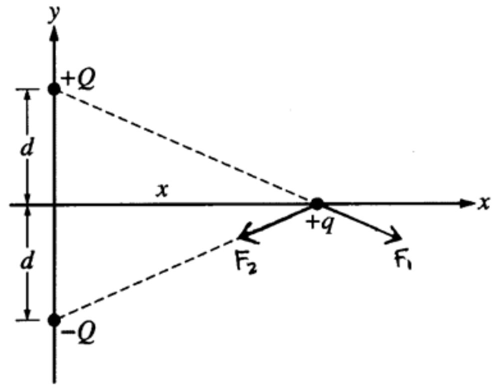
Question 3

15 points total | **Distribution of points**

(a) 2 points



For indicating the correct direction for the force due to the $+Q$ charge (F_1 as drawn above) — 1 point

For indicating the correct direction for the force due to the $-Q$ charge (F_2 as drawn above) — 1 point

(b) 6 points

For any indication that the magnitudes of F_1 and F_2 are the same — 1 point

The x-components of F_1 and F_2 cancel.

For any indication that the magnitude of the net force is the sum of the y-components of F_1 and F_2, which are equal — 1 point

Example: $F_{total} = F_1 \cos\theta + F_2 \cos\theta = 2F \cos\theta$, where θ is the angle between the y-axis and the dashed lines in the figure

For a correct expression for $\cos\theta$ — 1 point

$$\cos\theta = \frac{d}{\sqrt{x^2 + d^2}}$$

For a correct substitution for F into the above expression for F_{total} — 1 point

$$F = \frac{kqQ}{r^2} = \frac{kqQ}{x^2 + d^2}$$

$$F_{total} = 2\frac{kqQ}{x^2 + d^2}\frac{d}{\sqrt{x^2 + d^2}}$$

For the correct magnitude of the total force — 1 point

$$F_{total} = \frac{2kqQd}{\left(x^2 + d^2\right)^{3/2}}$$ or equivalent

For indicating the correct direction for the total force, e.g., negative y-direction, toward the bottom of the page, etc. — 1 point

Question 3 (continued)

Distribution of points

(c) 2 points

The field can be found from the force.

$E = F_{total}/q$

For the correct magnitude of the electric field — 1 point

$$E = \frac{2kQd}{\left(x^2 + d^2\right)^{3/2}}$$

For indicating the correct direction for the electric field, e.g., negative y-direction, toward the bottom of the page, etc. — 1 point

(d) 2 points

The total potential is the sum of the individual point charge potentials.

$$V = V_1 + V_2 = \frac{kQ}{\sqrt{x^2 + d^2}} + \frac{-kQ}{\sqrt{x^2 + d^2}}$$

For indicating that the electric potential is zero — 2 points

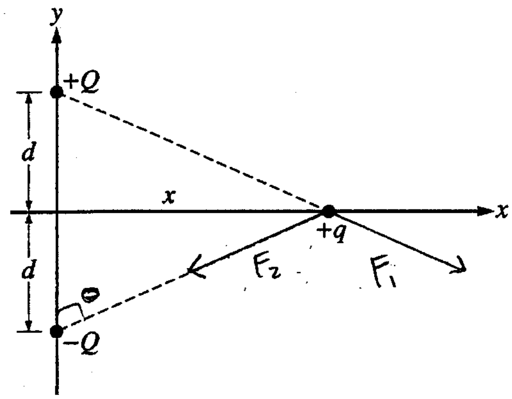
Note: One point partial credit could be earned for only recognizing that the potentials from the two charges must be added.

(e) 3 points

For any indication that as x gets large, the hypotenuse and x are approximately equal or d is negligible compared to x — 1 point

For indicating that the above implies that $\sqrt{x^2 + d^2} \approx x$ — 1 point

For indicating that substituting the approximate equality into the answer from part (b) yields $F_{total} = \dfrac{2kqQd}{x^3}$ — 1 point



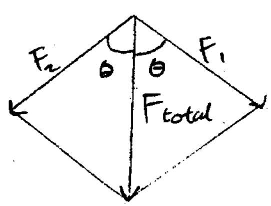
3. (15 points)

Three electric charges are arranged on an x-y coordinate system, as shown above. Express all algebraic answers to the following parts in terms of Q, q, x, d, and fundamental constants.

(a) On the diagram, draw vectors representing the forces F_1 and F_2 exerted on the $+q$ charge by the $+Q$ and $-Q$ charges, respectively.

(b) Determine the magnitude and direction of the total electric force on the $+q$ charge.

The strength of F_1 and F_2 is same.



$$|F_1| = |F_2| = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}$$

$$= \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{qQ}{d^2+x^2}\right)$$

$$\vec{F}_{total} = \vec{F}_1 + \vec{F}_2$$

$$= (|F_1|\sin\theta - |F_2|\sin\theta)(\hat{i}) + (|F_1|\cos\theta + |F_2|\cos\theta)(-\hat{j})$$

$$= 2|F_1|\cos\theta(-\hat{j})$$

$$= \left(\frac{1}{2\pi\varepsilon_0}\right)\left(\frac{qQ}{d^2+x^2}\right) \times \left(\frac{d}{\sqrt{x^2+d^2}}\right)(-\hat{j})$$

$$\therefore \; F_{total} = \left(\frac{qQd}{2\pi\varepsilon_0}\right)(x^2+d^2)^{-\frac{3}{2}}(-\hat{j})$$

to downward. (to negative y-axis)

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(c) Determine the electric field (magnitude and direction) at the position of the $+q$ charge due to the other two charges.

$$E = \frac{F}{q} = \frac{F_{total}}{q} = \left(\frac{Qd}{2\pi\varepsilon_0}\right)(x^2+d^2)^{-\frac{3}{2}}(-\hat{j})$$

$$\therefore E = \left(\frac{Qd}{2\pi\varepsilon_0}\right)(x^2+d^2)^{-\frac{3}{2}}(-\hat{j})$$

to downward (to negative y-axis)

(d) Calculate the electric potential at the position of the $+q$ charge due to the other two charges.

$$V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{Q_i}{r_i}$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{\sqrt{d^2+x^2}} + \frac{-Q}{\sqrt{d^2+x^2}}\right)$$

$$= 0$$

$$\therefore V = 0$$

(e) Charge $+q$ is now moved along the positive x-axis to a very large distance from the other two charges. The magnitude of the force on the $+q$ charge at this large distance now varies as $1/x^3$. Explain why this happens.

$$|F_{total}| = \frac{qQd}{2\pi\varepsilon_0}(x^2+d^2)^{-\frac{3}{2}}$$

If x is very large, d is much smaller than x,

$d \ll x$, so $(x^2+d^2) \simeq x^2$.

Then,

$$|F_{total}| \simeq \frac{Qqd}{2\pi\varepsilon_0}(x^2)^{-\frac{3}{2}} = \frac{Qqd}{2\pi\varepsilon_0}\left(\frac{1}{x^3}\right).$$

$\therefore$ So, $|F_{total}|$ varies as $\left(\frac{1}{x^3}\right)$

at large distance of x.

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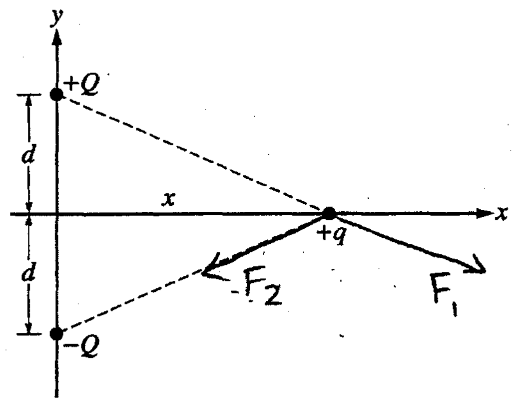


3. (15 points)

Three electric charges are arranged on an x-y coordinate system, as shown above. Express all algebraic answers to the following parts in terms of Q, q, x, d, and fundamental constants.

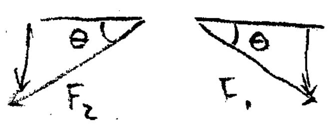
(a) On the diagram, draw vectors representing the forces F_1 and F_2 exerted on the $+q$ charge by the $+Q$ and $-Q$ charges, respectively.

(b) Determine the magnitude and direction of the total electric force on the $+q$ charge.

The x-components are equal and opposite; therefore the total electric force will be straight down ↓

$$F_{electric} = k_c \frac{q_1 q_2}{r^2}$$

$$= k_c \frac{Qq}{x^2+d^2}$$



$$\theta = \tan^{-1} \frac{d}{x}$$

$$\text{y-component} = F \sin\theta = k_c \frac{Qq}{x^2+d^2} \cdot \frac{d}{\sqrt{x^2+d^2}}$$

$$\frac{d}{\sqrt{x^2+d^2}}$$

$$\frac{k_c Qqd}{\left(\sqrt{x^2+d^2}\right)^3} \times 2 = \boxed{\frac{2 k_c Qqd}{\left(\sqrt{x^2+d^2}\right)^3} \text{ down}}$$

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(c) Determine the electric field (magnitude and direction) at the position of the $+q$ charge due to the other two charges.

$$E = \frac{F}{q} = \frac{2k_cQ}{\left(\sqrt{x^2+d^2}\right)^3}$$

(d) Calculate the electric potential at the position of the $+q$ charge due to the other two charges.

$$V = (-)Ed \quad \left(d \leftarrow \sqrt{x^2+d^2}\right)$$

$$= \frac{2k_cQ}{x^2+d^2}$$

(e) Charge $+q$ is now moved along the positive x-axis to a very large distance from the other two charges. The magnitude of the force on the $+q$ charge at this large distance now varies as $1/x^3$. Explain why this happens.

in $\frac{2k_cQq}{\sqrt{x^2+d^2}^3}$, d decreases so much compared to x that it is essentially zero, so this can be rewritten:

$$\frac{2k_cQq}{\sqrt{x^2}^3} \rightarrow \frac{2k_cQq}{x^3}$$

only the $1/x^3$ matters, d has shrunk to nothing.

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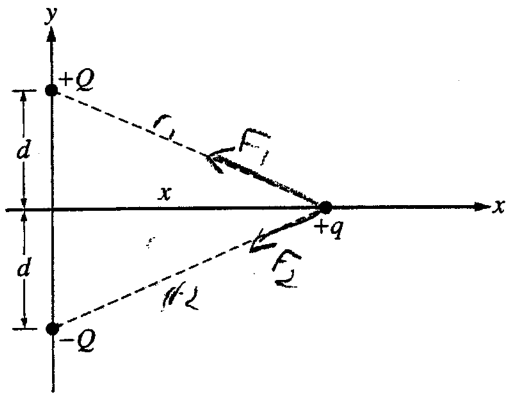

$(r_1)^2 = d^2 + x^2 = (r_2)^2$

3. (15 points)

Three electric charges are arranged on an x-y coordinate system, as shown above. Express all algebraic answers to the following parts in terms of Q, q, x, d, and fundamental constants.

(a) On the diagram, draw vectors representing the forces F_1 and F_2 exerted on the $+q$ charge by the $+Q$ and $-Q$ charges, respectively.

(b) Determine the magnitude and direction of the total electric force on the $+q$ charge.

$$F_1 = \frac{K(+Q)(q)}{r^2} = \frac{(9\times10^9)(Q)(q)}{d^2+x^2}$$

$$F_2 = \frac{k(-Q)(q)}{r^2} = \frac{(9\times10^9)(-Q)(q)}{d^2+x^2}$$

$$F_{total} = F_1 + F_2$$
$$= F_1 + (-F_2)$$
$$= \frac{9\times10^9 Qq}{d^2+x^2} + \frac{9\times10^9 Qq}{d^2+x^2} = \frac{2(9\times10^9 Qq)}{d^2+x^2}$$ West

(c) Determine the electric field (magnitude and direction) at the position of the $+q$ charge due to the other two charges.

$E_p = \frac{F}{q} = \frac{kq}{r^2} \Rightarrow E_1 = \frac{k(+Q)}{x^2+d^2}$

$E_2 = \frac{(-Q)k}{x^2+d^2} \Rightarrow E_T = E$

(d) Calculate the electric potential at the position of the $+q$ charge due to the other two charges.

$V = k \left[\frac{+Q}{r_1} + \frac{(-Q)}{r_2}\right]$

$= 9 \times 10^9 \left[\frac{Q}{\sqrt{d^2+x^2}} - \frac{Q}{\sqrt{d^2+x^2}}\right]$

$= 0$

(e) Charge $+q$ is now moved along the positive x-axis to a very large distance from the other two charges. The magnitude of the force on the $+q$ charge at this large distance now varies as $1/x^3$. Explain why this happens.

Since Electric force varies inversely proportional to the square of distances between the charges, force could vary at $1/x^3$ at a large distance

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Question 3

Sample: B3A
Score: 15

This very well-organized response earned full credit.

Sample: B3B
Score: 11

Parts (a) and (b) earned full credit. Note that the student's angle θ is the complimentary angle to that defined as θ in the scoring guidelines, so the use of the sine is correct. Part (c) earned no credit since the student loses the d in the numerator, and no direction is indicated. Part (d) has no correct work, but part (e) earned full credit.

Sample: B3C
Score: 5

Part (a) earned 1 point for the one correct vector. Part (b) earned 2 points for using the correct magnitude of the forces and work that implies these magnitudes are equal. Part (c) starts out correctly, but the student never arrives at a final value for the field and thus received no credit. Part (d) earned full credit, but part (e) earned nothing.