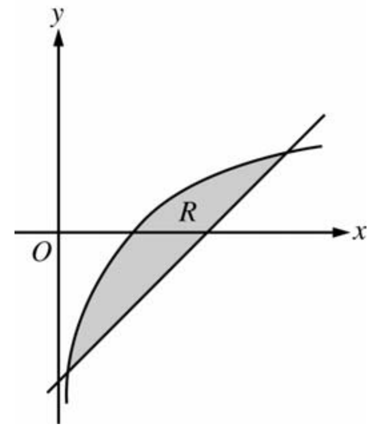
Question 1

Let R be the shaded region bounded by the graph of $y = \ln x$ and the line $y = x - 2$, as shown above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.

(c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about the y-axis.



$\ln(x) = x - 2$ when $x = 0.15859$ and 3.14619.
Let $S = 0.15859$ and $T = 3.14619$

(a) Area of $R = \int_S^T (\ln(x) - (x - 2))\, dx = 1.949$

$$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$$

(b) Volume $= \pi \int_S^T \left((\ln(x) + 3)^2 - (x - 2 + 3)^2 \right) dx$
$= 34.198$ or 34.199

$$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits, constant, and answer} \end{cases}$$

(c) Volume $= \pi \int_{S-2}^{T-2} \left((y + 2)^2 - \left(e^y\right)^2 \right) dy$

$$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$$

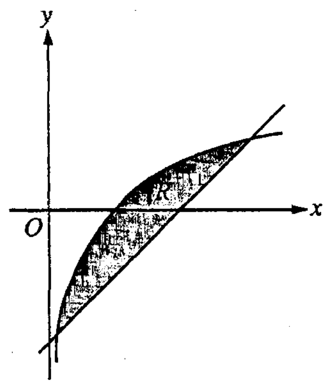
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$y = \ln x \qquad y = x - 2$

$\ln x = x - 2 \qquad x = .1586, \; x = 3.1462$

$$\int_{.1586}^{3.1462} (\ln x - x + 2)\, dx = \boxed{1.9491}$$

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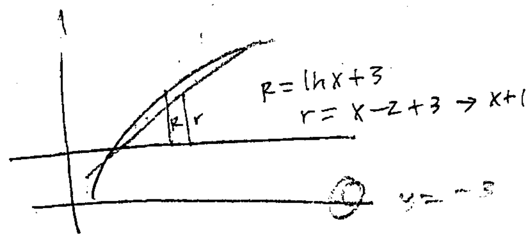
Do not write beyond this border.

Continue problem 1 on page 5.

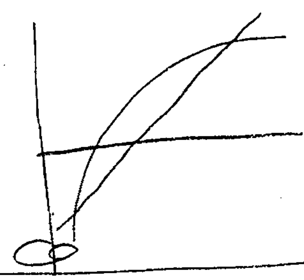
1A2

Work for problem 1(b)



$$\pi \int_{.1586}^{3.1462} \left[(\ln x + 3)^2 - (x - 2 + 3)^2 \right] dx = \boxed{34.1986}$$

Work for problem 1(c)



$$\boxed{2\pi \int_{.1586}^{3.1462} \left[x(\ln x - x + 2) \right] dx}$$

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1B,

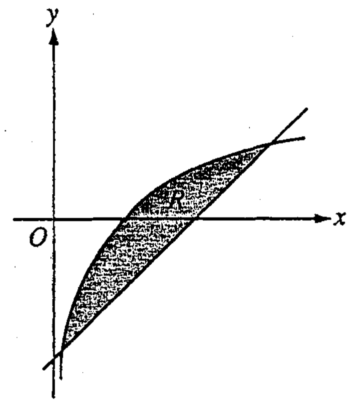
CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$y = \ln x \quad y = x - 2$

$\ln x = x - 2 \quad x = .158594 + 3.14619$

$A = \int_{.158594}^{3.14619} (\ln x - (x-2))\,dx$

$A = 1.949$

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Continue problem 1 on page 5.

1B2

Work for problem 1(b)

$V = \pi \int_{.158594}^{3.14619} ((-3 - \ln x)^2 - (-3 - (x-2))^2)\,dx$

$V = 10.886\pi$

$V = 34.199$

Work for problem 1(c)

$x - 2 = 0$
$x = 2$

$\ln x = 0$
$x = 1$

$V = \pi \int_{1}^{3.14619} ((\ln x)^2 - (x-2)^2)$

$V = \pi \int_{.158594}^{2} ((x-2)^2 - (\ln x)^2)$

$V = \pi \int_{1}^{3.14619} ((\ln x)^2 - (x-2)^2) + \pi \int_{.158594}^{2} ((x-2)^2 - (\ln x)^2)\,dx$

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1C1

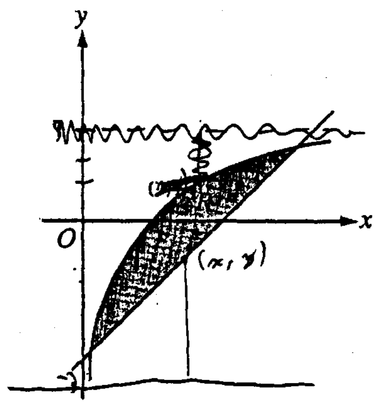
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$a =$ ~~[illegible]~~ $\int_0^{3.138} \ln x - (x-2)\,dx$

$\boxed{a = 1.80}$

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Continue problem 1 on page

Work for problem 1(b)

$r = \cancel{\#}3 - y$

$v = 2\pi \int_0^{3,138} (3-(x-2))(\ln x - (x-2))\, dx$

$v = 2\pi \cdot 6,299$

$\boxed{v = 12,599\,\pi}$

Work for problem 1(c)

$v = 2\pi \int_0^{3,138} x(\ln x - (x-2))\, dx$

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Question 1

Overview

This problem gave two graphs that intersect at $x = 0.15859$ and $x = 3.14619.$ A graphing calculator was required to find these two intersection values. Students needed to use integration to find an area and two volumes. In part (a) students had to find the area of the region bounded by the two graphs. In part (b) students had to calculate the volume of the solid generated by rotating the region about the horizontal line $y = -3,$ a line that lies below the given region. Part (c) tested the students' ability to set up an integral for the volume of a solid generated by rotating the given region around a vertical axis, in this case the y-axis. The given functions could be solved for x in terms of y, leading to the use of horizontal cross sections in the shape of washers and an integral in terms of the variable y. Although no longer included in the *AP Calculus Course Description*, the method of cylindrical shells could also be used to write an integral expression for the volume in terms of the variable x.

Sample: 1A
Score: 9

The student earned all 9 points.

Sample: 1B
Score: 6

The student earned 6 points: 3 points in part (a) and 3 points in part (b). In part (a) the student has the correct integrand, which earned the first point. The definite integral has the correct limits to three decimal places, which earned the second point. The answer is correct to three decimal places and earned the third point. In part (b) the student has the correct integrand, which earned the first 2 points. The extra factor of -1 in each integrand term does not cause a problem since it is an equivalent form to the standard. The student correctly evaluates the integral and produces the correct answer to three decimal places. In part (c) the student does have a difference of squares, which provides entry into the problem, but since neither term is correct the student did not earn either integrand point. The student was not eligible for the limits/constant point since no integrand point was earned.

Sample: 1C
Score: 3

The student earned 3 points: 1 point in part (a) and 2 points in part (c). In part (a) the student has the correct integrand, which earned the first point. The limit point was not awarded because the student's limits are not correct to three decimal places. Since the lower limit of 0 is not within the acceptable range for limits, the student was not eligible for the answer point. In part (b) the student attempts a cylindrical shell setup, but the integrand is incorrect, so neither point was earned. Since neither integrand point was earned, the student was not eligible for the limits/constant/answer point. In part (c) the student correctly provides the integrand for the cylindrical shells method and earned the first 2 points. The student's limits, in particular the lower value of 0, are not in the acceptable range, so the limits/constant point was not earned.