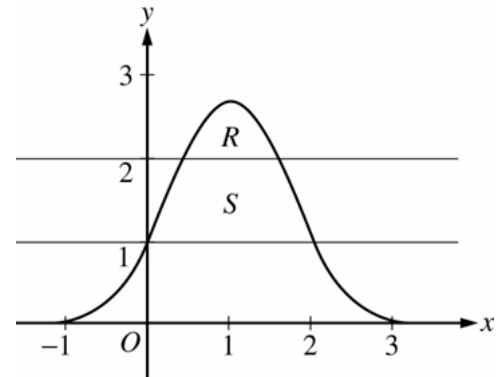


**AP<sup>®</sup> CALCULUS AB**  
**2007 SCORING GUIDELINES (Form B)**

**Question 1**

Let  $R$  be the region bounded by the graph of  $y = e^{2x-x^2}$  and the horizontal line  $y = 2$ , and let  $S$  be the region bounded by the graph of  $y = e^{2x-x^2}$  and the horizontal lines  $y = 1$  and  $y = 2$ , as shown above.



- (a) Find the area of  $R$ .  
 (b) Find the area of  $S$ .  
 (c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when  $R$  is rotated about the horizontal line  $y = 1$ .

$e^{2x-x^2} = 2$  when  $x = 0.446057, 1.553943$   
 Let  $P = 0.446057$  and  $Q = 1.553943$

(a) Area of  $R = \int_P^Q (e^{2x-x^2} - 2) dx = 0.514$

3 : { 1 : integrand  
 1 : limits  
 1 : answer

(b)  $e^{2x-x^2} = 1$  when  $x = 0, 2$

Area of  $S = \int_0^2 (e^{2x-x^2} - 1) dx - \text{Area of } R$   
 $= 2.06016 - \text{Area of } R = 1.546$

3 : { 1 : integrand  
 1 : limits  
 1 : answer

OR

$\int_0^P (e^{2x-x^2} - 1) dx + (Q - P) \cdot 1 + \int_Q^2 (e^{2x-x^2} - 1) dx$   
 $= 0.219064 + 1.107886 + 0.219064 = 1.546$

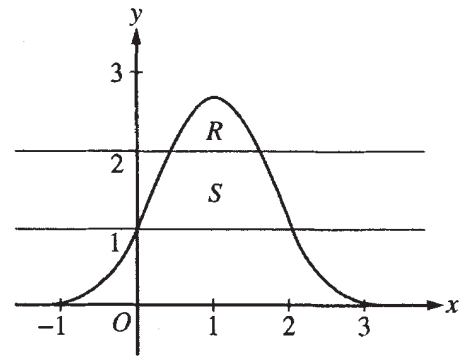
(c) Volume =  $\pi \int_P^Q \left( (e^{2x-x^2} - 1)^2 - (2 - 1)^2 \right) dx$

3 : { 2 : integrand  
 1 : constant and limits

CALCULUS AB  
SECTION II, Part A  
Time—45 minutes  
Number of problems—3

1A<sub>1</sub>

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$$e^{2x-x^2} = 2 \Rightarrow x = 0.446 \text{ and } x = 1.554$$

let  $a = 0.446$  and  $b = 1.554$

$$\text{Area} = \int_a^b e^{2x-x^2} - 2 \, dx$$

$$= \int_{0.446}^{1.554} e^{2x-x^2} - 2 \, dx$$

$$= 0.514 \text{ unit}^2$$

Do not write beyond this border.

Do not write beyond this border.

Continue problem 1 on page 5.

Work for problem 1(b)

$$\begin{aligned}\text{Area of } S &= \int_0^2 e^{2x-x^2} - 1 \, dx - \text{Area of } R \\ &= 2.060 - 0.514 \\ &= 1.546 \text{ unit}^2\end{aligned}$$

Work for problem 1(c)

$$\begin{aligned}V &= \pi \int_a^b (e^{2x-x^2} - 1)^2 - (2-1)^2 \, dx \\ \Rightarrow V &= \pi \int_{0.446}^{1.554} (e^{2x-x^2} - 1)^2 - 1 \, dx\end{aligned}$$

Do not write beyond this border.

Do not write beyond this border.

GO ON TO THE NEXT PAGE.

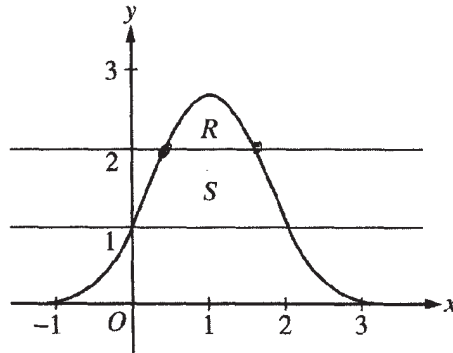
CALCULUS BC  
SECTION II, Part A

1B<sub>1</sub>

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$$(a) y = 2 = e^{2x - x^2}$$

$$\ln 2 = 2x - x^2 \Rightarrow x^2 - 2x + \ln 2 = 0$$

$$x = 1 \pm \sqrt{1 - \ln 2}$$

$$x = 1.554, 0.446$$

$$R = \int_{0.446}^{1.554} e^{2x - x^2} dx - 2x[1.554 - 0.446]$$

$$= 2.730 - 2.216$$

$$= 0.514$$

$$\therefore R = 0.514$$

Do not write beyond this border.

Do not write beyond this border.

Continue problem 1 on page 5.

Work for problem 1(b)

$$y = e^{2x-x^2} = 1$$

$$|m| = 2x - x^2$$

$$0 = x(2-x)$$

$$x = 2, 0$$

$$S = \int_0^2 e^{2x-x^2} dx - R - 2x|$$

$$= 4.060 - 0.514 - 2$$

$$= 1.546$$

$$\therefore S = 1.546$$

Work for problem 1(c)

$$V = \int_0^2 \pi (e^{2x-x^2} - 1)^2 dx$$

Do not write beyond this border.

Do not write beyond this border.

GO ON TO THE NEXT PAGE.

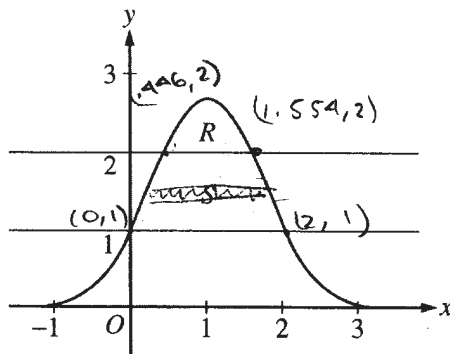
CALCULUS AB  
SECTION II, Part A

Time—45 minutes

Number of problems—3

101

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

a) Area R =  $\int_{0.446}^{1.554} (e^{2x-x^2} - 2) dx = 0.514$

Do not write beyond this border.

Do not write beyond this border.

Continue problem 1 on page 5.

1

1

1

1

1

1

1

1

1

1

102

Work for problem 1(b)

$$\text{Area } S = \int_{.446}^{1.554} (2.718 \cdot e^{2x-x^2}) - (e^{2x-x^2} - 1) dx =$$

Work for problem 1(c)

$$2\pi \int_{.446}^{1.554} (e^{2x-x^2} - 1)^2 dx$$

Do not write beyond this border.

Do not write beyond this border.

GO ON TO THE NEXT PAGE.

**AP<sup>®</sup> CALCULUS AB**  
**2007 SCORING COMMENTARY (Form B)**

**Question 1**

**Sample: 1A**  
**Score: 9**

The student earned all 9 points.

**Sample: 1B**  
**Score: 6**

The student earned 6 points: 3 points in part (a), 3 points in part (b), and no points in part (c). Correct work is presented in parts (a) and (b). Although the student attempts a correct solution by rotating the region  $R + S$  about  $y = 1$ , the response does not subtract the volume obtained when region  $S$  is rotated about  $y = 1$ . The integrand and the limits are incorrect, so the student did not earn any points in part (c).

**Sample: 1C**  
**Score: 3**

The student earned 3 points: 3 points in part (a), no points in part (b), and no points in part (c). The student presents correct work in part (a). Incorrect limits and an incorrect integrand are shown in part (b), so no points were earned. In part (c) the student has an incorrect integrand and so did not earn the first 2 points. The correct limits are shown, but the student did not earn the limits and constant point because of the extra factor of 2 multiplied by the integral.