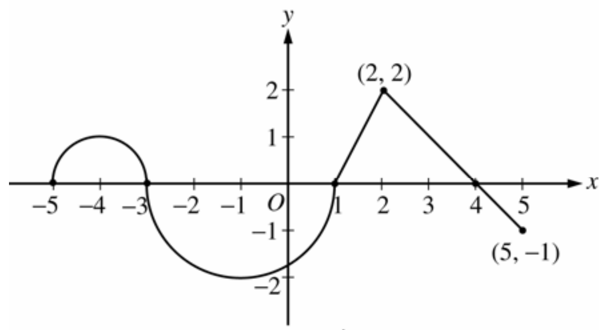
AP® CALCULUS AB
2007 SCORING GUIDELINES (Form B)

Question 4

Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.



Graph of f'

(a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.

(b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.

(c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.

(d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \le x \le 5$. Explain your reasoning.

(a) $f'(x) = 0$ at $x = -3,\ 1,\ 4$

f' changes from positive to negative at -3 and 4.

Thus, f has a relative maximum at $x = -3$ and at $x = 4$.

$2: \begin{cases} 1: x\text{-values} \\ 1: \text{justification} \end{cases}$

(b) f' changes from increasing to decreasing, or vice versa, at $x = -4,\ -1,$ and 2. Thus, the graph of f has points of inflection when $x = -4,\ -1,$ and 2.

$2: \begin{cases} 1: x\text{-values} \\ 1: \text{justification} \end{cases}$

(c) The graph of f is concave up with positive slope where f' is increasing and positive: $-5 < x < -4$ and $1 < x < 2$.

$2: \begin{cases} 1: \text{intervals} \\ 1: \text{explanation} \end{cases}$

(d) Candidates for the absolute minimum are where f' changes from negative to positive (at $x = 1$) and at the endpoints ($x = -5, 5$).

$$f(-5) = 3 + \int_1^{-5} f'(x)\,dx = 3 - \frac{\pi}{2} + 2\pi > 3$$

$$f(1) = 3$$

$$f(5) = 3 + \int_1^{5} f'(x)\,dx = 3 + \frac{3 \cdot 2}{2} - \frac{1}{2} > 3$$

The absolute minimum value of f on $[-5, 5]$ is $f(1) = 3$.

$3: \begin{cases} 1: \text{identifies } x = 1 \text{ as a candidate} \\ 1: \text{considers endpoints} \\ 1: \text{value and explanation} \end{cases}$

NO CALCULATOR ALLOWED

4A1

CALCULUS AB

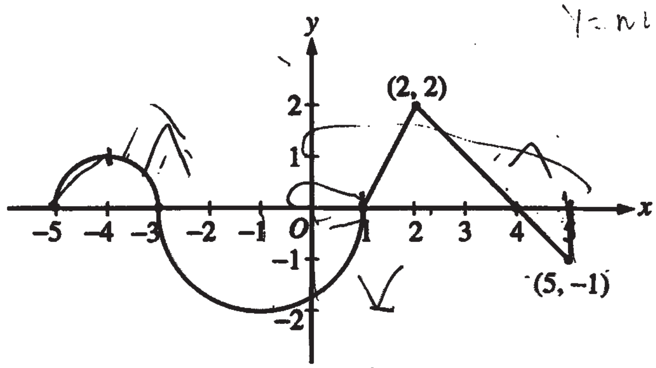
SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

y=mx+b



Graph of f'

Work for problem 4(a)

a) relative maximum at $x=-3,\ 4$

At $x=-3,\ 4$, the graph of f' change from positive to negative, which hints the graph of f change from increase to decrease. So at $x=-3,\ 4$, f has relative maximums

Work for problem 4(b)

points of inflection at $x=-4, -1, 2$

at all these x points, the graph of f' change from increase to decrease or from decrease to increase, which hints at these points, f change from concave up to concave down or concave down to concave up,

Continue problem 4 on page 11.

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4A2

Work for problem 4(c)

when $-5 < x < -4$, $1 < x < 2$,

the graph of f is concave up and also has positive slope.

From the graph of f', when $-5 < x < -4$ and $1 < x < 2$,

the graph of f' is both increasing and above x-axis,

which shows f' and f'' are both positive.

positive f' means the slope of f is positive

and positive f'' means f is concave upward.

Work for problem 4(d)

From the graph of f', the only local minimum of f is at $x = 1$., $f(1) = 3$

$$\int_{-5}^{5} f'(x)\,dx = F(5) - F(-5) = 2\pi - 8\pi + 3 - \frac{1}{2}$$

$$= \frac{5}{2} - 6\pi < 0$$

so $F(5) < F(-5)$

$$\int_{1}^{5} f'(x)\,dx = F(5) - F(1) = \frac{3 \times 2}{2} - \frac{1}{2} = \frac{5}{2} > 0$$

so $F(5) > F(1)$

thus the absolute minimum value of $f(x)$ over the close interval $-5 \le x \le 5$. is 3.

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NO CALCULATOR ALLOWED

4B1

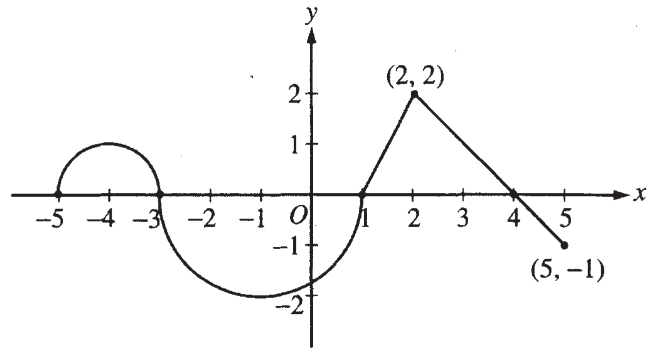
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of f'

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Work for problem 4(a)

~~at $x = -4$, and $x = 2$~~

~~at these points, f' changes from increasing to decreasing~~

at $x = -3$, $x = 4$

at these points f' changes from positive to negative

Work for problem 4(b)

at $x = -4$, $x = -1$,

at these points f' changes from increasing to decreasing

Continue problem 4 on page 11.

4B2

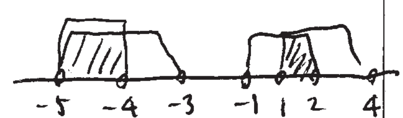
Work for problem 4(c)

f is concave up and has positive slope when $f''(x) > 0$ and $f'(x) > 0$

$f''(x) > 0$ means the slope of f' is positive.

so $f''(x) > 0$ when $(-5, -4)$, $(-1, 2)$,

$f'(x) > 0$ when $(-5, -3)$, $(1, 4)$,



the intervals are $(-5, -4)$, $(1, 2)$

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Work for problem 4(d)

$x^2 + y^2 = 1$ $\quad y^2 = 1 - x^2$ $\quad y = \sqrt{1 - x^2}$

$f(x)$ is minimum at the endpoints or at $x = 1$ because f' changes from negative to positive at $x = 1$.

$f(-5) =$

$f(1) = 3$

$f(5) = -\frac{5^2}{2} + 4 \cdot 5 - \frac{1}{2} = -14,$

$\frac{2+1}{2-5} = -1$ $\quad y - 2 = -(x - 2)$ $\quad f(1) = -\frac{1}{2} + 4 + C = 3$

$C = -\frac{1}{2}$

$y = -x + 4,$ $\quad f(x) = \int (-x+4)\,dx = -\frac{x^2}{2} + 4x + C$

$\frac{-25}{2} + \frac{40}{2} - \frac{1}{2} = -14$

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NO CALCULATOR ALLOWED

4C1

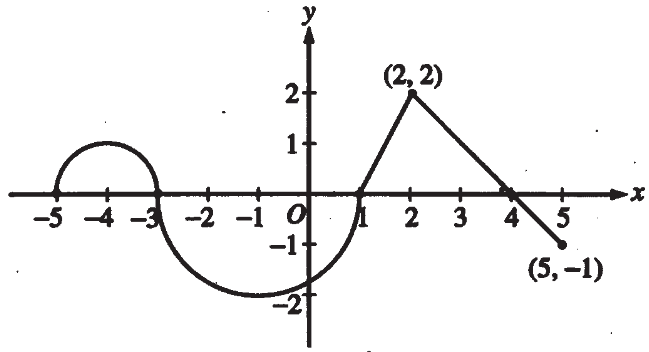
CALCULUS AB

SECTION II, Part B

Time—45 minutes

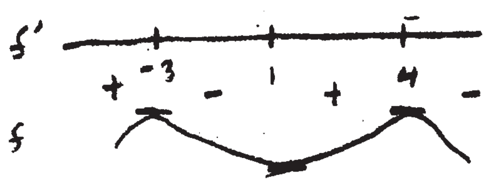
Number of problems—3

No calculator is allowed for these problems.



Graph of f'

Work for problem 4(a)



$f'(x)$ changes sign at $x = -3, 1, 4$

changes sign from − to + at $x = -3, 4$

$\therefore\ x = -3, 4$

Work for problem 4(b)

point of inflection occur when $f''(x) = 0$ or is undefined.

$\therefore\ x = -4, -3, -1, 1, 2$

Continue problem 4 on page 11.

4C 2

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Work for problem 4(c)

f': -3, 1, 4

$+ \quad - \quad + \quad -$

f'': -4, -3, -1, 1, 2

$+ \quad - \quad - \quad + \quad + \quad -$

$\therefore \ (-5,-4), \ (1,2)$

Work for problem 4(d)

$\frac{1}{2}\pi - \frac{1}{2}4\pi = -\frac{3}{2}\pi$

$f'(x) < 0$ at $-3<x<1$

so

$f(x)$ decrease at $-3<x<1$

$f'(x)<0$ also at $4<x<5$

but

$\left| \int_1^4 f(x) \right| > \left| \int_4^5 f'(x) \right|$

there fore

$f(x)$ have its absolute minimum value at $x=1$

$\boxed{\therefore \ -\frac{3}{2}\pi}$

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AP® CALCULUS AB
2007 SCORING COMMENTARY (Form B)

Question 4

Sample: 4A
Score: 9

The student earned all 9 points.

Sample: 4B
Score: 6

The student earned 6 points: 2 points in part (a), no points in part (b), 2 points in part (c), and 2 points in part (d). Correct work is presented in parts (a) and (c). In part (b) the student only finds two of the three values, so the first point was not earned. The justification point was not earned because it is not true that f' changes from increasing to decreasing at $x = -1.$ In part (d) the student earned the first 2 points since $x = 1$ is identified as a candidate and the endpoints are considered. Since the student never concludes that the absolute minimum is 3, the third point was not earned.

Sample: 4C
Score: 4

The student earned 4 points: 2 points in part (a), no points in part (b), 1 point in part (c), and 1 point in part (d). Correct work is presented in part (a). In part (b) the student gives two additional, incorrect values, so the first point was not earned. No justification is included. In part (c) the first point is earned because of the correct intervals. The student's sign chart alone did not earn the explanation point. It was necessary to explain the reasoning from the sign chart. In part (d) the student earned the first point since $x = 1$ is identified as a candidate. The student does not consider both endpoints and does not give a correct answer, so the last 2 points were not earned.