Question 4

Intent of Question

The primary goals of this question are to evaluate a student's ability to: (1) identify and compute an appropriate confidence interval, after checking the necessary conditions; (2) interpret the interval in the context of the question; and (3) use the confidence interval to conduct an appropriate test of significance.

Solution

Part (a):

Step 1: Identifies the appropriate confidence interval by name or formula and checks appropriate conditions.

Two sample *t* interval for $\mu_A - \mu_S$, the difference in mean waiting times, or

$(\bar{x}_A - \bar{x}_S) \pm t^*_{df}\sqrt{\frac{s_A^2}{n_A} + \frac{s_S^2}{n_S}}$ [See the next page for possible values of *df*.]

Conditions: 1. Independent random samples
2. Large samples or normal population distributions

One sample of 150 patients divided into two groups after sampling does not meet the condition of two independent random samples with fixed sample sizes. Nevertheless, it is reasonable to assume that mode of transportation splits the patients into two independent groups. Secondly, use of the two sample t interval is reasonable because each sample size is large (e.g., $n_A = 77 > 30$ and $n_S = 73 > 30$).

Alternatively, we could assume that the waiting times are (at least approximately) normally distributed, but we have no way to check this assumption with the information provided.

Step 2: Correct Mechanics

Degrees of freedom = $\min\{(77-1),(73-1)\} = 72$.

$(6.04 - 8.30) \pm 2.6459\sqrt{\frac{4.30^2}{77} + \frac{5.16^2}{73}}$

$-2.26 \pm 2.6459 \cdot (0.7777)$

-2.26 ± 2.0577

$(-4.3177, -0.2023)$

Step 3: Interpretation

Based on this sample, we are 99 percent confident that the true difference in the populations' mean waiting times (ambulance – self) is between –4.3177 minutes and –0.2023 minutes.

Equivalently,

With 99 percent confidence, the true mean wait time for those who arrive by ambulance is shorter than those who are self transported by somewhere between 0.2 and 4.3 minutes.

Part (b):

Since zero is not in the 99 percent confidence interval of plausible values for the true difference in means, we can reject $H_0: \mu_A - \mu_S = 0$ in favor of the alternative $H_a: \mu_A - \mu_S \neq 0$ at the $\alpha = .01$ significance level.

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Question 4 (continued)

Thus, we have statistically significant evidence that there is a difference in the mean wait times for the two groups.

Scoring

Part (a) is scored according to the number of correct steps. Each step is scored as essentially correct (E) or incorrect (I). Part (b) is scored as essentially correct (E) or incorrect (I).

Step 1: Identification of method and check of conditions.
A score of essentially correct (E) requires each of the following; otherwise, the score is I.

- A correct confidence interval procedure should be named or a correct formula given.
- Normality must be assessed by either checking that *EACH* sample size is sufficiently large (e.g., >30) OR by stating that each population needs to be normally distributed but that the data are not given to check that.
- It must be stated that the two groups are assumed to be independent random samples.

Step 2: Mechanics
An identifiable minor error in Step 2 will not necessarily change a score from essentially correct to incorrect. The following confidence intervals are all scored E.

Solutions to Step 2			
Procedure	**d.f.**	**t***	**Confidence Interval**
Unequal Variances	140.37	2.61140	(−4.2910, −0.2291)
Large samples	∞	2.576	(−4.26, −0.26)
Pooled variance	148	2.6095	(−4.2797, −0.2404)
Conservative Approach	72	2.6459	(−4.3177, −0.2023)
Unequal Variances	100 (table)	2.626	(−4.302, −0.218)
Unequal Variances	1000 (table)	2.581	(−4.267, −0.253)
Conservative Approach	60 (table)	2.660	(−4.329, −0.191)
Conservative Approach	80 (table)	2.639	(−4.312, −0.208)

Step 3: Interpretation of Confidence Interval
For a score of essentially correct (E), the interpretation must be about a difference of population means AND be in context AND mention the 99 percent level of confidence.

Note: If the interpretation is correct and also explains the meaning of "confidence level," then that extra explanation must be correct for a score of E. If it is not, the score is I.
Note: The correct interpretation of the confidence interval for Step 3 of part (a) may be found in part (b).

Part (b) is essentially correct (E) if the student concludes that the mean times differ because zero is not contained in the 99 percent confidence interval. The confidence level or the significance level must be stated and the conclusion must be stated in context.

Part (b) is incorrect (I) if the student only says the mean wait time for patients transported via ambulance is significantly lower without indicating the significance level adjustment needed for a one-sided test.

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Question 4 (continued)

4 Complete Response

All three steps of the confidence interval in part (a) are essentially correct and part (b) is essentially correct.

3 Substantial Response

All three steps of the confidence interval in part (a) are essentially correct and part (b) is incorrect.
OR
Two steps of the confidence interval in part (a) are essentially correct and part (b) is essentially correct.

2 Developing Response

Two steps of the confidence interval in part (a) are essentially correct and part (b) is incorrect.
OR
One step of the confidence interval in part (a) is essentially correct and part (b) is essentially correct.

1 Minimal Response

One step of the confidence interval in part (a) is essentially correct and part (b) is incorrect.
OR
Part (b) is essentially correct.

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4Ai

4. Patients with heart-attack symptoms arrive at an emergency room either by ambulance or self-transportation provided by themselves, family, or friends. When a patient arrives at the emergency room, the time of arrival is recorded. The time when the patient's diagnostic treatment begins is also recorded.

An administrator of a large hospital wanted to determine whether the mean wait time (time between arrival and diagnostic treatment) for patients with heart-attack symptoms differs according to the mode of transportation. A random sample of 150 patients with heart-attack symptoms who had reported to the emergency room was selected. For each patient, the mode of transportation and wait time were recorded. Summary statistics for each mode of transportation are shown in the table below.

Mode of Transportation	Sample Size	Mean Wait Time (in minutes)	Standard Deviation of Wait Times (in minutes)
Ambulance	77	6.04	4.30
Self	73	8.30	5.16

(a) Use a 99 percent confidence interval to estimate the difference between the mean wait times for ambulance-transported patients and self-transported patients at this emergency room.

2 sample t-interval

$\bar{x}_1 = 6.04$ = avg. minutes in wait time for ambulance patients

$n_1 = 77$ $s_{x_1} = 4.3$

$\bar{x}_2 = 8.3$ = avg. minutes in wait time for self transported patients

$n = 73$ $s_{x_2} = 5.16$

random sample - given ✓

independence - assume ✓

nearly normal $n_1 > 40$ ✓ - central limit theorem

$n_2 > 40$ ✓ - central limit theorem

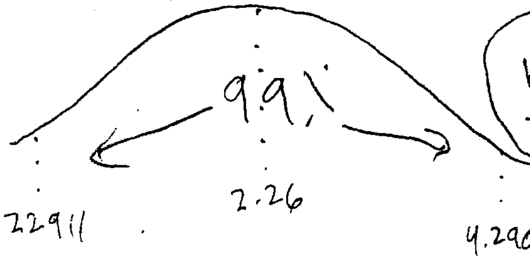
77 < 10% of all patients ✓

73 < 10% of all patients ✓

$t_{df}\left(\bar{x}_2 - \bar{x}_1 \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}\right)$

$t_{140.37176}\left((8.3 - 6.04) \pm t^* \sqrt{\frac{4.3^2}{77} + \frac{5.16^2}{73}}\right) =$

$(.22911, 4.2909)$



We are 99% confident based on our sample that the true average wait time in minutes is between .22911 minutes and 4.2909 minutes longer for self transported patients than for ambulance patients.

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If you need more room for your work for part (a), use the space below.

(b) Based only on this confidence interval, do you think the difference in the mean wait times is statistically significant? Justify your answer.

Yes, because 0 does not appear in ~~the~~ confidence interval this would suggest statistical significance, especially with a confidence level as high as 99%.

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An administrator of a large hospital wanted to determine whether the mean wait time (time between arrival and diagnostic treatment) for patients with heart-attack symptoms differs according to the mode of transportation. A random sample of 150 patients with heart-attack symptoms who had reported to the emergency room was selected. For each patient, the mode of transportation and wait time were recorded. Summary statistics for each mode of transportation are shown in the table below.

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Ambulance	77	6.04	4.30
Self	73	8.30	5.16

(a) Use a 99 percent confidence interval to estimate the difference between the mean wait times for ambulance-transported patients and self-transported patients at this emergency room.

Two - Sample T Interval (for a difference in means)

Tech Conditions

$n_1 > 30$ $n_2 > 30$

$77 > 30$ $73 > 30$

✓ SRS (stated above)

μ_A = mean wait time for patients who arrive by ambulance

μ_S = mean wait time for patients who arrive by self transportation

99% Confidence Interval for a difference in means

$df = 140.37$

$\bar{x}_1 = 6.04$

$\bar{x}_2 = 8.3$

$s_{x_1} = 4.3$

$s_{x_2} = 5.16$

$-4.291 < \mu_A - \mu_S < -.2291$

We can be 99% confident that the true population mean for the difference between average wait time for self transportation and Ambulance is between -.2291 minutes and -4.291 minutes. This means that we are 99% confident that the average wait time for ambulance is between .2291 and 4.291 minutes less than that of self transportation

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4B2

If you need more room for your work for part (a), use the space below.

(b) Based only on this confidence interval, do you think the difference in the mean wait times is statistically significant? Justify your answer.

yes, because 0 is not included in the 99% confidence interval, the duality of confidence intervals and hypothesis testing allows us to reject the null ($\mu_R - \mu_S = 0$) at the $\alpha = .01$ level. ~~[illegible]~~ Assuming the average wait times were equal, we would expect to see results this extreme < 1% of the time. Thus, there is strong evidence suggesting that the mean wait times is statistically significant

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Mode of Transportation	Sample Size	Mean Wait Time (in minutes)	Standard Deviation of Wait Times (in minutes)
Ambulance	77	6.04	4.30
Self	73	8.30	5.16

(a) Use a 99 percent confidence interval to estimate the difference between the mean wait times for ambulance-transported patients and self-transported patients at this emergency room.

μ_1 = mean wait times for ambulance-transported patients in this emergency rm

μ_2 = mean wait times for self-transported patients in this emerge room

* the distributions are approximately normal ($n \geq 30$) ✓✓
* random sample of patients with heart attack symptoms ✓
* independent (ambulance vs self) ✓

$$\bar{X}_1 - \bar{X}_2 \pm t\left(\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right)$$

$$(6.04 - 8.30) \pm 2.626\left(\sqrt{\frac{4.3^2}{77} + \frac{5.16^2}{73}}\right)$$

$$-4.291 \leq \mu_1 - \mu_2 \leq -.2291$$

* The method used successfully captures the population characteristic 99% of the time.

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If you need more room for your work for part (a), use the space below.

(b) Based only on this confidence interval, do you think the difference in the mean wait times is statistically significant? Justify your answer.

Yes, the interval shows only negative values when the difference of the means is taken, which means the mean waiting time for ambulance-transported patients is significantly less than for self-transported patients.

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AP® STATISTICS
2006 SCORING COMMENTARY

Question 4

Overview

The primary goals of this question were to evaluate a student's ability to: (1) identify and compute an appropriate confidence interval, after checking the necessary conditions; (2) interpret the interval in the context of the question; and (3) use that confidence interval to conduct an appropriate test of significance.

Sample: 4A
Score: 4

The correct procedure is identified in addition to the formula on which that method is based. The approximate normality of each sample mean's sampling distribution is appropriately assessed, $(n_1 > 40, n_2 > 40)$, and the central limit theorem is given as the reason why large sample sizes are needed. Independence (of the samples) is invoked and then it is noted that this condition is assumed to be true. As a nice addition, the essay states that the populations from which the two datasets were generated must be large (so that no finite population correction factor is needed). The confidence interval for the difference of population means is correctly computed with intermediate steps shown. An interpretation of the confidence interval is correctly stated—in context, with level stated, and for a difference of population means. While the accompanying picture was of minor concern, overall this interpretation was judged to be essentially correct. A correct test inference is drawn from the fact that the confidence interval excludes 0. The response also states the level of confidence for this inference. This essay earned a score of 4.

Sample: 4B
Score: 3

The correct procedure is identified. The approximate normality of each sample mean's sampling distribution is appropriately assessed although there is no linkage to the central limit theorem. The important assumption that the two samples are independent is not stated or addressed. A correct confidence interval, apparently from a calculator, is provided. A model interpretation of the confidence interval is given in a way that would be accessible to a person who has not studied statistics. An ideal explanation of the duality between confidence intervals and tests is provided, including an explanation of *p*-values. The confidence level and the significance level, as well as the role of 0 being excluded from the interval, are stated, and the conclusion is given in context. This essay earned a score of 3.

Sample: 4C
Score: 2

The confidence interval method is indicated by showing a correct formula. The approximate normality of each sample mean's sampling distribution is appropriately assessed, although there is some ambiguity about exactly what distributions are approximately normal. The assumption of independence is mentioned. A confidence interval is correctly computed, with work nicely shown. The interpretation does not directly communicate what the confidence interval suggests about the difference of mean wait times for the two types of patients. The conclusion about difference of means incorrectly asserts that the two-sided interval can be used for a one-tailed test. This essay earned a score of 2.

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