Question 4

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table above.

(a) Find the average acceleration of rocket A over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_{10}^{70} v(t)\,dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)\,dt$.

(c) Rocket B is launched upward with an acceleration of $a(t) = \frac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.

(a) Average acceleration of rocket A is

$$\frac{v(80) - v(0)}{80 - 0} = \frac{49 - 5}{80} = \frac{11}{20} \text{ ft/sec}^2$$

1 : answer

(b) Since the velocity is positive, $\int_{10}^{70} v(t)\,dt$ represents the distance, in feet, traveled by rocket A from $t = 10$ seconds to $t = 70$ seconds.

A midpoint Riemann sum is

$$20[v(20) + v(40) + v(60)]$$
$$= 20[22 + 35 + 44] = 2020 \text{ ft}$$

$$3 : \begin{cases} 1 : \text{explanation} \\ 1 : \text{uses } v(20), v(40), v(60) \\ 1 : \text{value} \end{cases}$$

(c) Let $v_B(t)$ be the velocity of rocket B at time t.

$$v_B(t) = \int \frac{3}{\sqrt{t+1}}\,dt = 6\sqrt{t+1} + C$$
$$2 = v_B(0) = 6 + C$$
$$v_B(t) = 6\sqrt{t+1} - 4$$
$$v_B(80) = 50 > 49 = v(80)$$

Rocket B is traveling faster at time $t = 80$ seconds.

$$4 : \begin{cases} 1 : 6\sqrt{t+1} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{finds } v_B(80), \text{ compares to } v(80), \\ \quad \text{and draws a conclusion} \end{cases}$$

Units of ft/sec^2 in (a) and ft in (b)

1 : units in (a) and (b)

NO CALCULATOR ALLOWED

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

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Work for problem 4(a)

$$\text{avgval}_{a(t)} = \frac{v(b) - v(a)}{b-a}$$

$$\text{avgval}_{a(t)} = \frac{v(80) - v(0)}{80-0}$$

$$\text{avgval}_{a(t)} = \frac{49-5}{80-0}$$

$$\boxed{\text{avgval}_{a(t)} = 11/20 \text{ ft/s}^2}$$

Work for problem 4(b)

$\int_{10}^{70} v(t)\,dt$ gives the distance the rocket has traveled from time 10 seconds to 70 seconds in feet.

$$\Delta x = \frac{70-10}{3} = 20$$

$$p(t) = 20\left[v(20) + v(40) + v(60)\right]$$

$$p(t) = 20\left[22 + 35 + 44\right] = 20\left[101\right]$$

$$\boxed{p(t) = 2020 \text{ feet}}$$

Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

Work for problem 4(c)

Rocket B

$a(t) = \frac{3}{\sqrt{t+1}}$

$v(t) = \int \frac{3}{\sqrt{t+1}}\, dt$

let $u = t+1$

$\frac{du}{dx} = 1$

$du = dx$

$v(t) = 3 \int u^{-1/2}\, du$

$v(t) = \frac{3u^{1/2}}{1/2} + c$

$v(t) = 6(t+1)^{1/2} + c$

$2 = 6(0+1)^{1/2} + c$

$c = -4$

$v(t) = 6(t+1)^{1/2} - 4$

$v(80) = 6(80+1)^{1/2} - 4$

$v(80) = 6\sqrt{81} - 4$

$v(80) = 54 - 4$

$\boxed{v(80) = 50 \text{ ft/sec}}$

Rocket A

$\boxed{v(80) = 49 \text{ ft/sec}}$

Rocket B is traveling faster at $t = 80$ sec. Rocket B's velocity was found by $v(t) = \int a(t)\, dt$ and is 50 ft/sec. Rocket A's velocity was 49 ft/sec

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4B,

NO CALCULATOR ALLOWED

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

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Work for problem 4(a)

$$\frac{1}{80}\left(9+8+7+6+5+4+3+2\right)=$$

$$=\frac{46}{80}=\frac{23}{40}\ ft/sec^2$$

Work for problem 4(b)

$\int_{10}^{70} v(t)\,dt$ indicates the distance the rocket traveled over the interval $10 \le t \le 70$. In this case, it indicates total distance because the rocket's velocity is only increasing during this time period.

$$(22+2(35)+44)=136\ ft.$$

$$\begin{array}{r} 70 \\ 22 \\ -44 \\ \hline 136 \end{array}$$

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Work for problem 4(c)

$3(t+1)^{-1/2}$

$v(t)=6\sqrt{t+1}+C$

$2=6\sqrt{0+1}+c$

$2=6+c$

$-4=c$

$v(t)=6\sqrt{t+1}-4$

$v(80)=6\sqrt{80+1}-4$

$6\cdot 9-4$

$v(80)=52$ ft/sec

Rocket B is traveling faster at t=80. The table states that Rocket A is traveling at 49 ft/sec. By taking the antideravitive for a(t) of Rocket B and solving with initial conditions for C, then substituting t=80, we find v(80)=52 ft/sec, faster than Rocket A.

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NO CALCULATOR ALLOWED

CALCULUS AB

SECTION II, Part B

Time—45 minutes

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Work for problem 4(a)

acceleration$_{ave} = \frac{11}{20}$ feet/second^2

$49 - 5 = 44$ $\quad \frac{44}{80}$ $\quad \frac{11}{20}$

Work for problem 4(b)

The integral of the velocity is the position. $\int_{10}^{70} v(t)\,dt$ means the position of the rocket from 10 sec to 70 sec.

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Work for problem 4(c)

$a(t) = \frac{3}{\sqrt{t+1}}$

$t+1 = u$

$1\,dt = du$

$\int \frac{3}{\sqrt{t+1}}\,dt$

$= \int \frac{3\,du}{\sqrt{u}}$

$= \int 3u^{-\frac{1}{2}}\,du$

$= 3 \cdot 2u^{\frac{1}{2}}$

$= 6u^{\frac{1}{2}}$

$v(t) = 6\sqrt{t+1}$

$v(80) = 6\sqrt{80+1}$

$= 6\sqrt{81}$

$= 6 \times 9$

$= 54$ feet/second.

Rocket B is traveling faster at time = 80 sec.

The antiderivative of the acceleration gives the velocity. Using this, the velocity of Rocket B was discovered to be 54 feet per second at time = 80 seconds. Compared to the velocity of Rocket A, which is 49 feet per second, Rocket B is traveling faster.

Question 4

Overview

This problem presented students with a table of velocity values for rocket A at selected times. In part (a) students needed to recognize the connection between the average acceleration of the rocket over the given time interval and the average rate of change of the velocity over this interval. In part (b) students had to recognize the definite integral as the total change, in feet, in rocket A's position from time $t = 10$ seconds to time $t = 70$ seconds and then approximate the value of this definite integral using a midpoint Riemann sum and the data in the table. Units of measure were important in both parts (a) and (b). Part (c) introduced a second rocket and gave its acceleration in symbolic form. The students were asked to compare the velocities of the two rockets at time $t = 80$ seconds. The velocity of rocket B could be determined by finding the antiderivative of the acceleration and using the initial condition or by using the Fundamental Theorem of Calculus and computing a definite integral.

Sample: 4A
Score: 9

The student earned all 9 points.

Sample: 4B
Score: 6

The student earned 6 points: 2 points in part (b), 3 points in part (c), and the units point. The first line in part (a) is a correct but uncommon method. An error was made in the addition. In part (b) the explanation is acceptable; the student correctly identifies the midpoint values but the method is incorrect. In part (c) the fourth point was not earned due to an error in the computation of $v(80)$ for rocket B.

Sample: 4C
Score: 3

The student earned 3 points: 1 point in part (a) and 2 points in part (c). The units are correct in part (a) but missing in part (b). For the explanation in part (b), the integral does not represent the position but rather the displacement over the time interval $[10, 70]$. The constant of integration does not appear in part (c) so the student earned the antiderivative and comparison points only.