AP® CALCULUS BC
2007 SCORING GUIDELINES (Form B)

Question 3

The wind chill is the temperature, in degrees Fahrenheit $(°F)$, a human feels based on the air temperature, in degrees Fahrenheit, and the wind velocity v, in miles per hour (mph). If the air temperature is $32°F$, then the wind chill is given by $W(v) = 55.6 - 22.1v^{0.16}$ and is valid for $5 \le v \le 60$.

(a) Find $W'(20)$. Using correct units, explain the meaning of $W'(20)$ in terms of the wind chill.

(b) Find the average rate of change of W over the interval $5 \le v \le 60$. Find the value of v at which the instantaneous rate of change of W is equal to the average rate of change of W over the interval $5 \le v \le 60$.

(c) Over the time interval $0 \le t \le 4$ hours, the air temperature is a constant $32°F$. At time $t = 0$, the wind velocity is $v = 20$ mph. If the wind velocity increases at a constant rate of 5 mph per hour, what is the rate of change of the wind chill with respect to time at $t = 3$ hours? Indicate units of measure.

(a) $W'(20) = -22.1 \cdot 0.16 \cdot 20^{-0.84} = -0.285$ or -0.286

When $v = 20$ mph, the wind chill is decreasing at 0.286 °F/mph.

$2 : \begin{cases} 1 : \text{value} \\ 1 : \text{explanation} \end{cases}$

(b) The average rate of change of W over the interval $5 \le v \le 60$ is $\dfrac{W(60) - W(5)}{60 - 5} = -0.253$ or -0.254.

$W'(v) = \dfrac{W(60) - W(5)}{60 - 5}$ when $v = 23.011$.

$3 : \begin{cases} 1 : \text{average rate of change} \\ 1 : W'(v) = \text{average rate of change} \\ 1 : \text{value of } v \end{cases}$

(c) $\left.\dfrac{dW}{dt}\right|_{t=3} = \left(\dfrac{dW}{dv} \cdot \dfrac{dv}{dt}\right)\Big|_{t=3} = W'(35) \cdot 5 = -0.892$ °F/hr

OR

$W = 55.6 - 22.1(20 + 5t)^{0.16}$

$\left.\dfrac{dW}{dt}\right|_{t=3} = -0.892$ °F/hr

$3 : \begin{cases} 1 : \dfrac{dv}{dt} = 5 \\ 1 : \text{uses } v(3) = 35, \text{ or uses } v(t) = 20 + 5t \\ 1 : \text{answer} \end{cases}$

Units of °F/mph in (a) and °F/hr in (c)

1 : units in (a) and (c)

Work for problem 3(a)

$W(V) = 55.6 - 22.1 V^{0.16}$

$W'(V) = -22.1(0.16) V^{0.16-1}$

$= -3.536 V^{-0.84}$

$W'(20) \approx -3.536 (20)^{-0.84}$

≈ -0.286 °F/mph

It means that the wind chill is decreasing at a rate of 0.286 °F/mph when $V = 20$ mph.

Work for problem 3(b)

$W'(V) = -3.536 V^{-0.84}$

avg. rate of change of W

$= \frac{1}{60-5} \int_5^{60} W'(V)\, dV$

$= \frac{1}{55} \int_5^{60} -3.536 V^{-0.84}\, dV$

$= \frac{1}{55} (-13.95882)$

≈ -0.254 °F/mph

$W'(V) = -0.254$

$-3.536 V^{-0.84} = -0.254$

$V = 22.989$ mph

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Continue problem 3 on page 9.

Work for problem 3(c)

$\frac{dv}{dt} = 5$

$\int dv = \int 5\,dt$

$v = 5t + c$

$20 = 5(0) + c$

$c = 20$

$\therefore v(t) = 5t + 20$

@ $t = 3$.

$v(3) = 15 + 20 = 35$ mph.

$\frac{dw}{dv} = -3.536 v^{-0.84}$

$\frac{dv}{dt} = 5$

$\frac{dw}{dt} = \frac{dw}{dv} \cdot \frac{dv}{dt}$

$= (-3.536 v^{-0.84})(5)$

$\left.\frac{dw}{dt}\right|_{t=3} = [-3.536(35)^{-0.84}](5)$

≈ -0.892 °F/h

END OF PART A OF SECTION II
IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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Work for problem 3(a)

$W(v) = 55.6 - 22.1v^{.16}$

$W'(v) = -22.1(.16)v^{-.84}$

$W'(20) = -.286$ °F/mph. the rate of change of the windchill at $(v = 20)$

Work for problem 3(b)

$$\frac{F(60) - F(5)}{60 - 5} = \frac{13.0503 - 27.0091}{60 - 5} = \boxed{-.254}$$

$W'(v) = -22.1(.16)v^{-.84} = -.254$

$\frac{1}{-.84}(v^{-.84}) = (.7177)^{1/-.84}$

$\boxed{v = 23.011}$

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Continue problem 3 on page 9.

Work for problem 3(c)

$W(v) = 55.6 - 22.1v^{.16}$

$W'(v) = -22.1(.16)v^{-.84}$

$W'(35) = -.178 \frac{\text{degrees}}{\text{mhp}}$

$\frac{dv}{dt} = 5$

$v \text{ at } t = 3 = 35$

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END OF PART A OF SECTION II
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Work for problem 3(a)

$w'(v) = .16(-22.1)v^{-.84}$ $\quad v = 20$

$w'(20) = .16(-22.1)(20^{-.84})$

$= -.2855$

$\boxed{w'(20) = -.286 \text{ m/hr.}}$

$w'(20)$ is how fast and in which direction the wind chill is moving when the air temperature is felt when the wind is traveling at a velocity of 20 mph

Work for problem 3(b)

$$\text{average rate change} = \frac{w(b) - w(a)}{b - a}$$

$$= \frac{w(60) - w(5)}{b - a}$$

$$= \frac{(55.6 - 22.1(60)^{.16}) - (55.6 - 22.1(5)}{60 - 5}$$

$$\frac{13.0503 - 27.0091}{55}$$

$$= -.2537$$

avg rate of change $\boxed{= -.254 \text{ m/hr}}$

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Continue problem 3 on

Work for problem 3(c)

$\frac{dv}{dt} = 5$ m/sec $t=0$, $V=20$

$V = 20 + 5t \quad x = 3$

$@ V_{t=3} = 20 + 15$

$= 45$

$$\frac{w(45) - w(20)}{45-20}$$

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END OF PART A OF SECTION II

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AP® CALCULUS BC
2007 SCORING COMMENTARY (Form B)

Question 3

Sample: 3A
Score: 9

The student earned all 9 points. The answer of 22.989 in part (b) is acceptable. In this case the student sets $W'(v)$ equal to the correct average rate of change rounded to three decimal places and correctly solves for v.

Sample: 3B
Score: 6

The student earned 6 points: 1 point in part (a), 3 points in part (b), 2 points in part (c), and no units point. In part (a) $W'(20)$ is correct, but the student does not give a complete explanation. It was necessary for the student to appeal to the fact that the wind chill is decreasing and not merely changing. In part (b) the student calls the function F instead of W but correctly finds the average rate of change. In part (c) the student earned the first 2 points but does not apply the chain rule to come up with the required answer. The student does not use correct units.

Sample: 3C
Score: 3

The student earned 3 points: 1 point in part (a), 1 point in part (b), 1 point in part (c), and no units point. In part (a) $W'(20)$ is correct, but the student does not explain that the wind chill is decreasing. In part (b) the student earned the first point for the average rate of change. In part (c) the student earned the first point but makes a mistake in calculating the velocity at $t = 3$, so the second point was not earned. Although the student was eligible for the third point, it was not earned since $\frac{dW}{dt}$ was not found at $t = 3$. The student does not use correct units.