Question 2

15 points total | **Distribution of points**

(a) 4 points

From Kirchhoff's loop rule, the sum of the potential differences around the circuit is zero.

For any statement of the loop rule — 1 point

$$\mathcal{E} - V_R - V_C = 0$$

For correct substitution of the potential differences across both resistor and capacitor — 1 point

$$\mathcal{E} - iR_1 - \frac{q}{C} = 0$$

For substituting the differential definition of current — 1 point

$$i = \frac{dq}{dt}$$

For a correct answer including correct signs (a correct answer with no supporting work was awarded full credit) — 1 point

$$\mathcal{E} - R_1\frac{dq}{dt} - \frac{q}{C} = 0$$

Alternate solution — *Alternate points*

For a correct exponential expression for current as a function of time — *1 point*

$$I = I_0 e^{-t/R_1C}$$

For applying Ohm's law to determine the initial current — *1 point*

$$I_0 = \frac{\mathcal{E}}{R_1}$$

For substituting the differential definition of current — *1 point*

$$I = \frac{dq}{dt}$$

For a correct answer (a correct answer with no supporting work was awarded full credit) — *1 point*

$$\frac{dq}{dt} = \frac{\mathcal{E}}{R_1}e^{-t/R_1C}$$

Question 2 (continued)

Distribution of points

(b) 3 points

Using the differential equation from part (a)

$$\mathcal{E} - \frac{dq}{dt}R_1 - \frac{q}{C} = 0$$

For separating the variables of the differential equation — 1 point

$$\frac{dq}{\mathcal{E}C - q} = \frac{dt}{R_1 C}$$

$$\int_0^q \frac{dq}{q - \mathcal{E}C} = \int_0^t -\frac{dt}{R_1 C}$$

For integrating the expression — 1 point

$$\ln(q - \mathcal{E}C)\Big|_0^q = -\frac{t}{R_1 C}\Big|_0^t$$

$$\ln(q - \mathcal{E}C) - \ln(-\mathcal{E}C) = \ln\frac{q - \mathcal{E}C}{-\mathcal{E}C} = -\frac{1}{R_1 C}(t - 0)$$

$$\frac{q - \mathcal{E}C}{-\mathcal{E}C} = e^{-t/R_1 C}$$

$$q - \mathcal{E}C = -\mathcal{E}Ce^{-t/R_1 C}$$

For a correct answer (a correct answer without supporting work in parts (a) or (b) was awarded 1 point) — 1 point

$$q = \mathcal{E}C\left(1 - e^{-t/R_1 C}\right)$$

Alternate solution — *Alternate points*

Using the differential equation from part (a) (alternate)

$$\frac{dq}{dt} = \frac{\mathcal{E}}{R_1}e^{-t/R_1 C}$$

For separating the variables of the differential equation — *1 point*

$$dq = \frac{\mathcal{E}}{R_1}e^{-t/R_1 C}dt$$

$$\int_0^q dq = \int_0^t \frac{\mathcal{E}}{R_1}e^{-t/R_1 C}dt$$

For integrating the expression — *1 point*

$$q\Big|_0^q = \frac{\mathcal{E}}{R_1}\left(-R_1 Ce^{-t/R_1 C}\right)\Big|_0^t$$

$$q = -R_1 C\left(\frac{\mathcal{E}}{R_1}e^{-t/R_1 C} - \frac{\mathcal{E}}{R_1}\right)$$

For a correct answer (a correct answer without supporting work in parts (a) or (b) was awarded 1 point) — *1 point*

$$q = \mathcal{E}C\left(1 - e^{-t/R_1 C}\right)$$

Question 2 (continued)

Distribution of points

(c) 3 points

Using the charge on the capacitor in terms of potential difference and capacitance
$q = CV$

For setting the expression for charge from part (b) equal to the charge on the capacitor — 1 point

$\mathcal{E}C\left(1 - e^{-t/R_1C}\right) = CV$

Solving for the time

$t = R_1C\ln\left(\dfrac{\mathcal{E}}{\mathcal{E} - V}\right)$

Substituting given values into the time expression

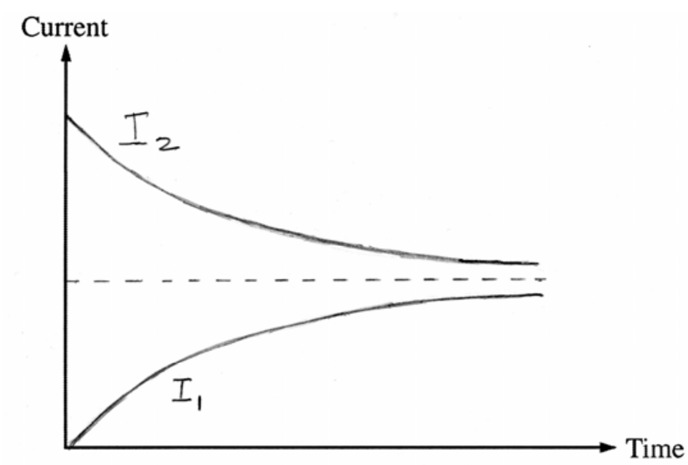
$t = (4700\ \Omega)(0.060\ \text{F})\ln\left(\dfrac{12\ \text{V}}{12\ \text{V} - 4\ \text{V}}\right)$

For the correct answer with units (a correct answer with units with no supporting work was awarded full credit) — 2 points

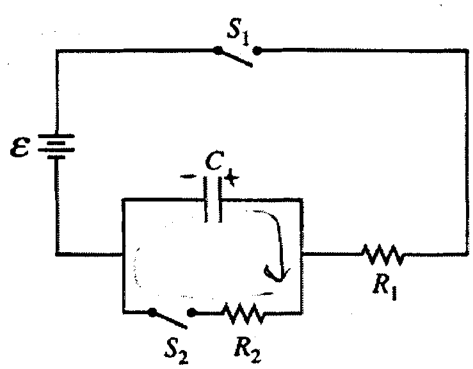
$t = 114\ \text{s}$ (or equivalent such as $t = 282\ln(3/2)\ \text{s}$)

Note: *An incorrect value of time greater than 0 s and less than 282 s (the time constant, R_1C) with correct units earned 1 point.*

(d) 5 points



For having I_2 start at a positive, nonzero value	1 point
For sketching I_2 as exponentially decreasing to a horizontal asymptote	1 point
For having I_1 starting at zero	1 point
For sketching I_1 as exponentially increasing to a horizontal asymptote	1 point
For showing the I_1 and I_2 converging at the same horizontal asymptote	1 point



$C = \frac{Q}{V}$ $\quad V = \frac{Q}{C}$

E&M 2.

The circuit above contains a capacitor of capacitance C, a power supply of emf $\mathcal{E}$, two resistors of resistances R_1 and R_2, and two switches, S_1 and S_2. Initially, the capacitor is uncharged and both switches are open. Switch S_1 then gets closed at time $t = 0$.

(a) Write a differential equation that can be solved to obtain the charge on the capacitor as a function of time t.

$$\mathcal{E} - IR_1 - \frac{Q}{C} = 0 \qquad I = \frac{\partial Q}{\partial t}$$

$$\mathcal{E} - \frac{\partial Q}{\partial t} R_1 - \frac{Q}{C} = 0$$

(b) Solve the differential equation in part (a) to determine the charge on the capacitor as a function of time t.

$$\mathcal{E} - \frac{\partial Q}{\partial t} R_1 - \frac{Q}{C} = 0$$

$$\frac{\mathcal{E}}{\cancel{R}} - \frac{Q}{C} = \frac{\partial Q}{\partial t} R_1$$

$$\frac{\mathcal{E}C - Q}{C} = \frac{\partial Q}{\partial t} R_1$$

$$\int_0^t \frac{1}{R_1 C}\,\partial t = \int_0^Q \frac{\partial Q}{\mathcal{E}C - Q} \qquad u = \mathcal{E}C - Q \quad \partial u = -\partial Q$$

$$\frac{t}{R_1C} = -\ln\left(\frac{\mathcal{E}C - Q}{\mathcal{E}C}\right)$$

$$e^{\frac{-t}{R_1C}} = \frac{\mathcal{E}C - Q}{\mathcal{E}C} \qquad Q = \mathcal{E}C - \mathcal{E}Ce^{\frac{-t}{R_1C}}$$

$$Q(t) = \mathcal{E}C\left(1 - e^{-\frac{t}{R_1C}}\right)$$

Numerical values for the components are given as follows:

$\mathcal{E} = 12\text{ V}$
$C = 0.060\text{ F}$
$R_1 = R_2 = 4700\ \Omega$

(c) Determine the time at which the capacitor has a voltage 4.0 V across it.

$$V = \frac{Q}{C} = \varepsilon\left(1 - e^{-t/R_1C}\right) = 4$$

$$1 - e^{-t/4700 \cdot .06} = \frac{4}{12} = \frac{1}{3}$$

$$1 - \frac{1}{3} = e^{-t/282}$$

$$\ln\left(\frac{2}{3}\right) = \frac{-t}{282}$$

$$t = -\ln\left(\frac{2}{3}\right) \cdot 282 = 114.34 \text{ seconds}$$

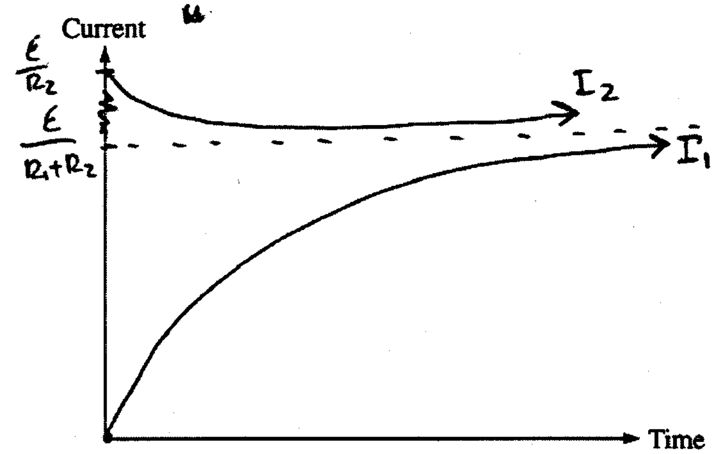
After switch S_1 has been closed for a long time, switch S_2 gets closed at a new time $t = 0$.

(d) On the axes below, sketch graphs of the current I_1 in R_1 versus time and of the current I_2 in R_2 versus time, beginning when switch S_2 is closed at new time $t = 0$. Clearly label which graph is I_1 and which is I_2.

As $t \to \infty$

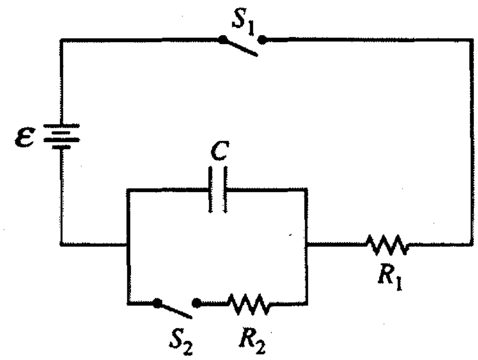
$I_{R_1} \to \dfrac{\varepsilon}{R_1 + R_2}$

because the capacitor acts like a broken circuit



Initially ~~both~~ R_1 begins at 0

Note: this is not drawn to scale — $\frac{\varepsilon}{R_2}$ is actually twice the value of $\frac{\varepsilon}{R_1+R_2}$.



E&M 2.

The circuit above contains a capacitor of capacitance C, a power supply of emf $\mathcal{E}$, two resistors of resistances R_1 and R_2, and two switches, S_1 and S_2. Initially, the capacitor is uncharged and both switches are open. Switch S_1 then gets closed at time $t = 0$.

(a) Write a differential equation that can be solved to obtain the charge on the capacitor as a function of time t.

$$\mathcal{E} - \frac{dq}{dt}R_1 - \frac{q}{C} = 0$$

(b) Solve the differential equation in part (a) to determine the charge on the capacitor as a function of time t.

$$V = \frac{Q}{C}$$
$$\mathcal{E} = \frac{Q}{C}$$
$$Q = \mathcal{E}C$$

$$\frac{dq}{dt} = \frac{\mathcal{E} - \frac{q}{c}}{R}$$

$$\int_0^q \frac{R\,dq}{(\mathcal{E} - \frac{q}{c})} = \int_0^t dt$$

$$-RC\int_0^q \frac{dq}{q - \mathcal{E}C} = t$$

$$-RC\left[\ln q - \mathcal{E}C\right]_0^q = t$$

$$-RC\left[\ln\left[\frac{q - \mathcal{E}C}{-\mathcal{E}C}\right]\right] = t$$

$$\cancel{e^{\ln}}\ \frac{q - \mathcal{E}C}{-\mathcal{E}} = e^{\left(-\frac{t}{RC}\right)}$$

$$q = \left[\mathcal{E}C - \mathcal{E}Ce^{-\frac{t}{RC}}\right]C$$

$$q(t) = C\mathcal{E}\left(1 - e^{-\frac{t}{RC}}\right)$$

GO ON TO THE NEXT PAGE.

Numerical values for the components are given as follows:

$\mathcal{E} = 12\text{ V}$
$C = 0.060\text{ F}$
$R_1 = R_2 = 4700\ \Omega$

(c) Determine the time at which the capacitor has a voltage 4.0 V across it.

$$V = \frac{q}{C}$$

$$4\cancel{(.06)} = \cancel{(.06)}(12)\left(1 - e^{\frac{-t}{(4700)(.06)}}\right)$$

$$\left(1 - e^{\frac{-t}{4700(.06)}}\right) = \frac{1}{3}$$

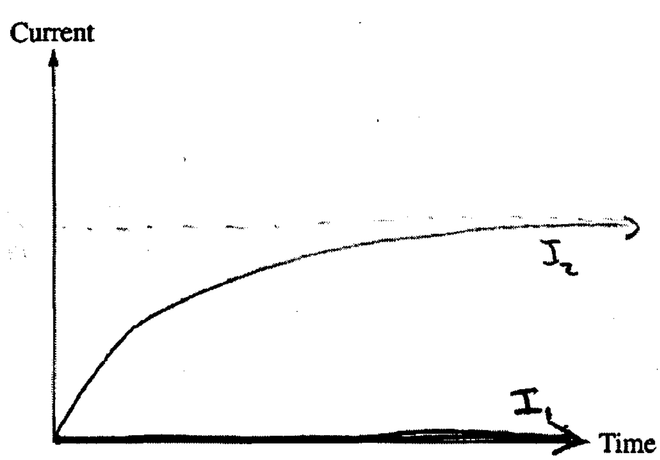
$$\frac{4}{3} = e^{\frac{-t}{4700(.06)}}$$

$$\ln\frac{4}{3} = \frac{-t}{4700(.06)}$$

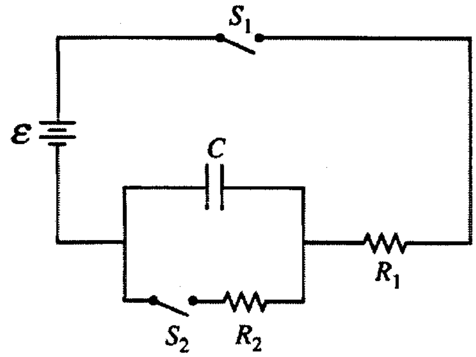
$$t = -4700(.06)\ \ln\frac{4}{3}$$

After switch S_1 has been closed for a long time, switch S_2 gets closed at a new time $t = 0$.

(d) On the axes below, sketch graphs of the current I_1 in R_1 versus time and of the current I_2 in R_2 versus time, beginning when switch S_2 is closed at new time $t = 0$. Clearly label which graph is I_1 and which is I_2.



$$\frac{Q}{C} = IR$$



E&M 2.

The circuit above contains a capacitor of capacitance C, a power supply of emf $\mathcal{E}$, two resistors of resistances R_1 and R_2, and two switches, S_1 and S_2. Initially, the capacitor is uncharged and both switches are open. Switch S_1 then gets closed at time $t = 0$.

(a) Write a differential equation that can be solved to obtain the charge on the capacitor as a function of time t.

$q = CV$

(b) Solve the differential equation in part (a) to determine the charge on the capacitor as a function of time t.

$q = C\mathcal{E}(1 - e^{-t/RC})$

Numerical values for the components are given as follows:

$\mathcal{E} = 12$ V
$C = 0.060$ F
$R_1 = R_2 = 4700\ \Omega$

(c) Determine the time at which the capacitor has a voltage 4.0 V across it.

$q = C\mathcal{E} = (.06)(4) = .24$

$.24 = (.06)(12V)(1 - e^{-t/4700 \cdot .06})$

$\frac{1}{3} = 1 - e^{-t/282}$

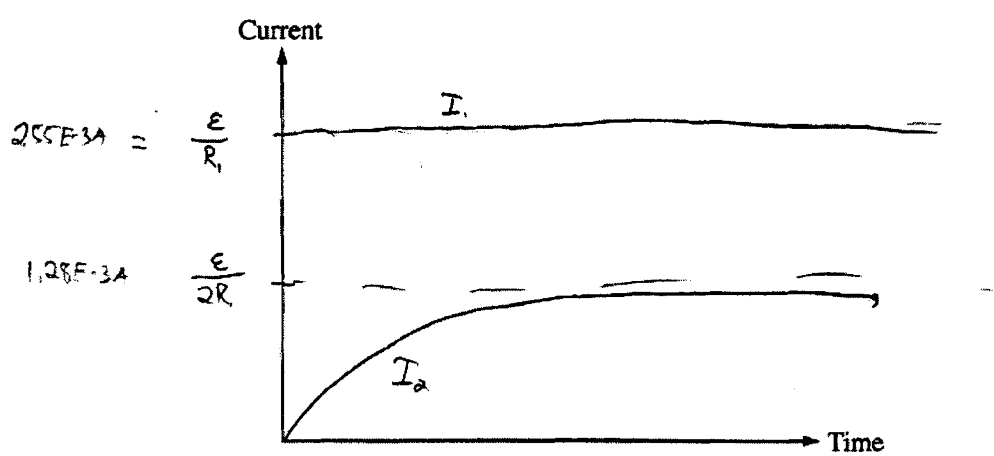
$\frac{2}{3} = e^{-t/282}$

$\ln\frac{2}{3} = \frac{-t}{282}$

$t = -282 \ln(\frac{2}{3}) = 114.34$ sec

After switch S_1 has been closed for a long time, switch S_2 gets closed at a new time $t = 0$.

(d) On the axes below, sketch graphs of the current I_1 in R_1 versus time and of the current I_2 in R_2 versus time, beginning when switch S_2 is closed at new time $t = 0$. Clearly label which graph is I_1 and which is I_2.



AP® PHYSICS C: ELECTRICITY AND MAGNETISM
2006 SCORING COMMENTARY

Question 2

Overview

This question was designed to test students' understanding of RC circuits, specifically with the capacitor in both series and parallel to the resistors. In part (a) students were asked to evaluate the circuit and write a differential equation that could be solved. Part (b) required students to solve the differential equation for the charge on the capacitor as a function of time. Part (c) required them to equate the expression for the charge determined in part (b) to the charge in the capacitor at a given voltage. Part (d) asked students to graph the currents in both the first and second resistor as a function of time (after the capacitor was charged and then switch S_2 was thrown).

Sample: E2A
Score: 15

This clearly written response earned full credit on all parts of the problem. Part (a) makes use of Kirchhoff's loop rule in setting up the differential equation, which is integrated in part (b) using limits of integration. Although it was not necessary in part (d) to scale the vertical axis of the graph, the student very clearly indicates the correct relationship between the two values of the current that are indicated.

Sample: E2B
Score: 9

This response earned 7 points full credit on parts (a) and (b). Part (c) earned the point for setting the expression for charge from part (b) equal to CV, where V is 4 volts, but failed to get the correct answer or an answer within the reasonable range, so no additional points were given. Part (c) earned only the point for showing the current I_1 starting at zero.

Sample: E2C
Score: 4

Part (a) received no credit. Part (b) received 1 point for a correct answer, even though no supporting work was shown, as this is a standard equation that may have been committed to memory. Part (c) received 3 points full credit, but no credit was given for the graphs in part (d).