General Notes About 2006 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

3. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution contains the application of that equation to the problem but the student does not write the basic equation, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the AP Physics exam equation sheet. See pages 21–22 of the *AP Physics Course Description* for a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each.

4. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

5. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

Question 3

15 points total | **Distribution of points**

(a) and (b)
These two parts were scored together because of the different approaches that could be used to answer them. The parts could be answered in either order.

Approach using translational and rotational dynamics

(a) 5 points

For use of Newton's 2nd law in both translational and rotational forms — 1 point
$\sum F = ma_{cm}$ and $\sum \tau = I\alpha_{cm}$

For a correct equation applying Newton's second law in translational form — 1 point
$Mg\sin\theta - f = Ma_{cm}$

For a correct equation applying Newton's second law in rotational form — 1 point
$fR = I\alpha_{cm}$

For a correct relationship between linear and angular acceleration for rolling without slipping — 1 point
$\alpha_{cm} = \dfrac{a_{cm}}{R}$

Substituting for I and α_{cm} into the rotational equation above
$fR = MR^2 \dfrac{a_{cm}}{R}$
$f = Ma_{cm}$

Substituting this expression for f into the equation for translational motion above
$Mg\sin\theta - Ma_{cm} = Ma_{cm}$

For the correct answer — 1 point
$a_{cm} = \dfrac{g}{2}\sin\theta$

(b) 3 points

For a correct kinematic equation containing a and υ — 1 point
$\upsilon^2 = \upsilon_0^2 + 2a\Delta x$, $\upsilon_0 = 0$

For correct substitution of the expression for acceleration from part (a) — 1 point
For correct substitution of the distance traveled — 1 point
$\upsilon^2 = 2\left(\dfrac{g}{2}\sin\theta\right)L$
$\upsilon = \sqrt{gL\sin\theta}$

Question 3 (continued)

Distribution of points

Approach using torque about point of contact between hoop and ramp and parallel axis theorem

(a) 5 points

For use of Newton's 2nd law in rotational form and the parallel axis theorem — 1 point

$\sum\tau = I\alpha_{cm}$ and $I = I_{cm} + Mh^2$

For a correct rotational inertia about the point of contact using the parallel axis theorem — 1 point

$I = MR^2 + MR^2 = 2MR^2$

For a correct torque about the point of contact — 1 point

$\sum\tau = RMg\sin\theta$

For a correct relationship between linear and angular acceleration for rolling without slipping — 1 point

$\alpha_{cm} = \dfrac{a_{cm}}{R}$

Substituting for $\sum\tau$, I, and α_{cm} into the rotational equation above

$RMg\sin\theta = 2MR^2\dfrac{a_{cm}}{R}$

For the correct answer — 1 point

$a_{cm} = \dfrac{g}{2}\sin\theta$

(b) 3 points

For a solution to part (b) as in the previous approach with points allotted similarly — 3 points

Question 3 (continued)

Distribution of points

Approach using conservation of energy and kinematics, working part (b) first

(b) 5 points

For a statement of conservation of energy containing potential and kinetic energy terms — 1 point

$\Delta U = K_{rot} + K_{trans}$

For a correct expression for the potential energy change — 1 point

For correct translational and rotational kinetic energies — 1 point

$MgL\sin\theta = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$

For a correct relationship between linear and angular velocity for rolling without slipping — 1 point

$\omega = \frac{v}{R}$

Substituting expressions for I and ω into the energy equation above

$MgL\sin\theta = \frac{1}{2}Mv^2 + \frac{1}{2}\left(MR^2\right)\left(\frac{v}{R}\right)^2$

$gL\sin\theta = \frac{1}{2}v^2 + \frac{1}{2}v^2 = v^2$

For the correct answer — 1 point

$v = \sqrt{gL\sin\theta}$

(a) 3 points

For a correct kinematic relationship — 1 point

$v^2 = v_0^2 + 2a\Delta x$, $v_0 = 0$

For correct substitution of the expression for velocity — 1 point

For correct substitution of the distance traveled — 1 point

$gL\sin\theta = 2a_{cm}L$

$a_{cm} = \frac{g}{2}\sin\theta$

Question 3 (continued)

		Distribution of points
(c)	4 points	
	Applying the kinematic equation for distance as a function of time to the vertical motion $H = gt^2/2$	
	For a correct expression for the time between leaving the table and landing on the floor $t = \sqrt{2H/g}$	1 point
	For use of zero acceleration in calculation of the horizontal distance traveled $x = \upsilon_x t$	1 point
	For correct substitution of υ_x from part (b)	1 point
	For correct substitution of *t* from previous calculation $d = \sqrt{gL\sin\theta}\sqrt{2H/g}$ $d = \sqrt{2LH\sin\theta}$	1 point
(d)	3 points	
	For checking the space next to "Greater than"	1 point
	For a sufficiently detailed justification containing no incorrect statements. Such an answer logically concludes, at a minimum, that the linear speed or velocity at the bottom of the ramp is greater for the disk because the rotational inertia of the disk is less. It is not necessary to state that the time of fall is the same.	2 points

One point was awarded for a minimal or partially correct answer.
No justification points were awarded if the space next to "Greater than" was not checked.

Examples of 2-point answers:

A disk will have smaller rotational inertia and will therefore have a greater rotational velocity. This will lead to a greater translational velocity, and a greater distance *x*.

The rotational inertia is less than the hoop, causing greater acceleration and more final speed at the end of the table.

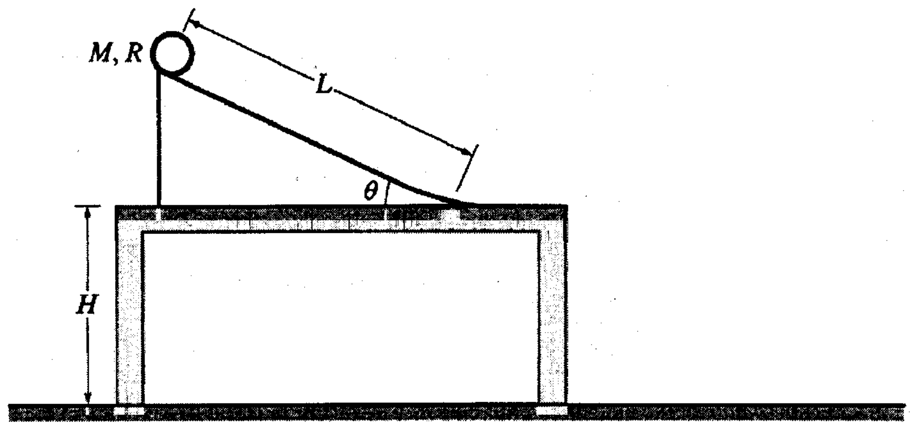
The acceleration when $I = MR^2/2$ is $(2/3)g\sin\theta$, so the disk will be moving faster at the bottom of the ramp and will travel farther.

Examples of 1-point answers:

A disk has a larger rotational inertia, so it will have a greater kinetic energy and will therefore land farther from the ramp.

The moment of inertia for the disk is smaller, thus its rotational velocity is bigger, causing it to go further.

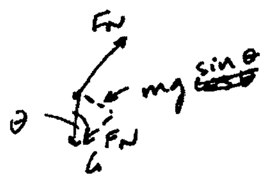
Less energy will be used to spin the disk than the hoop, and *I* of the disk is less than *I* of the hoop.



Mech 3.

A thin hoop of mass M, radius R, and rotational inertia MR^2 is released from rest from the top of the ramp of length L above. The ramp makes an angle θ with respect to a horizontal tabletop to which the ramp is fixed. The table is a height H above the floor. Assume that the hoop rolls without slipping down the ramp and across the table. Express all algebraic answers in terms of given quantities and fundamental constants.

(a) Derive an expression for the acceleration of the center of mass of the hoop as it rolls down the ramp.



F_G along ramp is $M \cdot g \cdot \sin\theta$

~~$Ma = Mg \sin\theta - I$ ... T is the force which~~

$a = \alpha r$, since there's no slipping

$M \cdot a = Mg \sin\theta - F_{FR}$ ← force of static friction which spins wheel

$F_{FR} \cdot R = I\alpha$

~~$F_{FR} \cdot R = MR^2 \frac{a}{R}$~~

~~$F_{FR} = M \cdot a$~~

$F_{FR} \cdot R = MR^2 \cdot \frac{a}{R}$

$F_{FR} = M \cdot a$

$M \cdot a = Mg \sin\theta - M \cdot a$

$2\cancel{M}a = \cancel{M}g \sin\theta$

$a = \boxed{\frac{g \sin\theta}{2}}$

GO ON TO THE NEXT PAGE.

(b) Derive an expression for the speed of the center of mass of the hoop when it reaches the bottom of the ramp.

$$v_f^2 = v_i^2 + 2ad$$

$$v_f^2 = 0 + 2ad$$

$$v_f = \sqrt{2 \cdot a \cdot d}$$

$$= \sqrt{2 \cdot \frac{g \cdot \sin\theta}{2} \cdot L}$$

$$= \boxed{\sqrt{Lg\sin\theta}}$$

(c) Derive an expression for the horizontal distance from the edge of the table to where the hoop lands on the floor.

$$\frac{1}{2}gt^2 = H$$

$$t^2 = \frac{2H}{g} \qquad t = \sqrt{\frac{2H}{g}}$$

v_f is entirely horizontal, so

$$d_x = v_f \cdot t = \sqrt{\frac{2H \cdot L \cdot g \cdot \sin\theta}{g}}$$

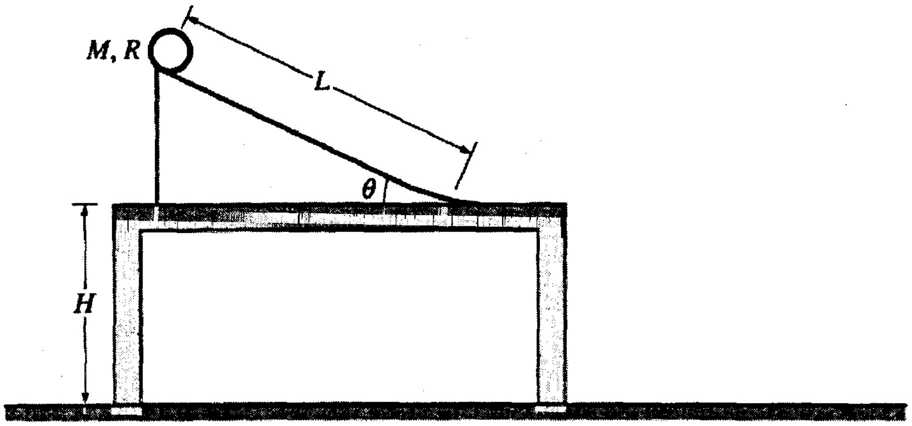
$$= \boxed{\sqrt{2HL\sin\theta}}$$

(d) Suppose that the hoop is now replaced by a disk having the same mass M and radius R. How will the distance from the edge of the table to where the disk lands on the floor compare with the distance determined in part (c) for the hoop?

____ Less than   ____ The same as   ✓ Greater than

Briefly justify your response.

the moment of inertia of a disk is less than that of a hoop, so less energy will go to spinning it up ($\frac{1}{2}I\omega^2$) and more to translational energy ($\frac{1}{2}mv^2$) so greater horizontal speed when it falls



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(a) Derive an expression for the acceleration of the center of mass of the hoop as it rolls down the ramp.

$\alpha = \frac{v^2}{R}$ $\qquad \tau = R \cdot F \qquad \tau = I\alpha$

$\tau = R \cdot ma \qquad \tau = MR^2\alpha$

$\frac{a}{\cancel{R}} = \frac{v^2}{\cancel{R}} \qquad \cancel{R}\cancel{M}a = \cancel{M}R^{\cancel{2}}\alpha$

$a = v^2 \qquad a = R\alpha$

$\alpha = \frac{a}{R}$

from (b): $v = \sqrt{gL\sin\theta}$

$a = gL\sin\theta$

(b) Derive an expression for the speed of the center of mass of the hoop when it reaches the bottom of the ramp.

$h = L\sin\theta$ $\quad ME_1 = ME_2$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2 + \frac{1}{2}(\cancel{M}\cancel{R^2})\frac{v^2}{\cancel{R^2}}$

$gh = \frac{1}{2}v^2 + \frac{1}{2}v^2$

$gh = v^2$

$gL\sin\theta = v^2$

$\sqrt{gL\sin\theta} = v$

(c) Derive an expression for the horizontal distance from the edge of the table to where the hoop lands on the floor.

$H = \frac{1}{2}at^2$ $\quad v = \frac{x}{t}$ - from (a): $a = gL\sin\theta$

$\sqrt{\frac{2H}{a}} = t$ $\quad x = vt$ - from (b): $v = \sqrt{gL\sin\theta}$

$\sqrt{\frac{2H}{gL\sin\theta}} = t$ $\quad x = \left(\sqrt{gL\sin\theta}\right)\left(\sqrt{\frac{2H}{gL\sin\theta}}\right)$

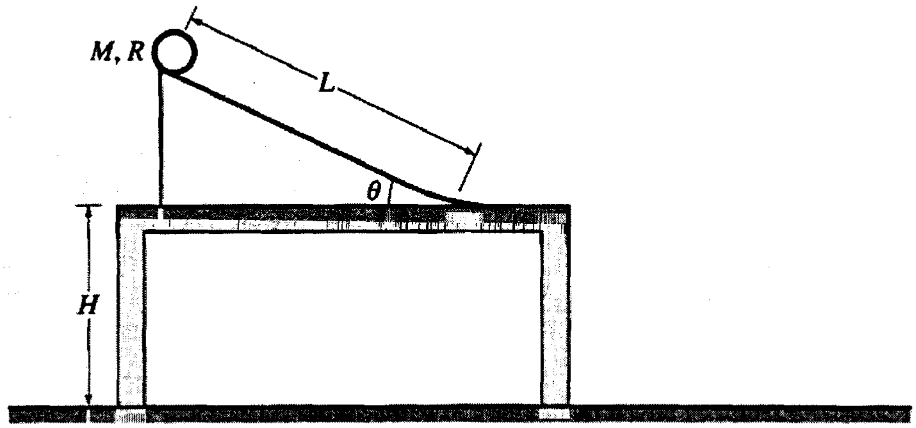
$x = \sqrt{2H}$

(d) Suppose that the hoop is now replaced by a disk having the same mass M and radius R. How will the distance from the edge of the table to where the disk lands on the floor compare with the distance determined in part (c) for the hoop?

____ Less than $\quad$ ____ The same as $\quad$ ✓ Greater than

Briefly justify your response.

for the disc, $I = \frac{1}{2}MR^2$.
when put in part (b), velocity
becomes greater



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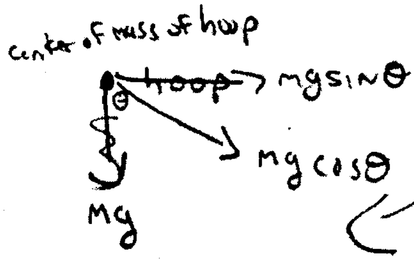


C.M not rotating

$\Sigma F_x = ma$

$mg\cos\theta = ma$

$a = \frac{\cancel{m}g\cos\theta}{\cancel{m}}$

$\boxed{a = g\cos\theta}$

(b) Derive an expression for the speed of the center of mass of the hoop when it reaches the bottom of the ramp.

$a = g\cos\theta$

$\Delta x = L$

$v_0 = 0$

$V_f^2 = V_0^2 + 2a(\Delta x)$

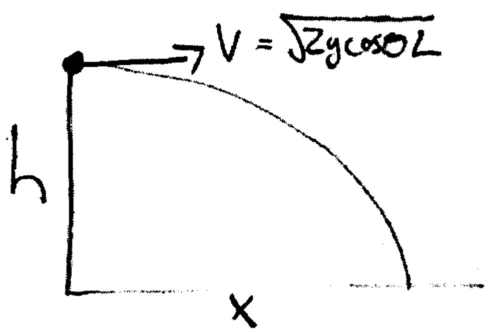
$V_f^2 = 0^2 + 2g\cos\theta L$

C.m. is not rotating

$V_f^2 = 2g\cos\theta L$

$\boxed{V_f = \sqrt{2g\cos\theta L}}$

(c) Derive an expression for the horizontal distance from the edge of the table to where the hoop lands on the floor.



$x = x_0 + V_{0x}t + \cancel{\tfrac{1}{2}at^2}$ $\quad a_x = 0$

$x = 0 + \sqrt{2g\cos\theta L}\, t$

$\boxed{x = (\sqrt{2g\cos\theta L})t}$

(d) Suppose that the hoop is now replaced by a disk having the same mass M and radius R. How will the distance from the edge of the table to where the disk lands on the floor compare with the distance determined in part (c) for the hoop?

____ Less than ✓ The same as ____ Greater than

Briefly justify your response.

If the disk has the same Mass + the same radius, the velocity at the edge of the table will not be affected, therefore the distance it falls will not be affected.

AP® PHYSICS C: MECHANICS
2006 SCORING COMMENTARY

Question 3

Overview

Parts (a) and (b) involved a hoop rolling down an incline and asked students to derive expressions for the acceleration and velocity of the center of mass of the hoop. These parts of the question were intended to test students' understanding of the physical behavior of an object that is rolling down an incline without slipping. Student papers were straightforward to analyze. There were two basic approaches, one based on rotational and translational dynamics using Newton's second law, and a simpler one based on conservation of mechanical energy. After the hoop rolled to the bottom of the incline in parts (a) and (b), it proceeded to roll off a table of a given height in part (c). Students were required to perform a straightforward analysis using only the horizontal velocity of the hoop to find how far from the table it would land. Part (d) asked students to make a prediction regarding a similar situation in which a disk rather than a hoop was rolled down the incline. To analyze part (d) correctly, students needed to understand how the rotational inertia would change if the mass distribution were altered and how this would impact the translational velocity of the object after it rolled down the incline, and hence the horizontal distance it would travel as it fell. This derivation problem required students to show how they arrived at the answer from a recognizable and fundamental starting point.

Sample: M3A
Score: 15

This response uses the dynamics approach to part (a) and the kinematics approach to part (b). All work is done correctly, and full credit was awarded for all the parts to the question. The student wisely crosses off work that is not to be graded.

Sample: M3B
Score: 11

Part (b) is a correct derivation of the expression for the speed using conservation of energy and received all 5 possible points for using this approach. Part (a) is an independent attempt at a dynamics approach to finding the acceleration, but it does not result in the correct answer. Therefore it is eligible to receive at most 2 of the remaining 3 points for parts (a) and (b). But none of the 3 points were awarded since the only point that could be applied using the dynamics approach (for rolling without slipping concept) was awarded in part (b). Since the student indicated in part (a) the answer to part (b), full credit for part (a) could have been achieved had the student correctly applied the kinematic equation relating speed and acceleration. Part (c) lost 1 point for the incorrect expression for the time to fall, but part (d) received full credit.

Sample: M3C
Score: 5

A dynamics approach is attempted in part (a), but Newton's second law in translational form is applied incorrectly and there is no recognition of the need to use Newton's second law in rotational form, so part (a) received no credit. However, the acceleration from part (a) is correctly used in part (b) to calculate the speed, so this part received 3 points full credit. Part (c) received 2 points partial credit; 1 for using the correct equation for nonaccelerated horizontal motion and 1 for correct substitution of the speed from part (b). However, there is no derivation of an expression for the time of fall in terms of the given quantities and fundamental constants. Part (d) is incorrect and received no credit.