



## **AP<sup>®</sup> Calculus AB 2006 Student Responses**

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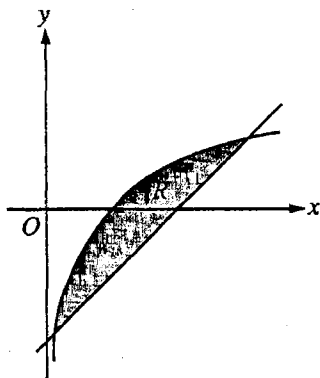
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CALCULUS AB  
SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$$y = \ln x \quad y = x - 2$$

$$\ln x = x - 2 \quad x = .1506, x = 3.1462$$

$$\int_{.1506}^{3.1462} (\ln x - x + 2) dx = \boxed{1.9491}$$

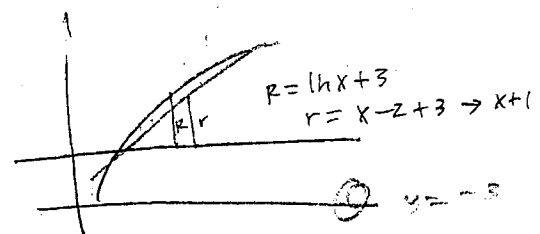
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Continue problem 1 on page 5.

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Work for problem 1(b)

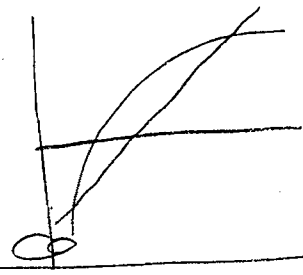


$$\pi \int_{-1.586}^{3.1462} \left[ (\ln(x+3))^2 - (x-2+3)^2 \right] dx = 34.1986$$

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Work for problem 1(c)



$$2\pi \int_{-1.586}^{3.1462} \left[ x(\ln(x) - x + 2) \right] dx$$

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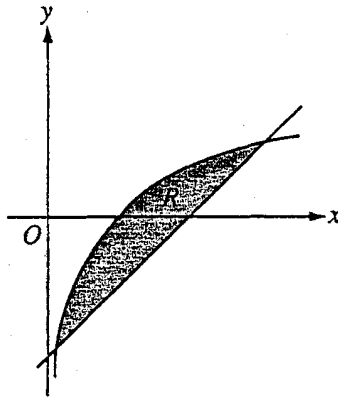
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CALCULUS BC  
SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$$y = \ln x \quad y = x - 2$$

$$\ln x = x - 2 \quad x = .158594 + 3.14619$$

$$A = \int_{.158594}^{3.14619} (\ln x - (x - 2)) dx$$

$$A = 1.949$$

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Continue problem 1 on page 5.

Work for problem 1(b)

$$V = \pi \int_{.158594}^{3.14619} ((-3 - \ln x))^2 - (-3 - (x-2))^2 dx$$

$$V = 10.886 \pi$$

$$V = 34.199$$

Work for problem 1(c)

$$\begin{aligned} x-2=0 & \quad \ln x = 0 \\ x=2 & \quad x=1 \end{aligned}$$

$$V = \pi \int_1^{3.14619} ((\ln x)^2 - (x-2)^2)$$

$$V = \pi \int_{.158594}^2 ((x-2)^2 - (\ln x)^2)$$

$$V = \pi \int_1^{3.14619} ((\ln x)^2 - (x-2)^2) + \pi \int_{.158594}^2 ((x-2)^2 - (\ln x)^2) dx$$

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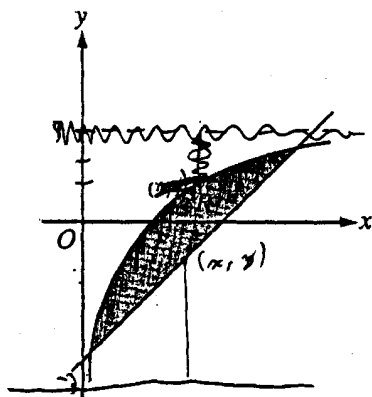
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CALCULUS AB  
SECTION II, Part A  
Time—45 minutes  
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$$a = \int_0^{3.138} \ln x - (x-2) dx$$

$$a \approx 1.80$$

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Continue problem 1 on page



Work for problem 1(b)

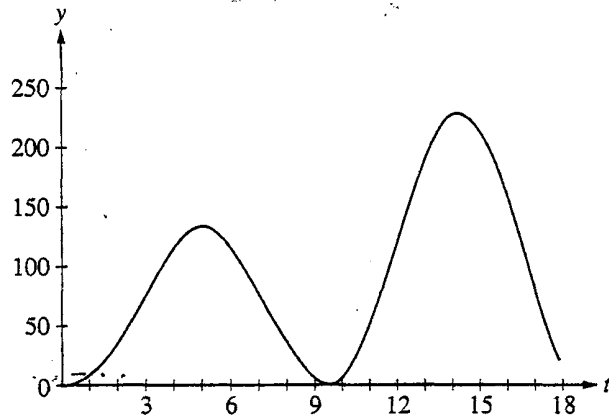
$$\begin{aligned}
 & \sqrt{2} \int_0^{2.138} (3 - (x-2)) (\ln x - (x-2)) dx \\
 & \sqrt{2} \approx 6.299 \\
 & \boxed{\sqrt{2} \approx 12.599 \pi}
 \end{aligned}$$

Work for problem 1(c)

$$\sqrt{2} \int_0^{3.138} x (\ln x - (x-2)) dx$$

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Work for problem 2(a)

$$L(t) = 60\sqrt{t} \sin^2\left(\frac{t}{3}\right)$$

$$\int_0^{18} 60\sqrt{t} \sin^2\left(\frac{t}{3}\right) dt$$

$$= 1657.8237$$

$$= \text{1658 cars}$$

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Continue problem 2 on page 7.



Work for problem 2(b)

$$L(t) = 150 = 60\sqrt{t} \sin^2\left(\frac{t}{3}\right)$$

$$60\sqrt{t} \sin^2\frac{t}{3} - 150 = 0$$

$$t = 12.42831 \quad \text{or} \quad 16.121657$$

$$\frac{1}{b-a} \int_a^b L(t) dt = \frac{1}{16.121657 - 12.42831} \int_{12.42831}^{16.121657} 60\sqrt{t} \sin^2\frac{t}{3} dt$$

$$= \frac{1}{3.693347} [736.54986] = 199.4261195$$

$$12.428 \leq t \leq 16.121657$$

$$199.426 \text{ cars/hr}$$

Work for problem 2(c)

Cars turning left x incoming cars going straight

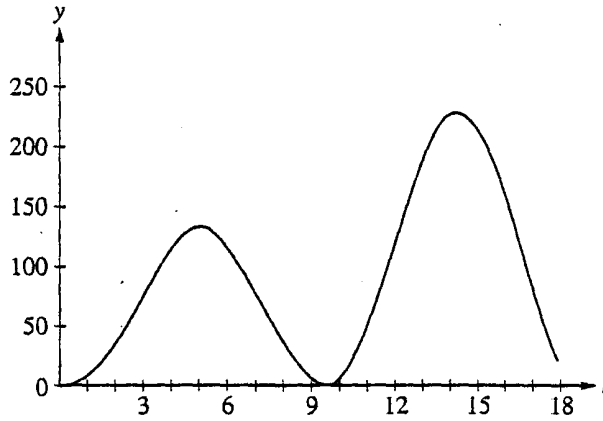
$$(\text{Cars turning left}) 500 \leq 200,000$$

$$\text{cars turning left} \leq 400$$

$$\int_{14}^{16} L(t) dt = \int_{14}^{16} 60\sqrt{t} \sin^2\left(\frac{t}{3}\right) dt = 412.26$$

Yes, there will need to be a signal because between the interval  $t=14$  and  $t=16$ , 412 cars turn left. When you multiply that by 500, it exceeds 200,000

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Work for problem 2(a)

$$\int_0^{18} L(t) dt$$

$$\int_0^{18} [607t \sin^2(t/3)] dt$$

$$1658$$

1658 total cars turn left through the intersection between the hours of 0 hours and 18 hours.

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Continue problem 2 on page 7.

Work for problem 2(b)

$$L(t) > 150$$

$$607t \sin^2\left(\frac{t}{3}\right) > 150$$

$$t = [12.42831, 16.121657]$$

$L(t)$  is greater than 150 at all hours between 12.42831 hours and 16.121657 hours.

Average value:

$$\frac{1}{b-a} \int_a^b L(t) dt$$

$$\frac{1}{16.121657 - 12.42831} \int_{12.42831}^{16.121657} [607t \sin^2\left(\frac{t}{3}\right)] dt$$

$$199.426$$

The average number of cars turning left between 12.42831 hours and 16.121657 hours is 199.426 cars.

Work for problem 2(c)

500 straight cars / 2 hours

$$200,000 = \text{straight cars} * \text{left cars} \\ (500)$$

$$\frac{200,000}{500} = 400$$

Therefore, 400 left turn cars during a 2-hour interval would require a traffic signal.

$$\int_a^{a+2} L(t) dt \geq 400?$$

$$13.253, 15.323$$

$$L(t) \geq 200?$$

$$L(t) \geq 200 \text{ at}$$

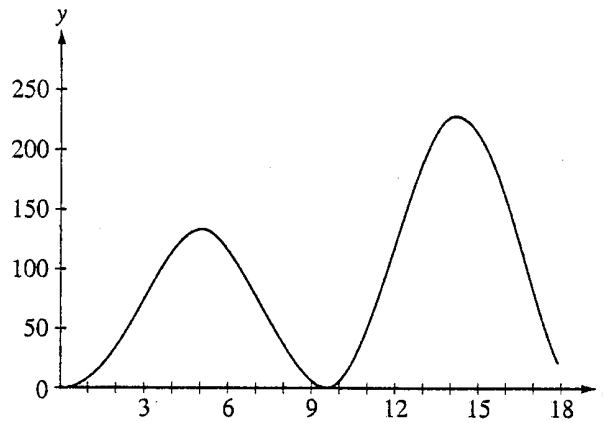
$$[13.253, 15.323]$$

Yes, the intersection does require a traffic signal. At 13.253 hours,  $L(t) = 200$  and increases upward. At at least 200 cars per hour, the product would exceed 200,000 therefore requiring a signal. The flow of cars does not drop below 200 cars/hr until 15.323 hours.

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Work for problem 2(a)

total cars  $\Rightarrow \int L(t)$   
 $= \int_0^{18} 60\sqrt{t} \sin^2\left(\frac{t}{3}\right)$   
 $\approx 1650$  cars

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Continue problem 2 on page 7.

Work for problem 2(b)

$$L(t) \geq 150 \text{ from } \approx 12.428 \text{ to } \approx 16.120$$

$$\frac{1}{b-a} \int_{12.428}^{16.120} 60\pi \sin^2\left(\frac{t}{3}\right)$$

$$\frac{1}{16.120-12.428} (736.34771)$$

$$\frac{1}{3.692} (736.34771) \approx 199.4 = \text{avg. \# of cars turning left}$$

Work for problem 2(c)

$$\left( \begin{array}{l} \text{total \# of cars} \\ \text{turning left} \end{array} \right) (500 \text{ straight cars}) \geq 200,000$$

$$\left( \int_{12.428}^{16.120} 60\pi \sin^2\left(\frac{t}{3}\right) \right) (500)$$

$$(736.34771)(500) = 368173.855$$

$$368174 \geq 200,000$$

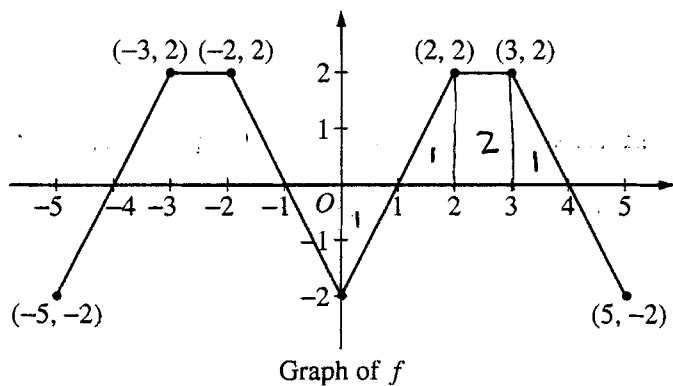
Yes

The product of cars turning  $\textcircled{L}$  and cars going straight is greater than 200,000, and requires the installation of a traffic signal.

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Work for problem 3(a)

$$g(4) = \int_0^4 f(t) dt = 3$$

$$g(4) = 3$$

$$g'(4) = f(4) = 0$$

$$g''(4) = f'(4) = \frac{2+2}{3-5} = \frac{4}{-2} = -2$$

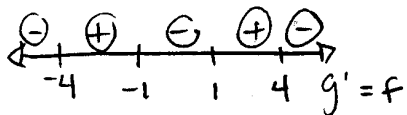
$$g''(4) = f'(4) = -2$$

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Continue problem 3 on page 9.

Work for problem 3(b)



$g$  has a relative minimum at  $x=1$  because  $g'(x)=f(x)$  changes from negative to positive at  $x=1$

Work for problem 3(c)

$$g'(x) = f(x)$$

$$g(x) = \int f(x) dx$$

- if  $g(5) = 2$  and  $f$  is periodic w/ a period length of 5, then  $g(10) = 4$

- $g(108) = ?$

$$g(108) = \int_0^{108} f(x) dx = 44$$

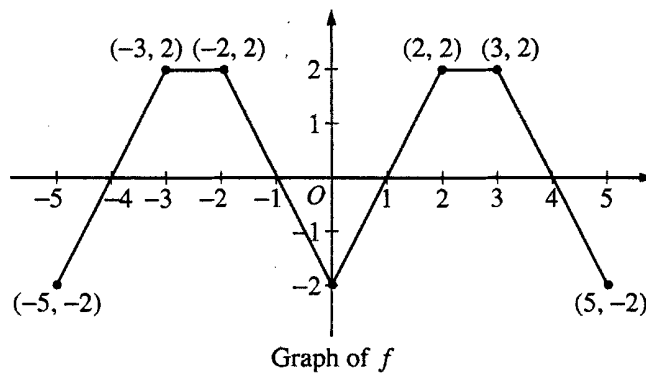
$$g'(108) = f(108) = 2$$

$$(y - 44) = 2(x - 108)$$

$$y = 2x - 72$$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.



Work for problem 3(a)

$$g(4) = \int_0^4 f(t) dt = 3$$

$$g'(4) = \frac{d}{dx} \int_0^x f(t) dt = f(x)$$

$$= f(4) = 0$$

$$g''(4) = f'(4) = \frac{-2 - 2}{5 - 3} = \frac{-4}{2} = -2$$

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Continue problem 3 on page 9.



Work for problem 3(b)

$$g(1) = \int_0^1 f(x) dt = -1$$

$$g(-5) = \int_0^{-5} f(x) dt = -2$$

$$g(-1) = \int_0^{-1} f(x) dt = +1$$

$$g(4) = \int_0^4 f(x) dt = 3$$

$$g(5) = \int_0^5 f(x) dt = 2$$

$g$  has neither a max or min

Ⓐ  $x=1$  because max occurs

Ⓑ  $x=4$  and min Ⓐ

$$x = -5$$

Work for problem 3(c)

$$\frac{10}{5} = 2 \quad g(5) = \int_0^5 f(t) dt = 2$$

$$1 \quad g(10) = 2 \int_0^5 f(t) dt$$

$$g(10) = 2(2) = 4$$

$$g(108) = \frac{108}{5} \int_0^5 f(t) dt = \frac{216}{5}$$

$$\frac{108}{5} = 21 \frac{3}{5} \quad f(108) = f(3)$$

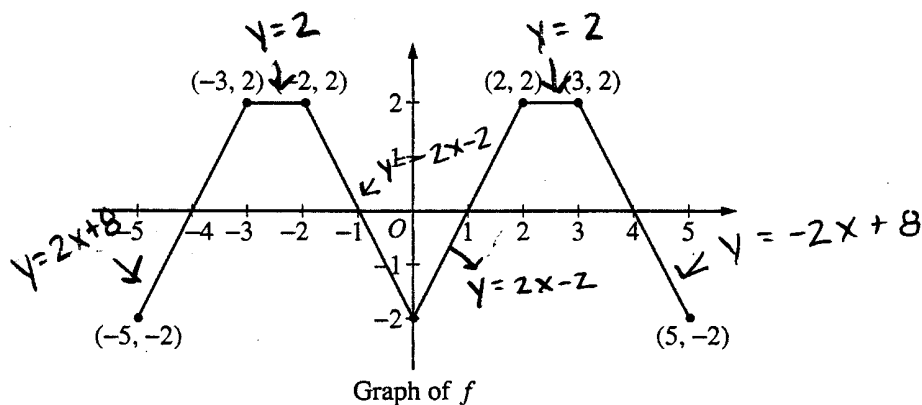
$$g'(108) = f(108) = f(3) = 2$$

$$y - \frac{216}{5} = 2(x - 108)$$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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Work for problem 3(a)

$$\frac{2 - (-2)}{-3 + 5} = 2 \quad y = 2x + 8$$

$$\frac{2 - (-2)}{-2} = -2 \quad y = -2x - 2$$

$$\frac{2 + 2}{2} = 2 \quad y = 2x - 2$$

$$\frac{2 + 2}{3 - 5} = -2 \quad y = -2x + 8$$

$$\begin{aligned} g(4) &= \int_0^4 (-2x + 8) dx \\ &= \left[ -x^2 + 8x \right]_0^4 \\ &= 16 \end{aligned}$$

$$\therefore g(4) = 16$$

$$g'(4) = 0$$

$$g''(4) = -2$$

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Continue problem 3 on page 9.

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Work for problem 3(b)

$$\begin{array}{cccc} - & + & - & + \\ \frac{0}{-4} & \frac{0}{-1} & \frac{0}{1} & \frac{0}{4} \end{array}$$

There is a relative minimum at  $x=1$  because the sign analysis shows that slope is negative from  $(-1, 1)$  and then there is 0 slope at  $x=1$  and slope is positive from  $(1, 4)$ .

Work for problem 3(c)

$$\text{slope} = -2$$

$$2 = -2(5) + b$$

$$b = 12$$

$$\therefore y = -2x + 12$$

The equation of the line tangent to the curve is  $y = -2x + 12$

$$y|_{x=10} = -8$$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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## NO CALCULATOR ALLOWED

## CALCULUS AB

## SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

$t$ (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

Work for problem 4(a)

$$\text{avg val}_{a(t)} = \frac{v(b) - v(a)}{b - a}$$

$$\text{avg val}_{a(t)} = \frac{v(80) - v(0)}{80 - 0}$$

$$\text{avg val}_{a(t)} = \frac{49 - 5}{80 - 0}$$

$$\text{avg val}_{a(t)} = 11/20 \text{ ft/s}^2$$

Work for problem 4(b)

$\int_{10}^{70} v(t) dt$  gives the distance the rocket has traveled from time 10 seconds to 70 seconds in feet.

$$\Delta x = \frac{70 - 10}{3} = 20$$

$$P(t) = 20 [v(20) + v(40) + v(60)]$$

$$P(t) = 20 [22 + 35 + 44] = 20 [101]$$

$$P(t) = 2020 \text{ feet}$$

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Continue problem 4 on page 11.

Work for problem 4(c)

Rocket B

$$a(t) = \frac{3}{\sqrt{t+1}}$$

$$v(t) = \int \frac{3}{\sqrt{t+1}} dt$$

$$\begin{aligned} \text{let } u &= t+1 \\ \frac{du}{dx} &= 1 \\ du &= dx \end{aligned}$$

$$v(t) = 3 \int u^{-1/2} du$$

$$v(t) = 3 \frac{u^{1/2}}{1/2} + C$$

$$v(t) = 6(t+1)^{1/2} + C$$

$$2 = 6(0+1)^{1/2} + C$$

$$C = -4$$

$$v(t) = 6(t+1)^{1/2} - 4$$

$$v(80) = 6(80+1)^{1/2} - 4$$

$$v(80) = 6\sqrt{81} - 4$$

$$v(80) = 54 - 4$$

$$v(80) = 50 \text{ ft/sec}$$

Rocket A

$$v(80) = 49 \text{ ft/sec}$$

Rocket B is traveling faster at  $t = 80$  sec. Rocket B's velocity was found by  $v(t) = \int a(t) dt$  and is 50 ft/sec. Rocket A's velocity was 49 ft/sec

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NO CALCULATOR ALLOWED

CALCULUS AB  
SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

$t$ (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

Work for problem 4(a)

$$\frac{1}{80} (9 + 8 + 7 + 6 + 5 + 4 + 3 + 2) =$$

$$= \frac{46}{80} = \frac{23}{40} \text{ ft/sec}^2$$

Work for problem 4(b)

$\int_{10}^{70} v(t) dt$  indicates the distance the rocket traveled over the interval  $10 \leq t \leq 70$ . In this case, it indicates total distance because the rocket's velocity is only increasing during this time period.

$$\begin{array}{r} 70 \\ 22 \\ -44 \\ \hline 136 \end{array}$$

$$(22 + 2(35) + 44) = 136 \text{ ft.}$$

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Continue problem 4 on page 11.

Work for problem 4(c)

$$3(t+1)^{-1/2}$$

$$v(t) = 6\sqrt{t+1} + C$$

$$2 = 6\sqrt{0+1} + C$$

$$2 = 6 + C$$

$$-4 = C$$

$$v(t) = 6\sqrt{t+1} - 4$$

$$v(80) = 6\sqrt{80+1} - 4$$

$$6 \cdot 9 - 4$$

$$v(80) = 52 \text{ ft/sec}$$

Rocket B is traveling faster at  $t=80$ . The table states that Rocket A is traveling at 49 ft/sec. By taking the antiderivative for  $a(t)$  of Rocket B and solving with initial conditions for C, then substituting  $t=80$ , we find  $v(80) = 52 \text{ ft/sec}$ , faster than Rocket A.

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**NO CALCULATOR ALLOWED**

**CALCULUS AB  
SECTION II, Part B**

**Time—45 minutes**

**Number of problems—3**

**No calculator is allowed for these problems.**

$t$ (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

Work for problem 4(a)

acceleration<sub>ave</sub> =  $\frac{11}{20}$  feet/second<sup>2</sup>

$$\begin{array}{r} 49 \\ -5 \\ \hline 44 \end{array} \qquad \begin{array}{r} 44 \\ -80 \\ \hline 11 \\ \hline 20 \end{array}$$

Work for problem 4(b)

The integral of the velocity is the position.  $\int_{10}^{70} v(t) dt$  means the position of the rocket from 10 sec to 70 sec.

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Continue problem 4 on page 11.



Work for problem 4(c)

$$a(t) = \frac{3}{\sqrt{t+1}}$$

$$t+1 = u$$

$$| dt = du$$

$$\int \frac{3}{\sqrt{t+1}} dt$$

$$= \int \frac{3 du}{\sqrt{u}}$$

$$= \int 3u^{-\frac{1}{2}} du$$

$$= 3 \cdot 2u^{\frac{1}{2}}$$

$$= 6u^{\frac{1}{2}}$$

$$v(t) = 6\sqrt{t+1}$$

$$v(80) = 6\sqrt{80+1}$$

$$= 6\sqrt{81}$$

$$= 6 \cdot 9$$

$$= 54 \text{ feet/second.}$$

Rocket B is traveling faster at time = 80 sec.

The antiderivative of the acceleration gives the velocity. Using this, the velocity of Rocket B was discovered to be 54 feet per second at time = 80 seconds. Compared to the velocity of Rocket A, which is 49 feet per second, Rocket B is traveling faster.

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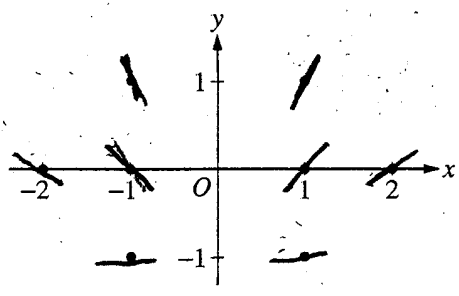
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NO CALCULATOR ALLOWED

Work for problem 5(a)



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NO CALCULATOR ALLOWED

Work for problem 5(b)

$$\frac{dy}{dx} = -\frac{1+y}{x}$$

$$\int \frac{dy}{1+y} = \int \frac{dx}{x}$$

$$e^{\ln|1+y|} = e^{\ln|x| + C}$$

$$1+y = Cx \quad 1+y > 0$$

$$y = Cx - 1, \quad y > -1$$

$$f(x) = Cx - 1, \quad f(x) > -1$$

$$f(-1) = C(-1) - 1 = 1$$

$$C(-1) = 2$$

$$C = -2$$

$$f(x) = -2x - 1, \quad f(x) > -1$$

$$D = \{x \in \mathbb{R} \mid x < 0\} \quad -2x - 1 > -1$$

$$D = (-\infty, 0) \quad \begin{array}{l} \text{or} \\ -2x > 0 \\ 2x < 0 \end{array}$$

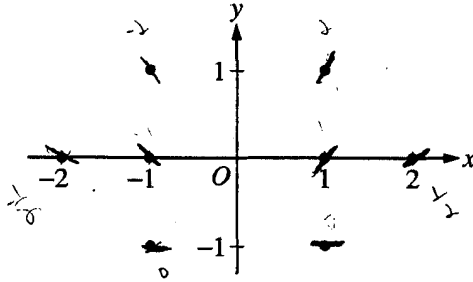
$$x < 0$$

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Work for problem 5(a)



$$y' = \frac{1+y}{x}$$

$$(1, 0) \quad \frac{1}{1} = 1$$


---


$$m = 1$$

$$(1, 1) \quad \frac{2}{1} = 2$$


---


$$m = 2$$

$$(1, -1) \quad \frac{0}{1} = 0$$


---


$$m = 0$$

$$(2, 0) \quad \frac{1}{2} = \frac{1}{2}$$


---


$$m = \frac{1}{2}$$

$$(-1, 0) \quad \frac{1}{-1} = -1$$


---


$$m = -1$$

$$(-1, 1) \quad \frac{2}{-1} = -2$$


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$$m = -2$$

$$(-1, -1) \quad \frac{0}{-1} = 0$$


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$$m = 0$$

$$(-2, 0) \quad \frac{1}{-2} = -\frac{1}{2}$$


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$$m = -\frac{1}{2}$$

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Continue problem 5 on page 13.

NO CALCULATOR ALLOWED

Work for problem 5(b)

$$y' = \frac{1+y}{x}$$

$$\int \frac{dy}{1+y} = \int \frac{dx}{x}$$

$$\ln(1+y) = \ln x + C$$

$$e^{\ln(1+y)} = e^{\ln x + C}$$

$$e^{\ln(1+y)} = 1+y$$

$$y = Ce^{2\ln x} - 1 \quad \text{where } x \neq 0$$

$$Ce^{2\ln x} \neq 1$$

$$x = 0$$

$$1 = Ce^{2\ln 1} - 1 \quad 2 = Ce^{2\ln 1}$$

$$C = \frac{2}{e^{2\ln(1)}}$$

$$y = \frac{2}{e^{2\ln(1)}} (e^{2\ln x}) - 1$$

$$\text{Domain: } (-\infty, 0) \cup (0, \infty)$$

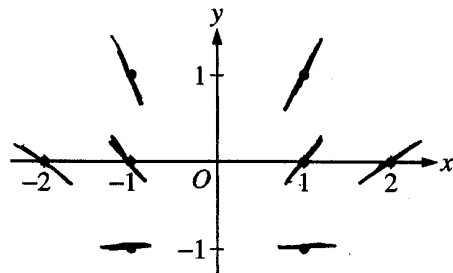
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Work for problem 5(a)



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Continue problem 5 on page 13.

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Work for problem 5(b)

$$u = 1+y$$

$$du = 1 \cdot dy$$

$$\int \frac{1}{1+y} dy = \int \frac{1}{u} du$$

$$\int \frac{1}{u} du = \int \frac{1}{x} dx$$

$$\ln u = \ln x$$

$$\ln(1+y) = \ln 2$$

$$\ln(1) = \ln 2$$

$$\ln 2 = \ln 2$$

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Work for problem 6(a)

$$g(x) = e^{ax} + f(x)$$

$$g'(x) = a e^{ax} + f'(x) \Rightarrow g'(0) = a e^0 + f'(0)$$

$$= a - 4$$

$$g''(x) = a^2 e^{ax} + f''(x) \Rightarrow g''(0) = a^2 e^0 + f''(0)$$

$$= a^2 + 3$$

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Continue problem 6 on page 15.



Work for problem 6(b)

$$h(x) = \cos(kx) \cdot f(x)$$

$$h'(x) = -\sin(kx) \cdot k \cdot f(x) + \cos(kx) \cdot f'(x)$$

$$= -k \cdot \sin(kx) \cdot f(x) + \cos(kx) \cdot f'(x)$$

$$h'(0) = -k \cdot \sin 0 \cdot f(0) + \cos(0) \cdot f'(0)$$

$$= 0 + 1 \cdot (-4) = -4$$

$$y = mx + b$$

$$y = -4x + b$$

$$2 = -4 \cdot 0 + b$$

$$b = 2$$

$$h(0) = \cos 0 \cdot f(0) = 1 \cdot 2 = 2$$

the line tangent to the graph of  $h$  is  $y = -4x + 2$

STOP

END OF EXAM

THE FOLLOWING INSTRUCTIONS APPLY TO THE COVERS OF THE SECTION II BOOKLET.

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- CHECK TO SEE THAT YOUR AP NUMBER LABEL APPEARS IN THE BOX(ES) ON THE COVER(S).
- MAKE SURE YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMS YOU HAVE TAKEN THIS YEAR.

NO CALCULATOR ALLOWED

Work for problem 6(a)

$$u = ax$$

$$du = a$$

$$g(x) = e^{ax} + f(x)$$

$$g'(x) = ae^{ax} + f'(x)$$

$$g'(0) = ae^{a \cdot 0} + 4$$

$$g'(0) = a + 4$$

$$g''(0) = 2ae^{ax} + f''(0)$$

$$g''(0) = 2ae^{a(0)} + 3$$

$$g''(0) = 2a + 3$$

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Continue problem 6 on page 15.

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Work for problem 6(b)

$$h(x) = \cos(kx) f(x)$$

$$h'(x) = \cos(kx) f'(x) + k \sin(kx) f(x)$$

$$h'(0) = \cos(0) \cdot -4 + k \sin(0) \cdot 2$$

$$h'(0) = -4 + 0$$

$$h'(0) = -4$$

$$h(0) = \cos(k \cdot 0) f(0)$$

$$h(0) = 1 \cdot 2$$

$$h(0) = 2$$

$$y = 2 = -4(x - 0)$$

STOP

END OF EXAM

THE FOLLOWING INSTRUCTIONS APPLY TO THE COVERS OF THE SECTION II BOOKLET.

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Work for problem 6(a)

$$f(0)=2 \quad f'(0)=-4 \quad f''(0)=3$$

$$g(x) = e^{ax} + f(x)$$

$$g'(x) = e^{ax} \cdot a + f'(x)$$

$$e^x \frac{d}{dx} = e^x$$

$$e^{f(x)} \frac{d}{dx} = e^{f(x)} f'(x) \frac{d}{dx}$$

$$g'(x) = ae^{ax} + -4$$

$$g'(x) = ae^{ax} - 4$$

$$e^0=1 \quad g'(0) = a e^{a(0)} - 4$$

$$g'(0) = a - 4$$

$$g''(x) = [ae^{ax} \cdot a]$$

$$g''(x) = a^2 e^{ax}$$

$$g''(0) = a^2 e^{a(0)} = 1$$

$$g''(0) = a^2$$

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Continue problem 6 on page 15.

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Work for problem 6(b)

$$h(x) = \cos(kx) f(x)$$

$$h'(x) = -\sin(kx) \cdot k \cdot f'(x)$$

$$h'(x) = -k f(x) \sin(kx) \quad (0,$$

$$h(x) = \cos(k(0)) f(0)$$

$$= \cos 0 \cdot 2$$

$$= 2$$

point (0, 2)

$$y - 2 = (-k f(x) \sin(kx)) (x - 0)$$

$$y = [-k f(x) \sin(kx)] (x) + 2$$

STOP

END OF EXAM

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