Consider the differential equation \( \frac{dy}{dx} = 3x + 2y + 1 \).

(a) Find \( \frac{d^2y}{dx^2} \) in terms of \( x \) and \( y \).

(b) Find the values of the constants \( m, b, \) and \( r \) for which \( y = mx + b + e^{rx} \) is a solution to the differential equation.

(c) Let \( y = f(x) \) be a particular solution to the differential equation with the initial condition \( f(0) = -2 \). Use Euler’s method, starting at \( x = 0 \) with a step size of \( \frac{1}{2} \), to approximate \( f(1) \). Show the work that leads to your answer.

(d) Let \( y = g(x) \) be another solution to the differential equation with the initial condition \( g(0) = k \), where \( k \) is a constant. Euler’s method, starting at \( x = 0 \) with a step size of 1, gives the approximation \( g(1) \approx 0 \). Find the value of \( k \).

\[
\begin{align*}
(a) \quad \frac{d^2y}{dx^2} &= 3 + 2 \frac{dy}{dx} = 3 + 2(3x + 2y + 1) = 6x + 4y + 5 \\
(b) \quad \text{If } y &= mx + b + e^{rx} \text{ is a solution, then} \\
&= m + re^{rx} = 3x + 2(mx + b + e^{rx}) + 1. \\
&\text{If } r \neq 0: m = 2b + 1, \ r = 2, \ 0 = 3 + 2m, \\
&\quad \text{so } m = -\frac{3}{2}, \ r = 2, \text{ and } b = -\frac{5}{4}. \\
&\text{OR} \\
&\quad \text{If } r = 0: m = 2b + 3, \ r = 0, \ 0 = 3 + 2m, \\
&\quad \text{so } m = -\frac{3}{2}, \ r = 0, \text{ and } b = -\frac{9}{4}. \\
(c) \quad f\left(\frac{1}{2}\right) &\approx f(0) + f'(0) \cdot \frac{1}{2} = -2 + (-3) \cdot \frac{1}{2} = -\frac{7}{2} \\
f'\left(\frac{1}{2}\right) &\approx 3 \left(\frac{1}{2}\right) + 2 \left(-\frac{7}{2}\right) + 1 = -\frac{9}{2} \\
f(1) &\approx f\left(\frac{1}{2}\right) + f'\left(\frac{1}{2}\right) \cdot \frac{1}{2} = -\frac{7}{2} + \left(-\frac{9}{2}\right) \cdot \frac{1}{2} = -\frac{23}{4} \\
(d) \quad g'(0) &= 3 \cdot 0 + 2 \cdot k + 1 = 2k + 1 \\
g(1) &\approx g(0) + g'(0) \cdot 1 = k + (2k + 1) = 3k + 1 = 0 \\
k &= -\frac{1}{3}
\end{align*}
\]
Work for problem 5(a)

\[
\frac{dy}{dx} = \frac{d}{dx} \left( \frac{dy}{dx} \right) = 3 + 2 \cdot \frac{dy}{dx} = 3 + 2 (3x + 4y + 1).
\]

\[
= 3 + 6x + 4y + 2 = 6x + 4y + 5
\]

Work for problem 5(b)

\[y = mx + b + e^{rx} \quad e^{rx} = y - mx - b\]

\[
\frac{dy}{dx} = m + re^{rx} = m + r(y - mx - b)
\]

\[
= -rmx + ry + (m - br) = 3x + 2y + 1
\]

\[-rm = 3, \quad r = 2, \quad m - br = 1\]

\[m = \frac{3}{-2} = -\frac{3}{2}\]

\[br = m - 1 \Rightarrow b = \frac{m - 1}{r} = \left( -\frac{3}{2} - 1 \right). \quad \frac{1}{2} = \frac{-5}{2}, \quad \frac{1}{2} = -\frac{5}{4}\]

\[\therefore m = -\frac{3}{2}, \quad r = 2, \quad \frac{1}{2} = -\frac{5}{4}\]
Work for problem 5(c)

\[ f(\frac{1}{2}) = f(0 + \frac{1}{2}) \approx f(0) + \frac{1}{2} f'(0) = -2 + \frac{1}{2} \left( 3 \cdot 0 + 2 \cdot (-2) + 1 \right) \]
\[ = -2 + \frac{1}{2} \left( -4 + 1 \right) = -2 - \frac{3}{2} = -\frac{7}{2} \]

\[ f(1) = f(\frac{1}{2} + \frac{1}{2}) \approx f(\frac{1}{2}) + \frac{1}{2} f'(\frac{1}{2}) = -\frac{7}{2} + \frac{1}{2} \left( 3 \cdot \frac{1}{2} + 2 \cdot (-\frac{7}{2}) + 1 \right) \]
\[ = -\frac{7}{2} + \frac{1}{2} \left( \frac{3}{2} - 7 + 1 \right) = -\frac{7}{2} + \frac{1}{2} \cdot (\frac{9}{2}) = -\frac{7}{2} - \frac{9}{4} \]
\[ = -\frac{23}{4}, \quad \therefore f(1) \approx -\frac{23}{4} \]

Work for problem 5(d)

\[ g(1) = g(0 + 1) \approx g(0) + 1 \cdot g'(0) = k + 1 \cdot (3 \cdot 0 + 2 \cdot k + 1) \]
\[ = k + 2k + 1 = 3k + 1 = 0 \quad \Rightarrow \quad 3k = -1 \]
\[ k = -\frac{1}{3}, \quad \therefore k = -\frac{1}{3} \]
Work for problem 5(a)

\[ \frac{d^2y}{dx^2} = 3 + 2 \frac{dy}{dx} \]

\[ = 3 + 2 \left( 3x^2 + 2y + 1 \right) \]

\[ = 3 + 6x + 4y + 2 \]

\[ = 6x + 4y + 5 \]

Work for problem 5(b)

\[ \frac{dy}{dx} = m + ce^{rx} \]

\[ = 3x + 2y + 1 \]

Continue problem 5 on page 13.
Work for problem 5(c)

\[ f(0) = -2 \]
\[ f(0.5) = f(0) + f'(0) \times 0.5 \]
\[ = -2 + (-3) \times 0.5 \]
\[ = -3.5 \]
\[ f(1) = f(0.5) + f'(0.5) \times 0.5 \]
\[ = 3.5 + (-3) \times 0.5 \]
\[ = -5 \]
\[ \therefore f(1) = -5 \]

Work for problem 5(d)

\[ g(u) = k \]
\[ g(0.5) = g(0) + g'(0) \times 1 \]
\[ = k + (2k - 4) \]
\[ = 0 \]
\[ 3k + 1 = 0 \]
\[ \therefore k = -\frac{1}{3} \]
Work for problem 5(a) \[
\frac{dy}{dx} = 3x + 2y + 1
\]
\[
\frac{d^2 y}{dx^2} = 3 + 2 \frac{dy}{dx} = 3 + 2(3x + 2y + 1)
\]
\[
= 3 + 6x + 4y + 2
\]
\[
= 6x + 4y + 5
\]

Work for problem 5(b) \[
dy = (3x + 2y + 1) dx \Rightarrow 5dy = 5(3x + 2y + 1) dx
\]
\[
y = \frac{3}{2} x^2 + 2xy + x
\]
\[
\gamma - 2xy = \gamma (1-2x) = \frac{3}{2} x^2 + x
\]
\[
y = \frac{3x^2 + 2x}{(1-2x)^2} = \frac{3x^2 + 2x}{2-4x} = mx + b + e^{rx}
\]
\[
3x^2 + 2x = 2mx + 2b + 2e^{rx}
\]
\[
-4mx^2 - 4bx - 4xe^{rx}
\]
\[
2b + 2e^{rx} = 0
\]
\[
-4b - 4e^{rx} = \frac{14}{3}
\]
\[
2x = 2mx - 4bx - 4xe^{rx}
\]

Continue problem 5 on page 13.
Work for problem 5(c)

\[ x_0 = -2 \]
\[ x_0 = -2 \]

\[ y_0 = -\frac{4}{3} \quad \text{from 5(b)} \]

\[ x_1 = x_0 + \frac{1}{2} (y_0') \]
\[ y_1 = y_0 + \frac{1}{2} (y_0') \]

\[ x_1 = x_0 + \frac{1}{2} \left(-\frac{4}{3}\right) \]
\[ y_1 = y_0 + \frac{1}{2} \left(-\frac{4}{3}\right) \]

\[ x_1 = x_0 + \frac{1}{2} \left(-\frac{4}{3}\right) \]
\[ y_1 = y_0 + \frac{1}{2} \left(-\frac{4}{3}\right) \]

\[ y_1 = -\frac{8}{3} \]
\[ y_1 = -\frac{8}{3} \]

\[ x_2 = x_1 + \frac{1}{2} \]
\[ y_2 = y_1 + \frac{1}{2} \]

\[ x_2 = x_1 + \frac{1}{2} \]
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\[ y_2 = \frac{1}{2} \]

Work for problem 5(d)

\[ x_0 = 0 \]
\[ y_0 = k \]

\[ x_0 = 0 \]
\[ y_0 = k \]

\[ x_1 = x_0 + \frac{1}{2} \]
\[ y_1 = y_0 + \frac{1}{2} (y_0') \]

\[ x_1 = x_0 + \frac{1}{2} \]
\[ y_1 = y_0 + \frac{1}{2} (y_0') \]

\[ x_1 = x_0 + \frac{1}{2} \]
\[ y_1 = y_0 + \frac{1}{2} (y_0') \]

\[ x_1 = 0 + \frac{1}{2} \]
\[ y_1 = k + \frac{1}{2} \]

\[ x_1 = 0 + \frac{1}{2} \]
\[ y_1 = k + \frac{1}{2} \]

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\[ y_2 = k + \frac{1}{2} + \frac{1}{2} \]

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\[ y_2 = k + \frac{1}{2} + \frac{1}{2} \]

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AP® CALCULUS BC
2007 SCORING COMMENTARY (Form B)

Question 5

Sample: 5A
Score: 9

The student earned all 9 points.

Sample: 5B
Score: 6

The student earned 6 points: 2 points in part (a), 1 point in part (b), 1 point in part (c), and 2 points in part (d). The student presents correct work in parts (a) and (d). In part (b) the student only finds the correct \( \frac{dy}{dx} \), and so the first point was earned. In part (c) the student earned the first point by the use of Euler’s method with two steps to approximate \( f(1) \). The student makes an error in calculating \( f\left(\frac{1}{2}\right) \), so the second point was not earned.

Sample: 5C
Score: 3

The student earned 3 points: 2 points in part (a), no points in part (b), 1 point in part (c), and no points in part (d). The student presents correct work in part (a). In part (b) the student does not find \( \frac{dy}{dx} \). In part (c) the student earned the first point by the use of Euler’s method with two steps to approximate \( f(1) \). The student makes an error in calculating \( f\left(\frac{1}{2}\right) \), so the second point was not earned. In part (d) the student uses \( \frac{1}{2} \) instead of 1 for \( \Delta x \) and makes computational errors, so no points were awarded.