part (a) the student sets up the definite integral using symmetry and earned the region point. The student has the correct integrand, which earned the first point in part (a). The answer is correct to three decimal places and earned the second point. In part (b) the student continues to use symmetry in defining the limits of integration. The correct integrand earned the first 2 points. The answer is correct to three decimal places and earned the third point. In part (c) the student did not earn any points because the radius is incorrect.

Sample: 1C
Score: 3

The student earned 3 points: the region point, 2 points in part (a), no points in part (b), and no points in part (c). In part (a) the student earned the region point by using the correct limits of integration. The student has the correct integrand, which earned the first point in part (a). The answer is correct to three decimal places and earned the second point. In part (b) the student did not earn any points because the washer method is not used correctly. In part (c) the student did not earn any points because the radius is incorrect.