# Question 2

<table>
<thead>
<tr>
<th>15 points total</th>
<th>Distribution of points</th>
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(a) 4 points

For any use of conservation of energy

For example, initially the small block has only potential energy, and it is all converted
to kinetic energy when it reaches the bottom of the ramp.

For a correct expression for the initial potential energy

For a correct expression for the kinetic energy at the bottom of the ramp

\[ Mgh = \frac{1}{2}M(3.5v_0)^2 \]

For the correct answer

\[ h = \frac{3.5^2 v_0^2}{2g} \] or equivalent

(b) 4 points

For any use of conservation of momentum

For a correct expression for the initial momentum of the blocks

For a correct expression for the final momentum of the blocks

\[ M(3.5v_0) = Mv + (1.5M)(2v_0) \]

\[ v = 3.5v_0 - 3v_0 \]

For the correct answer

\[ v = 0.5v_0 \]

(c) 4 points

For a correct relationship between friction and the acceleration of the block

\[ \sum F = ma = f_{fric} \]

For a correct kinematic equation relating acceleration and distance that does not
contain time

\[ v_f^2 = v_i^2 - 2aD \]

For using the correct initial speed of the block

\[ 0 = 4v_0^2 - 2aD \]

\[ a = 2v_0^2/D \]

Substituting expressions for \( a \) and \( f \) into the first equation above

\[ (1.5M)2v_0^2/D = \mu(1.5M)g \]

For the correct answer

\[ \mu = 2v_0^2/Dg \]
Alternate solution

For any indication that the work done on the block as it slides is equal to its initial kinetic energy

\[ f_{\text{fric}}d = \frac{1}{2}mv_i^2 \]

For a correct expression for the work done on the block

For a correct expression for the initial kinetic energy of the block

\[ \mu (1.5M)gD = \frac{1}{2}(1.5M)(2v_0)^2 \]

For the correct answer

\[ \mu = \frac{2v_0^2}{Dg} \]

(d) 3 points

For indicating that the collision is inelastic

For indicating that the reason it is inelastic is because the change in kinetic energy is not zero, or because kinetic energy is lost in the collision

For showing that the change in kinetic energy is not zero

\[ \Delta K = K_f - K_i = \left[ \frac{1}{2}M(0.5v_0)^2 + \frac{1}{2}(1.5M)(2v_0)^2 \right] - \frac{1}{2}M(3.5v_0)^2 \]

\[ \Delta K = -3Mv_0^2 \]
2. (15 points)
A small block of mass \( M \) is released from rest at the top of the curved frictionless ramp shown above. The block slides down the ramp and is moving with a speed \( 3.5v_0 \) when it collides with a larger block of mass \( 1.5M \) at rest at the bottom of the incline. The larger block moves to the right at a speed \( 2v_0 \) immediately after the collision. Express your answers to the following questions in terms of the given quantities and fundamental constants.

(a) Determine the height \( h \) of the ramp from which the small block was released.

By the conservation of energy:

The decrease in potential energy = The increase in kinetic energy

\[ Mgh = \frac{1}{2} M v^2 \Rightarrow Mgh = \frac{1}{2} M (3.5v_0)^2 \]

\[ \therefore \ h = \frac{\frac{1}{2} (3.5v_0)^2}{g} = \frac{4.9v_0^2}{8g} \]

(b) Determine the speed of the small block after the collision.

Since there is no external force, the total linear momentum is conserved:

\[ M \cdot v_1 + 1.5M \cdot v_2 = M \cdot v_3 + 1.5M \cdot v_4 \]

Sub in the values: \( v_1 = 3.5v_0 \), \( v_2 = 0 \), \( v_4 = 2v_0 \)

\[ 3.5Mv_0 = 3Mv_0 + MV_3 \]

\[ \therefore \ v_3 = 0.5v_0 \]

\[ \therefore \ The \ speed \ of \ the \ small \ block \ after \ the \ collision \ is \ 0.5v_0 \]

GO ON TO THE NEXT PAGE.
(c) The larger block slides a distance $D$ before coming to rest. Determine the value of the coefficient of kinetic friction $\mu$ between the larger block and the surface on which it slides.

By the conservation of energy, the decrease in kinetic energy = work done against friction

\[
\frac{1}{2} \cdot (1.5M) \cdot (2V_0)^2 = f \cdot D
\]

\[
f = \frac{3M \cdot V_0^2}{D}
\]

The normal force $N = Mg$ as the block is at rest in vertical direction.

\[
f = N \cdot \mu = (1.5Mg) \cdot \mu
\]

\[
\therefore \mu = \frac{f}{N} = \frac{3M \cdot V_0^2}{D \cdot 1.5Mg} = \frac{2V_0^2}{Dg}
\]

(d) Indicate whether the collision between the two blocks is elastic or inelastic. Justify your answer.

The total kinetic energy before collision:

\[
\frac{1}{2} M (3.5V_0)^2 = \frac{49}{8} M \cdot V_0^2
\]

The total kinetic energy after collision:

\[
\frac{1}{2} M (0.5V_0)^2 + \frac{1}{2} (1.5M) (2V_0)^2 = \frac{25}{8} M \cdot V_0^2
\]

\[
\frac{49}{8} M \cdot V_0^2 > \frac{25}{8} M \cdot V_0^2.
\]

\[
\therefore \text{There is a loss of energy during the collision.}
\]

\[
\text{Hence the collision is inelastic.}
\]

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2. (15 points)

A small block of mass $M$ is released from rest at the top of the curved frictionless ramp shown above. The block slides down the ramp and is moving with a speed $3.5v_0$ when it collides with a larger block of mass $1.5M$ at rest at the bottom of the incline. The larger block moves to the right at a speed $2v_0$ immediately after the collision. Express your answers to the following questions in terms of the given quantities and fundamental constants.

(a) Determine the height $h$ of the ramp from which the small block was released.

\[ Mgh = \frac{1}{2} Mv^2 \]

\[ 0 = \frac{1}{2} (3.5v_0)^2 \]

\[ h = \frac{12.25v_0^2}{5} \]

(b) Determine the speed of the small block after the collision.

\[ Mv = M_1v_1 + M_2v_2 \]

\[ M(3.5v_0) = Mv + (1.5M)(2v_0) \]

\[ 3.5v_0 = v + 3v_0 \]

\[ 0.5v_0 = v_f \]
(c) The larger block slides a distance $D$ before coming to rest. Determine the value of the coefficient of kinetic friction $\mu$ between the larger block and the surface on which it slides.

\[
\frac{1}{2} (1.5M) (2v)^2 = \frac{1}{2} (1.5M) 4v^2
\]

\[
(2v)^2 = 2aD
\]

\[
\sum F = ma = F - F_f
\]

\[
\sum F = (1.5M) \left( \frac{2v^2}{D} \right) = F - (1.5g) M \cdot k
\]

\[
\frac{3Mv^2 - F}{D} = -M \cdot k
\]

\[
\alpha = \frac{2v^2}{D}
\]

(d) Indicate whether the collision between the two blocks is elastic or inelastic. Justify your answer.

\[
K_{E1} = \frac{1}{2} M v_0^2 = \frac{1}{2} M (2.5v_0)^2 = \frac{1}{2} M (12.25v_0^2)
\]

\[
= 6.125Mv_0^2
\]

$K_E$ of larger block = $3Mv^2$

$K_E$ of smaller block after the collision = \[\frac{1}{2} (M)(0.5v_0)^2\]

\[= \frac{1}{2} (0.25) Mv_0^2\]

\[= 0.125Mv_0^2\]

It is elastic, because the speed of the larger after the collision is $2v_0$ but the speed of the smaller block after the collision is $0.5v_0$. Thus, the two masses do not travel at the same speed after the collision.
2. (15 points)

A small block of mass $M$ is released from rest at the top of the curved frictionless ramp shown above. The block slides down the ramp and is moving with a speed $3.5v_0$ when it collides with a larger block of mass $1.5M$ at rest at the bottom of the incline. The larger block moves to the right at a speed $2v_0$ immediately after the collision. Express your answers to the following questions in terms of the given quantities and fundamental constants.

(a) Determine the height $h$ of the ramp from which the small block was released.

\[ \frac{1}{2} m_0 v_0^2 + m_0 g h_0 = \frac{1}{2} M v_1^2 + m_0 g h_1 \]

\[ M 	imes 9.8 \times h = \frac{1}{2} M \times 3.5 v_0 \]

\[ h = \frac{3.5 v_0}{2(9.8)} \]

\[ h = 0.1786 v_0 \]

(b) Determine the speed of the small block after the collision.

\[ m_0 v_0 + m_1 v_1 = m_0 v'_0 + m_1 v'_1 \]

\[ 3.5 v_0 x M = M v'_0 + 1.5 M x 2v'_1 \]

\[ \frac{3.5 v_0}{3 v'_1} = v'_0 \]

\[ 1.167 v_0 / v'_1 = v'_0 \]

GO ON TO THE NEXT PAGE.
(c) The larger block slides a distance $D$ before coming to rest. Determine the value of the coefficient of kinetic friction $\mu$ between the larger block and the surface on which it slides.

\[ F_t = F_n \cdot \mu \]
\[ F_n = 1.5 \times 9.8 \]
\[ = 14.7 \text{ N} \]

(d) Indicate whether the collision between the two blocks is elastic or inelastic. Justify your answer.

elastic collision because as block 1 collides with bigger mass block 2 both blocks has moved along opposite direction to each other.
Question 2

Sample: B2A
Score: 15

This response is well organized, and the work is very easy to follow. Part (c) uses the alternate solution.

Sample: B2B
Score: 10

Part (a) earned 3 points since the cancellation on the second line is wrong and results in an incorrect answer. Part (b) received full credit. Part (c) earned 2 points for the kinematic equation and use of the correct initial speed. Part (d) earned 1 point for showing that the change in kinetic energy is not zero but does not have a correct conclusion or justification.

Sample: B2C
Score: 4

Part (a) earned 2 points for using conservation of energy and having the correct potential energy. The other 2 points were earned in part (b) for using conservation of momentum and having the correct initial momentum.