Question 2

Intent of Question

The primary goals of this question are to evaluate a student's ability to: (1) identify and compute an appropriate confidence interval, after checking the necessary conditions; (2) interpret the interval in the context of the question; and (3) use the confidence interval to conduct an appropriate test of significance.

Solution

Part (a):

Step 1: Identifies the appropriate confidence interval by name or formula and checks appropriate conditions.

Two sample *z* confidence interval for $p_D - p_N$, the difference in the proportions of parts meeting specifications for the two shifts OR $\left(\hat{p}_D - \hat{p}_N\right) \pm z^* \sqrt{\dfrac{\hat{p}_D\left(1-\hat{p}_D\right)}{n_D} + \dfrac{\hat{p}_N\left(1-\hat{p}_N\right)}{n_N}}$.

Conditions:
1. Independent random samples from two separate populations
2. Large samples, so normal approximation can be used

The problem states that random samples of parts were selected from the two different shifts. We need to assume that these parts were produced independently. That is, each employee works the day shift or night shift, but not both, and the machine quality does not vary over time. Since the sample sizes are both 200 and the number of successes (188 and 180) and the number of failures (12 and 20) for each shift are larger than 10, it is reasonable to use the large sample procedures.

Step 2: Correct mechanics

$$\hat{p}_D = \frac{188}{200} = 0.94 \text{ and } \hat{p}_N = \frac{180}{200} = 0.90$$

$$(0.94 - 0.9) \pm 2.0537\sqrt{\frac{0.94 \times 0.06}{200} + \frac{0.9 \times 0.1}{200}}$$

$$0.04 \pm 2.0537 \times 0.0271$$

$$0.04 \pm 0.0556$$

$$(-0.0156, 0.0956)$$

Step 3: Interpretation

Based on these samples, we can be 96 percent confident that the difference in the proportions of parts meeting specifications for the two shifts is between -0.0156 and 0.0956.

Question 2 (continued)

Part (b):

Since zero is in the 96 percent confidence interval, zero is a plausible value for the difference $p_D - p_N$, and we do not have evidence to support the manager's belief. In other words, there does not appear to be a statistically significant difference between the proportions of parts meeting specifications for the two shifts at the $\alpha = 0.04$ level.

Scoring

Part (a) is scored according to the number of correct steps. Each of the first three steps is scored as essentially correct (E) or incorrect (I). Part (b) is scored as essentially correct (E) or incorrect (I).

Notes for Step 1:

The student must identify an appropriate confidence interval and comment on both independence and large sample sizes in order to get this step essentially correct.
Minimum amount of information on independence and large sample sizes needed for an essentially correct response: independence with a check mark AND an indication that the number of successes and the number of failures is larger than 10 (or larger than 5) for both samples.
The student does not need to restate the fact that these are random samples.

Notes for Step 2: An identifiable minor arithmetic error in Step 2 will not necessarily change a score from essentially correct to incorrect.

Alternative Solutions for Step 2	
Procedure	**96% Confidence Interval**
Calculator	(−0.0155652, 0.0955652)
Wilson Estimator	(−0.0169858, 0.0961937)

Part (b) is essentially correct (E) if the student comments on the fact that zero is contained in the confidence interval and the justification links this outcome to a 96 percent confidence level, or a 0.04 significance level, and includes a statement indicating that the data do not support the manager's belief that there is a difference in the proportion of parts that meet specifications produced by the two shifts.

Part (b) is incorrect (I) if the student says no because zero is in the confidence interval OR simply says no without providing relevant justification.

Note: If a 95 percent confidence interval is used, then the maximum score is 3.

Question 2 (continued)

4 Complete Response

All three steps of the confidence interval in part (a) are essentially correct, and part (b) is essentially correct.

3 Substantial Response

All three steps of the confidence interval in part (a) are essentially correct, and part (b) is incorrect.
OR
Two steps of the confidence interval in part (a) are essentially correct and part (b) is essentially correct.

2 Developing Response

Two steps of the confidence interval in part (a) are essentially correct, and part (b) is incorrect.
OR
One step of the confidence interval in part (a) is essentially correct, and part (b) is essentially correct.

1 Minimal Response

One step of the confidence interval in part (a) is essentially correct, and part (b) is incorrect.
OR
Part (b) is essentially correct.

2. A large company has two shifts—a day shift and a night shift. Parts produced by the two shifts must meet the same specifications. The manager of the company believes that there is a difference in the proportions of parts produced within specifications by the two shifts. To investigate this belief, random samples of parts that were produced on each of these shifts were selected. For the day shift, 188 of its 200 selected parts met specifications. For the night shift, 180 of its 200 selected parts met specifications.

(a) Use a 96 percent confidence interval to estimate the difference in the proportions of parts produced within specifications by the two shifts.

Let P_1 = proportion of parts produced within the day shift
P_2 = proportion of parts produced within the night shift.

$p_1 = \frac{188}{200} = 0.94 \quad p_2 = \frac{180}{200} = 0.9$

I would use difference in population proportions z-interval

Conditions
a) random samples are taken ✓
b) samples are taken independently ✓
c) sample size is large
$n_1 p_1 = 200(0.94) = 188 > 5$ ✓
$n_1(1-p_1) = 200(1-0.94) = 12 > 5$ ✓
$n_2 p_2 = 200(0.9) = 180 > 5$ ✓
$n_2(1-p_2) = 200(0.1) = 20 > 5$ ✓

all conditions are satisfied.

$$p_1 - p_2 \pm z^* \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

$$= (0.94 - 0.9) \pm 2.054 \sqrt{\frac{(0.94)(1-0.94)}{200} + \frac{(0.9)(1-0.9)}{200}} = (-0.016, 0.0956)$$

We are 95% confident that the true difference in the proportion of parts produced within specifications by the two shifs is between -0.016 and 0.0956

(b) Based only on this confidence interval, do you think that the difference in the proportions of parts produced within specifications by the two shifts is significantly different from 0 ? Justify your answer.

I do not think that the difference in the proportions of parts produced within specifications by the two shifts is significantly different from zero. Zero lies in the 95% confidence interval and both the lower limit and upper limit of the 95% confidence interval are very close to 0.

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2. A large company has two shifts—a day shift and a night shift. Parts produced by the two shifts must meet the same specifications. The manager of the company believes that there is a difference in the proportions of parts produced within specifications by the two shifts. To investigate this belief, random samples of parts that were produced on each of these shifts were selected. For the day shift, 188 of its 200 selected parts met specifications. For the night shift, 180 of its 200 selected parts met specifications.

(a) Use a 96 percent confidence interval to estimate the difference in the proportions of parts produced within specifications by the two shifts.

n_1 (sample size of day shift) = 200

p_1 (proportion of selected parts met specifications) = $\frac{188}{200}$

n_2 (sample size of night shift) = 200

p_1 (proportion of selected parts met specifications) = $\frac{180}{200}$

P_d : difference in proportions of parts produced within specifications by two shifts

2-Proportion Z Interval will be used.
(C-Level : 0.96)

*Conditions ⇒ $n_1p_1 = 188$, $n_1(1-p_1) = 22$, $n_2p_2 = 180$, $n_2(1-p_2) = 20$ > 10
Random sample. (given), We can assume day shift and night shift are independent.

Left = −0.01557 Right : 0.095565
(−0.01557, 0.095565)

(b) Based only on this confidence interval, do you think that the difference in the proportions of parts produced within specifications by the two shifts is significantly different from 0 ? Justify your answer.

H_0 : $P_d = 0$
H_a : $P_d \neq 0$.

Since 0 is in the 96 percent confidence interval, we have not enough evidence to reject H_0. Therefore, the difference in the proportions of parts produced within specifications by the two shifts is not significantly different from 0.

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2C

2. A large company has two shifts—a day shift and a night shift. Parts produced by the two shifts must meet the same specifications. The manager of the company believes that there is a difference in the proportions of parts produced within specifications by the two shifts. To investigate this belief, random samples of parts that were produced on each of these shifts were selected. For the day shift, 188 of its 200 selected parts met specifications. For the night shift, 180 of its 200 selected parts met specifications.

(a) Use a 96 percent confidence interval to estimate the difference in the proportions of parts produced within specifications by the two shifts.

0.04 ± 0.56

$= -0.0156$ to 0.096 Confidence level 96%

Confidence interval for $p_1 - p_2$, where p_1 is the proportion for the day shift, and p_2 the night shift.

(b) Based only on this confidence interval, do you think that the difference in the proportions of parts produced within specifications by the two shifts is significantly different from 0 ? Justify your answer.

Since the confidence interval contains 0, which indicates there is no difference, the two proportions are not significantly different at $\alpha = 0.04$. Based on the confidence interval, there is not enough evidence to suggest that there is a statistically significant difference between the proportions of the product within specifications for two different shifts.

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AP® STATISTICS
2006 SCORING COMMENTARY (Form B)

Question 2

Sample: 2A
Score: 4

This essay identifies and computes a 96 percent confidence interval for the difference in two population proportions, uses the confidence interval to test the null hypothesis that the proportions are equal, and reaches appropriate conclusions. A correct formula for the confidence interval is provided and correct numerical substitutions into the formula are made. The assumption of independent samples is checked. The quantity $\sqrt{\frac{p_1(1-p_1)}{n_1}+\frac{p_1(1-p_1)}{n_1}}$ may not provide an appropriate standard error if the samples are not independent. The essay also shows that the sample sizes are large enough to accurately use the 98th percentile of the standard normal distribution in constructing the 96 percent confidence interval. The essay provides a good interpretation of the confidence interval with respect to estimating the difference in the proportions of parts that meet specifications for the two shifts. Part (b) clearly indicates that there is not sufficient evidence to support the manager's belief that the proportions are different, because the 96 percent confidence interval for the difference in the proportions contains zero. Although this essay lacks a direct connection to a significance level in part (b), and it switches to a 95 percent confidence level, it was scored as essentially correct.

Sample: 2B
Score: 3

In part (a) the essay clearly identifies the two proportions and corresponding sample sizes. A two-sample Z confidence is specified, and the lower and upper limits are correctly evaluated. Further, the essay presents the assumption that the day shift and night shift results are independent, and it explicitly checks for sufficiently large sample sizes. However, it fails to provide any interpretation of the confidence interval in part (a). The appropriate null and alternative hypotheses are stated in part (b), and an appropriate conclusion is reached by noting that the confidence interval includes zero. This essay also would have been stronger if it had made a connection between the 96 percent level of confidence and the $\alpha = 0.04$ level of significance.

Sample: 2C
Score: 2

This essay provides a good response to part (b) that uses the result that the confidence interval contains zeros to conclude that at the $\alpha = 0.04$ level of significance there is not enough evidence to conclude there is a difference between proportions of parts that meet specifications for the two shifts. In part (a) a correct 96 percent confidence interval is provided, but the essay does not identify the procedure for constructing the confidence, either by formula or in words; it does not address the assumptions of independent samples and large sample sizes; and it does not provide any interpretation of the confidence interval.