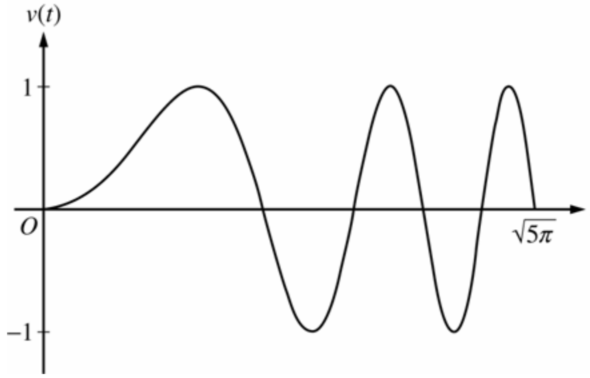
AP® CALCULUS AB
2007 SCORING GUIDELINES (Form B)

Question 2

A particle moves along the x-axis so that its velocity v at time $t \ge 0$ is given by $v(t) = \sin\left(t^2\right)$. The graph of v is shown above for $0 \le t \le \sqrt{5\pi}$. The position of the particle at time t is $x(t)$ and its position at time $t = 0$ is $x(0) = 5$.



(a) Find the acceleration of the particle at time $t = 3$.

(b) Find the total distance traveled by the particle from time $t = 0$ to $t = 3$.

(c) Find the position of the particle at time $t = 3$.

(d) For $0 \le t \le \sqrt{5\pi}$, find the time t at which the particle is farthest to the right. Explain your answer.

(a) $a(3) = v'(3) = 6\cos 9 = -5.466$ or -5.467

$1 : a(3)$

(b) Distance $= \int_0^3 |v(t)|\, dt = 1.702$

OR

For $0 < t < 3$, $v(t) = 0$ when $t = \sqrt{\pi} = 1.77245$ and $t = \sqrt{2\pi} = 2.50663$

$x(0) = 5$

$x\left(\sqrt{\pi}\right) = 5 + \int_0^{\sqrt{\pi}} v(t)\, dt = 5.89483$

$x\left(\sqrt{2\pi}\right) = 5 + \int_0^{\sqrt{2\pi}} v(t)\, dt = 5.43041$

$x(3) = 5 + \int_0^3 v(t)\, dt = 5.77356$

$\left|x\left(\sqrt{\pi}\right) - x(0)\right| + \left|x\left(\sqrt{2\pi}\right) - x\left(\sqrt{\pi}\right)\right| + \left|x(3) - x\left(\sqrt{2\pi}\right)\right| = 1.702$

$2 : \begin{cases} 1 : \text{setup} \\ 1 : \text{answer} \end{cases}$

(c) $x(3) = 5 + \int_0^3 v(t)\, dt = 5.773$ or 5.774

$3 : \begin{cases} 2 \begin{cases} 1 : \text{integrand} \\ 1 : \text{uses } x(0) = 5 \end{cases} \\ 1 : \text{answer} \end{cases}$

(d) The particle's rightmost position occurs at time $t = \sqrt{\pi} = 1.772$.

The particle changes from moving right to moving left at those times t for which $v(t) = 0$ with $v(t)$ changing from positive to negative, namely at $t = \sqrt{\pi}, \sqrt{3\pi}, \sqrt{5\pi}$ $(t = 1.772, 3.070, 3.963)$.

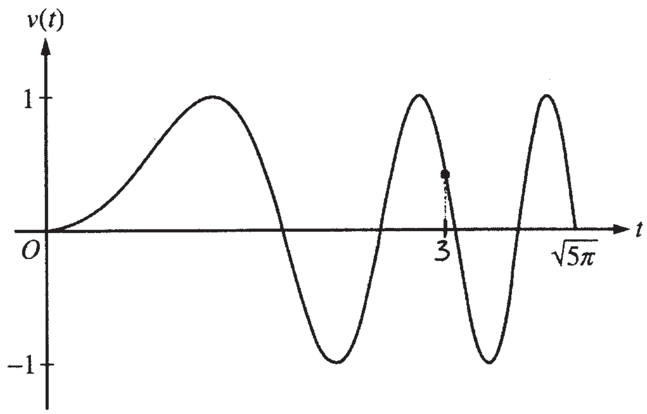
Using $x(T) = 5 + \int_0^T v(t)\, dt$, the particle's positions at the times it changes from rightward to leftward movement are:

T:	0	$\sqrt{\pi}$	$\sqrt{3\pi}$	$\sqrt{5\pi}$
$x(T)$:	5	5.895	5.788	5.752

The particle is farthest to the right when $T = \sqrt{\pi}$.

$3 : \begin{cases} 1 : \text{sets } v(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

AB
2 A_1



Work for problem 2(a)

$a(t) = v'(t) = \cos(t^2)(2t)$
$= 2t\cos(t^2)$

$a(3) = 6\cos 9 = -5.467$

Work for problem 2(b)

The object reverses direction twice before $t = 3$.

$v(t) = \sin(t^2) = 0$
$t = \{1.772, 2.507\}$

Distance traveled $= \left|\int_0^{1.772} v(t)\,dt\right| + \left|\int_{1.772}^{2.507} v(t)\,dt\right| + \left|\int_{2.507}^{3} v(t)\,dt\right|$

$= 0.895 + 0.464 + 0.343$
$= 1.702$

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Continue problem 2 on page 7.

AB
2A2

Work for problem 2(c)

Position at $t=3$ is the initial position plus net distance traveled.

$$x(3) = x(0) + \int_0^3 v(t)\,dt$$
$$= 5 + 0.774$$
$$= 5.774$$

Work for problem 2(d)

When the object reaches right and reverses direction, $x(t)$ has reached a relative maximum.

$\therefore$ $v(t) = 0$ for which $v(t)$ changes from +ve to −ve.

$t = \{1.772, 3.070, 3.963\}$

Comparing $x(t_1) = \int_0^{t_1} v(t)\,dt + 5$:

t_1	x
1.772	5.895
3.070	5.788
3.963	5.752

$\therefore$ As t increases, the maximum displacement to the right decreases.

$\therefore$ $x(t)$ has an absolute maximum at $t = 1.772$.

$\therefore$ The object is farthest right at $t = 1.772$.

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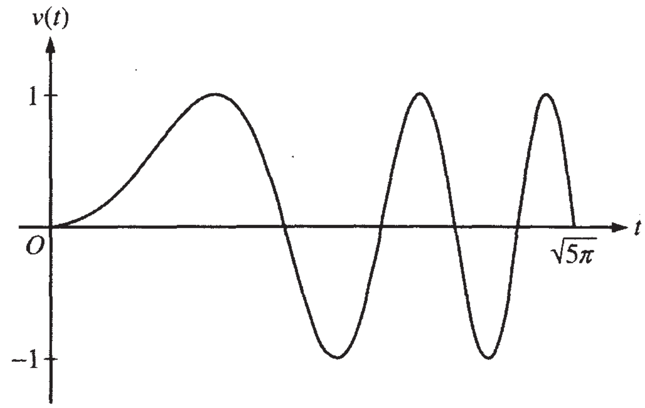
AB 2
2B$_1$



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Work for problem 2(a)

Acceleration: $a(t) \rightarrow$ rate of change of velocity $= v'(t)$

$\therefore a(t) = (\sin(t^2))' = 2t\cos t$

$\therefore a(3) = 2(3)\cdot\cos(3) = 6\cos 3 \approx -5.940$

Work for problem 2(b)

Total distance traveled accumulative velocity $\rightarrow \int_a^b v(t)\,dt$

$\therefore$ Total distance traveled $\cdot \int_0^{\sqrt{\pi}} \sin(t^2)\,dt - \int_{\sqrt{\pi}}^{\sqrt{2\pi}} \sin(t)\,dt + \int_{\sqrt{2\pi}}^{3} \sin(t^2)\,dt$

$= 1.702$

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Continue problem 2 on page 7.

AB2
2B₂

Work for problem 2(c)

velocity: v(t) → rate of change of position.

Because initial position = 5, the position at t=3 is

$5 + \int_0^3 \sin t^2 \, dt = 5.774$

Work for problem 2(d)

when $t = \sqrt{\pi}$, the value of $\int_0^t v(t)\,dt$ becomes the greatest

So, the particle is farthest to the right when $t = \sqrt{\pi}$

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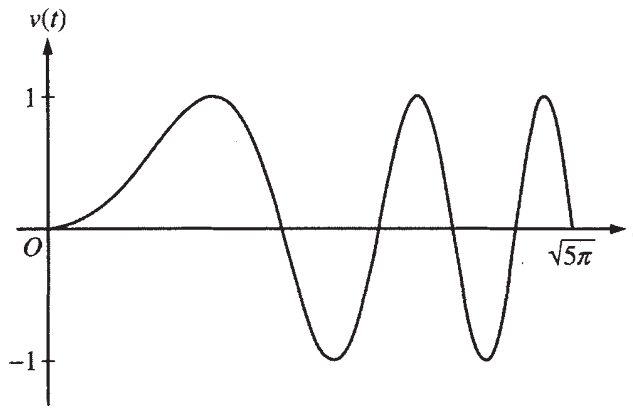

Work for problem 2(a)

$a = \frac{dv}{dt} = \frac{d \sin t^2}{dt} = 2t\cos t^2$

$a(3) = 6\cos 6^2 = -0.767$ unit/s²

the particle is decelerating

Work for problem 2(b)

$\text{dis} = x(t) = \int_0^3 \& \, v(t) = \int_0^3 \sin(t^2)\, dt$

$= 0.774$ unit

Continue problem 2 on page 7.

AB2
2C 1/2

Work for problem 2(c)

$x(t) = \int_0 \sin(t^2)\, dt$

$x(t) = \frac{-1}{2t}\cos(t^2) + C \qquad x(0) = 5$

$5 = C$

$x(t) = -\frac{\cos t^2}{2t} + 5$

$x(3) = 5.152$ unit.

Work for problem 2(d)

It is farthest to the right at $\sqrt{\pi}$ as area above the graph is greater than the area below the graph.

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Question 2

Sample: 2A
Score: 9

The student earned all 9 points.

Sample: 2B
Score: 6

The student earned 6 points: no points in part (a), 2 points in part (b), 3 points in part (c), and 1 point in part (d). Correct work is presented in parts (b) and (c). In part (a) the student did not earn the first point because the derivative of $v(t)$ is incorrect. The student could have used the graphing calculator to find the numerical derivative. In part (d) the student does not set $v(t) = 0,$ so the first point was not earned. The answer point was earned but not the reason point since the student does not explicitly rule out the other times for which $v(t) = 0.$

Sample: 2C
Score: 3

The student earned 3 points: no points in part (a), no points in part (b), 2 points in part (c), and 1 point in part (d). In part (a) the derivative of $v(t)$ is correct, but the student makes an error when evaluating the acceleration at $t = 3.$ In part (b) the student integrates the velocity to find displacement instead of integrating the speed to find distance traveled. In this case, since the particle changes direction on the interval from $t = 0$ to $t = 3,$ displacement is not the same as distance traveled. In part (c) the student has a correct integrand and uses $x(0) = 5,$ which earned the first 2 points. The student attempts to find the antiderivative of $v(t)$ but did not earn the last point. In part (d) the student does not set $v(t) = 0,$ so the first point was not earned. The answer point was earned but not the reason point since the student does not explicitly rule out the other times when $v(t) = 0.$