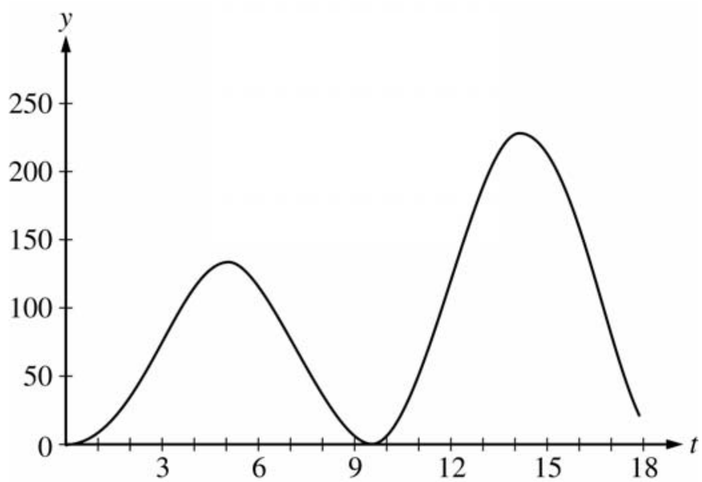
Question 2

At an intersection in Thomasville, Oregon, cars turn left at the rate $L(t) = 60\sqrt{t}\sin^2\left(\frac{t}{3}\right)$ cars per hour over the time interval $0 \le t \le 18$ hours. The graph of $y = L(t)$ is shown above.

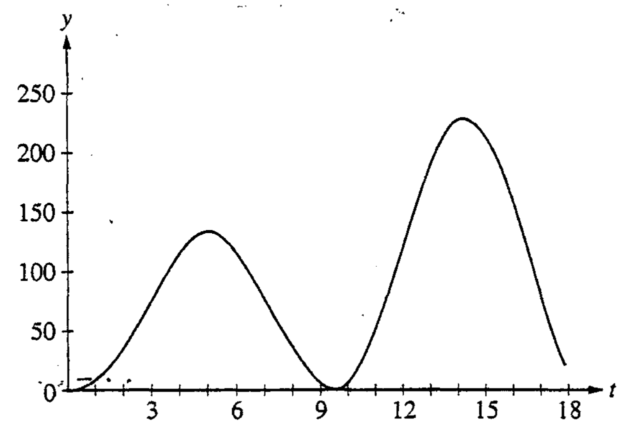


(a) To the nearest whole number, find the total number of cars turning left at the intersection over the time interval $0 \le t \le 18$ hours.

(b) Traffic engineers will consider turn restrictions when $L(t) \ge 150$ cars per hour. Find all values of t for which $L(t) \ge 150$ and compute the average value of L over this time interval. Indicate units of measure.

(c) Traffic engineers will install a signal if there is any two-hour time interval during which the product of the total number of cars turning left and the total number of oncoming cars traveling straight through the intersection is greater than 200,000. In every two-hour time interval, 500 oncoming cars travel straight through the intersection. Does this intersection require a traffic signal? Explain the reasoning that leads to your conclusion.

(a) $\int_0^{18} L(t)\,dt \approx 1658$ cars	$2: \begin{cases} 1: \text{setup} \\ 1: \text{answer} \end{cases}$
(b) $L(t) = 150$ when $t = 12.42831,\ 16.12166$ Let $R = 12.42831$ and $S = 16.12166$ $L(t) \ge 150$ for t in the interval $[R, S]$ $\frac{1}{S-R}\int_R^S L(t)\,dt = 199.426$ cars per hour	$3: \begin{cases} 1: t\text{-interval when } L(t) \ge 150 \\ 1: \text{average value integral} \\ 1: \text{answer with units} \end{cases}$
(c) For the product to exceed 200,000, the number of cars turning left in a two-hour interval must be greater than 400. $\int_{13}^{15} L(t)\,dt = 431.931 > 400$	$4: \begin{cases} 1: \text{considers 400 cars} \\ 1: \text{valid interval } [h, h+2] \\ 1: \text{value of } \int_h^{h+2} L(t)\,dt \\ 1: \text{answer and explanation} \end{cases}$
OR	OR
The number of cars turning left will be greater than 400 on a two-hour interval if $L(t) \ge 200$ on that interval. $L(t) \ge 200$ on any two-hour subinterval of $[13.25304, 15.32386]$. Yes, a traffic signal is required.	$4: \begin{cases} 1: \text{considers 200 cars per hour} \\ 1: \text{solves } L(t) \ge 200 \\ 1: \text{discusses 2 hour interval} \\ 1: \text{answer and explanation} \end{cases}$



Work for problem 2(a)

$$L(t) = 60\sqrt{t}\,\sin^2\left(\frac{t}{3}\right)$$

$$\int_0^{18} 60\sqrt{t}\,\sin^2\left(\frac{t}{3}\right)dt$$

$$= 1657.8237$$

$$= 1658 \text{ cars}$$

Do not write beyond this border.

Continue problem 2 on page 7.

Work for problem 2(b)

$L(t) = 150 = 60\sqrt{t}\,\sin^2\left(\frac{t}{3}\right)$

$60\sqrt{t}\,\sin^2\frac{t}{3} - 150 = 0$

$t = 12.42831$ or 16.121657

$$\frac{1}{b-a}\int_a^b L(t) = \frac{1}{16.121657-12.42831}\int_{12.42831}^{16.121657} 60\sqrt{t}\,\sin^2\frac{t}{3}\,dt$$

$$= \frac{1}{3.693347}\left[736.54986\right] = 199.426195$$

$12.428 \le t \le 16.121657$ $\quad$ 199.426 cars/hr

Work for problem 2(c)

Cars turning left x oncoming cars going straight

(cars turning left) 500 ≤ 200,000

cars turning left ≤ 400

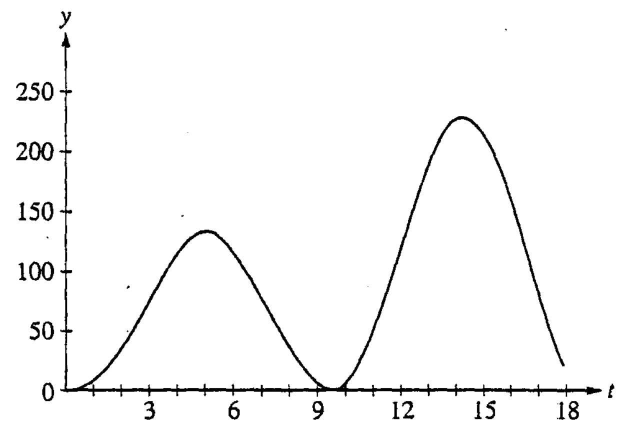
$$\int_{14}^{16} L(t) = \int_{14}^{16} 60\sqrt{t}\,\sin^2\left(\frac{t}{3}\right)dt = 412.26$$

Yes, there will need to be a signal because between the interval t=14 and t=16, 412 cars turn left. When you multiply that by 500, it exceeds 200,000

Do not write beyond this border.

Do not write beyond this border.

GO ON TO THE NEXT PAGE.



Work for problem 2(a)

$\int_0^{18} L(t)\,dt$

$\int_0^{18} [60\sqrt{t}\sin^2(t/3)]\,dt$

1658

1658 total cars turn left through the intersection between the hours of 0 hours and 18 hours.

Do not write beyond this border.

Continue problem 2 on page 7.

Work for problem 2(b)

$L(t) > 150$

$60\sqrt{t}\sin^2(t/3) > 150$

$t = [12.42831, 16.121657]$

L(t) is greater than 150 at all hours between 12.42831 hours and 16.121657 hours.

average value:

$\frac{1}{b-a}\int_a^b L(t)\,dt$

$\frac{1}{16.121657 - 12.42831}\int_{12.42831}^{16.121657} [60\sqrt{t}\sin^2(t/3)]\,dt$

199.426

The average number of cars turning left between 12.42831 hours and 16.121657 hours is 199.426 cars.

Work for problem 2(c)

500 straight cars/2 hours

200,000 = straight cars * left cars
(500)

$\frac{200{,}000}{500} = 400$

Therefore, 400 left turn cars during a 2-hour interval would require a traffic signal.

$\int_a^{a+2} L(t)\,dt \geq 400?$

13.253, 15.323

$L(t) \geq 200?$

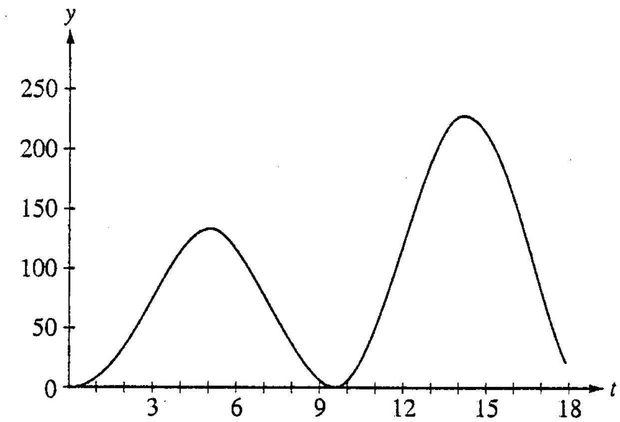
$L(t) \geq 200$ at
$[13.253, 15.323]$

Yes, the intersection does require a traffic signal. At 13.253 hours, L(t) = 200 and increases upward. At at least 200 cars per hour, the product would exceed 200000 therefore requiring a signal. The flow of cars does not drop below 200 cars/hr until 15.323 hours.

Do not write beyond this border.

Do not write beyond this border.

GO ON TO THE NEXT PAGE.



Work for problem 2(a)

total cars $\Rightarrow \int L(t)$

$= \int_0^{18} 60\sqrt{t}\, \sin^2\left(\frac{t}{3}\right)$

≈ 1658 cars

Do not write beyond this border.

Do not write beyond this border.

Continue problem 2 on page 7.

Do not write beyond this border.

Work for problem 2(b)

$L(t) \geq 150$ from ≈ 12.428 to ≈ 16.120

$\frac{1}{b-a} \int_{12.428}^{16.120} 60\sqrt{t} \sin^2(\frac{t}{3})$

$\frac{1}{16.120-12.428} (736.34771)$

$\frac{1}{3.692} (736.34771) \approx 199.4$ = avg. # of cars turning left

Work for problem 2(c)

(total # of cars turning left)(500 straight cars) $\geq$ 200,000

$(\int_{12.428}^{16.120} 60\sqrt{t} \sin^2(\frac{t}{2}))(500)$

$(736.34771)(500) = 368173.855$

$368174 \geq 200,000$

$\boxed{\text{Yes}}$

The product of cars turning (L) and cars going straight is greater than 200,000, and requires the installation of a traffic signal.

Do not write beyond this border.

GO ON TO THE NEXT PAGE.

Question 2

Overview

This problem gave students a function that modeled the rate, in cars per hour, at which cars turn left at a given intersection. In part (a) students had to use the definite integral to find the total number of cars that turned left in a given time period. In part (b) students had to use their graphing calculators to find the time interval during which the rate equaled or exceeded 150 cars per hour and then compute the average value of the rate over this time interval. Part (c) described a condition that would require the installation of a traffic signal at an intersection and asked students to decide if a signal was necessary at this particular intersection. Students could do this in several ways. For example, students could recognize that a signal would be required if the number of cars that turn left over a two-hour time interval exceeds 400 cars and then find an appropriate interval. Or students might recognize that a signal would be required if the average value of the rate at which cars turn left over any two-hour time interval exceeds 200 cars per hour. Since the rate itself exceeds 200 cars per hour during the interval $13.253 < t < 15.324,$ the average value of the rate will also exceed 200 cars per hour during this time interval of length greater than two hours, and thus a signal would be required.

Sample: 2A
Score: 9

The student earned all 9 points. In part (c) all 4 points were earned for a correct argument based on the value of the integral of $L(t)$ over a two-hour period.

Sample: 2B
Score: 6

The student earned 6 points: 2 points in part (a), 2 points in part (b), and 2 points in part (c). In part (a) the student provides the correct definite integral and its correct evaluation to the nearest whole number, which earned both points. In part (b) the student determines the correct interval, which earned the first point, and provides the correct average value setup to earn the second point. The third point was not earned because the units given for the computed average value are not correct. In part (c) the student earned the first point for observing that 400 left turns in a two-hour interval are needed. The second point was earned for correctly identifying the interval on which $L(t) \geq 200.$ The student never compares the length of the given interval to 2 and thus did not earn the third point. The student was not eligible for the fourth point.

Sample: 2C
Score: 4

The student earned 4 points: 2 points in part (a), 1 point in part (b), and 1 point in part (c). In part (a) the student provides the correct definite integral and its correct evaluation to the nearest whole number, which earned both points. In part (b) the upper limit for the student's interval is not correct in the third decimal place, so the first point was not earned. The second point was earned for the average value setup since the student provides the correct value for the given integral, divided by the length of the interval. The third point was not earned because the student rounds the answer to the first decimal place instead of the required three. In part (c) the first point was earned for the observation that the product of the total number of cars turning left and 500 needs to be greater than 200,000. The student does not consider a two-hour interval, so no further points were earned.