

# AP<sup>®</sup> Calculus AB 2007 Free-Response Questions

## The College Board: Connecting Students to College Success

The College Board is a not-for-profit membership association whose mission is to connect students to college success and opportunity. Founded in 1900, the association is composed of more than 5,000 schools, colleges, universities, and other educational organizations. Each year, the College Board serves seven million students and their parents, 23,000 high schools, and 3,500 colleges through major programs and services in college admissions, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT<sup>®</sup>, the PSAT/NMSQT<sup>®</sup>, and the Advanced Placement Program<sup>®</sup> (AP<sup>®</sup>). The College Board is committed to the principles of excellence and equity, and that commitment is embodied in all of its programs, services, activities, and concerns.

© 2007 The College Board. All rights reserved. College Board, Advanced Placement Program, AP, AP Central, SAT, and the acorn logo are registered trademarks of the College Board. PSAT/NMSQT is a registered trademark of the College Board and National Merit Scholarship Corporation.

Permission to use copyrighted College Board materials may be requested online at: www.collegeboard.com/inquiry/cbpermit.html.

Visit the College Board on the Web: www.collegeboard.com. AP Central is the official online home for the AP Program: apcentral.collegeboard.com.

# CALCULUS AB SECTION II, Part A Time—45 minutes Number of problems—3

#### A graphing calculator is required for some problems or parts of problems.

1. Let *R* be the region in the first and second quadrants bounded above by the graph of  $y = \frac{20}{1 + x^2}$  and below by the horizontal line y = 2.

(a) Find the area of *R*.

- (b) Find the volume of the solid generated when R is rotated about the x-axis.
- (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the *x*-axis are semicircles. Find the volume of this solid.

#### WRITE ALL WORK IN THE PINK EXAM BOOKLET.



- 2. The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval  $0 \le t \le 7$ , where t is measured in hours. In this model, rates are given as follows:
  - (i) The rate at which water enters the tank is  $f(t) = 100t^2 \sin(\sqrt{t})$  gallons per hour for  $0 \le t \le 7$ .
  - (ii) The rate at which water leaves the tank is

 $g(t) = \begin{cases} 250 & \text{for } 0 \le t < 3\\ 2000 & \text{for } 3 < t \le 7 \end{cases}$  gallons per hour.

The graphs of f and g, which intersect at t = 1.617 and t = 5.076, are shown in the figure above. At time t = 0, the amount of water in the tank is 5000 gallons.

- (a) How many gallons of water enter the tank during the time interval  $0 \le t \le 7$ ? Round your answer to the nearest gallon.
- (b) For  $0 \le t \le 7$ , find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.
- (c) For  $0 \le t \le 7$ , at what time *t* is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

#### WRITE ALL WORK IN THE PINK EXAM BOOKLET.

© 2007 The College Board. All rights reserved. Visit apcentral.collegeboard.com (for AP professionals) and www.collegeboard.com/apstudents (for students and parents).

x	f(x)	f'(x)	g(x)	g'(x)	
1	6	4	2	5	
2	9	2	3	1	
3	10	-4	4	2	
4	-1	3	6	7	

- 3. The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by h(x) = f(g(x)) 6.
  - (a) Explain why there must be a value r for 1 < r < 3 such that h(r) = -5.
  - (b) Explain why there must be a value c for 1 < c < 3 such that h'(c) = -5.
  - (c) Let w be the function given by  $w(x) = \int_{1}^{g(x)} f(t) dt$ . Find the value of w'(3).
  - (d) If  $g^{-1}$  is the inverse function of g, write an equation for the line tangent to the graph of  $y = g^{-1}(x)$  at x = 2.

#### WRITE ALL WORK IN THE PINK EXAM BOOKLET.

## **END OF PART A OF SECTION II**

# CALCULUS AB SECTION II, Part B Time—45 minutes Number of problems—3

No calculator is allowed for these problems.

- 4. A particle moves along the x-axis with position at time t given by  $x(t) = e^{-t} \sin t$  for  $0 \le t \le 2\pi$ .
  - (a) Find the time t at which the particle is farthest to the left. Justify your answer.
  - (b) Find the value of the constant A for which x(t) satisfies the equation Ax''(t) + x'(t) + x(t) = 0 for  $0 < t < 2\pi$ .

t (minutes)	0	2	5	7	11	12
r'(t) (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

5. The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t, where t is measured in minutes. For 0 < t < 12, the graph of r is concave down. The table above gives selected values of the rate of change, r'(t), of the radius of the balloon over the time interval  $0 \le t \le 12$ . The radius of the balloon is 30 feet when t = 5. (Note: The volume of a sphere of radius r is given by  $V = \frac{4}{3}\pi r^3$ .)

- (a) Estimate the radius of the balloon when t = 5.4 using the tangent line approximation at t = 5. Is your estimate greater than or less than the true value? Give a reason for your answer.
- (b) Find the rate of change of the volume of the balloon with respect to time when t = 5. Indicate units of measure.
- (c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate  $\int_0^{12} r'(t) dt$ . Using correct units, explain the meaning of  $\int_0^{12} r'(t) dt$  in terms of the radius of the balloon.
- (d) Is your approximation in part (c) greater than or less than  $\int_{0}^{12} r'(t) dt$ ? Give a reason for your answer.

#### WRITE ALL WORK IN THE PINK EXAM BOOKLET.

© 2007 The College Board. All rights reserved.

Visit apcentral.collegeboard.com (for AP professionals) and www.collegeboard.com/apstudents (for students and parents).

- 6. Let f be the function defined by  $f(x) = k\sqrt{x} \ln x$  for x > 0, where k is a positive constant.
  - (a) Find f'(x) and f''(x).
  - (b) For what value of the constant k does f have a critical point at x = 1? For this value of k, determine whether f has a relative minimum, relative maximum, or neither at x = 1. Justify your answer.
  - (c) For a certain value of the constant k, the graph of f has a point of inflection on the x-axis. Find this value of k.

### WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF EXAM