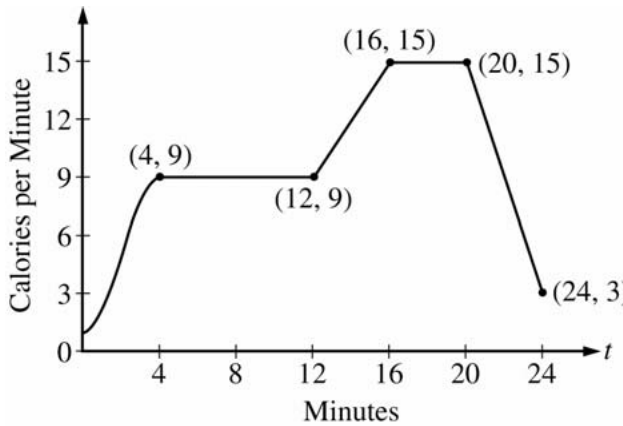
AP® CALCULUS AB
2006 SCORING GUIDELINES (Form B)

Question 4

The rate, in calories per minute, at which a person using an exercise machine burns calories is modeled by the function f. In the figure above, $f(t) = -\frac{1}{4}t^3 + \frac{3}{2}t^2 + 1$ for $0 \le t \le 4$ and f is piecewise linear for $4 \le t \le 24$.



(a) Find $f'(22)$. Indicate units of measure.

(b) For the time interval $0 \le t \le 24$, at what time t is f increasing at its greatest rate? Show the reasoning that supports your answer.

(c) Find the total number of calories burned over the time interval $6 \le t \le 18$ minutes.

(d) The setting on the machine is now changed so that the person burns $f(t) + c$ calories per minute. For this setting, find c so that an average of 15 calories per minute is burned during the time interval $6 \le t \le 18$.

(a) $f'(22) = \dfrac{15-3}{20-24} = -3$ calories/min/min

1 : $f'(22)$ and units

(b) f is increasing on $[0, 4]$ and on $[12, 16]$.

On $(12, 16)$, $f'(t) = \dfrac{15-9}{16-12} = \dfrac{3}{2}$ since f has constant slope on this interval.

On $(0, 4)$, $f'(t) = -\frac{3}{4}t^2 + 3t$ and

$f''(t) = -\frac{3}{2}t + 3 = 0$ when $t = 2$. This is where f' has a maximum on $[0, 4]$ since $f'' > 0$ on $(0, 2)$ and $f'' < 0$ on $(2, 4)$.

On $[0, 24]$, f is increasing at its greatest rate when $t = 2$ because $f'(2) = 3 > \frac{3}{2}$.

4 : $\begin{cases} 1 : f' \text{ on } (0, 4) \\ 1 : \text{shows } f' \text{ has a max at } t = 2 \text{ on } (0, 4) \\ 1 : \text{shows for } 12 < t < 16,\ f'(t) < f'(2) \\ 1 : \text{answer} \end{cases}$

(c) $\displaystyle\int_6^{18} f(t)\,dt = 6(9) + \frac{1}{2}(4)(9+15) + 2(15)$

$= 132$ calories

2 : $\begin{cases} 1 : \text{method} \\ 1 : \text{answer} \end{cases}$

(d) We want $\displaystyle\frac{1}{12}\int_6^{18} (f(t) + c)\,dt = 15$.

This means $132 + 12c = 15(12)$. So, $c = 4$.

OR

Currently, the average is $\dfrac{132}{12} = 11$ calories/min.

Adding c to $f(t)$ will shift the average by c.

So $c = 4$ to get an average of 15 calories/min.

2 : $\begin{cases} 1 : \text{setup} \\ 1 : \text{value of } c \end{cases}$

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4A

NO CALCULATOR ALLOWED

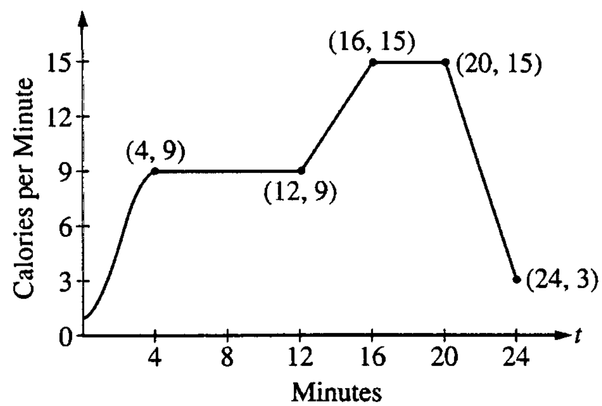
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



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Work for problem 4(a)

$f'(22) =$ Gradient of straight line from $t = 20$ to $t = 24$

$= \frac{3-15}{24-20}$

$= \frac{-12}{4}$

$= -3$ calories/minute^2

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Work for problem 4(b)

From graph, we see that f is only increasing in the intervals $0 \le t \le 4$ and $12 \le t \le 16$

Rate of increment for $12 \le t \le 16$

$= \frac{15-9}{16-12}$

$= \frac{6}{4} = 1.5$

For $0 \le t \le 4$,

$f(t) = -\frac{3}{4}t^2 + 3t$

$f'(t) = -\frac{3}{2}t + 3$

At greatest rate of increment, $f''(t) = 0 \Rightarrow t = 2$

$f'''(t) = -\frac{3}{2} < 0 \Rightarrow$ At $t = 2$, rate of increment is greatest and not smallest

$f(2) = -\frac{3}{4}(2)^2 + 3(2) = 3 > 1.5$

$\therefore f$ is increasing at its greatest rate ~~than~~ at $t = 2$

Work for problem 4(c)

~~For 6 to 12~~

~~For $6 \le t \le 12$~~,

Total number of calories burned $= \int_6^{18} f(t)\,dt$

~~$\int_6^{12} 9\,dt + \int_{12}^{16}$~~

$= 9(12-6) + \frac{1}{2}(9+15)(16-12) + 15(18-16)$

$= 54 + 48 + 30 = 132$ calories

Work for problem 4(d)

Before setting is changed, average calories in $6 \le t \le 18$

$= \frac{1}{18-6}\int_6^{18} f(t)\,dt = \frac{132}{12} = 11$ calories

Now, $\frac{1}{18-6}\int_6^{18} [f(t)+c]\,dt = 15$

$\Rightarrow \frac{1}{12}\int_6^{18} f(t)\,dt + \frac{1}{12}\int_6^{18} c\,dt = 15$

$\Rightarrow \frac{1}{12}[cx]_6^{18} = 15 - 11 = 4$

$\frac{1}{12}(18-6)c = 4$

$\Rightarrow c = 4$

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CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

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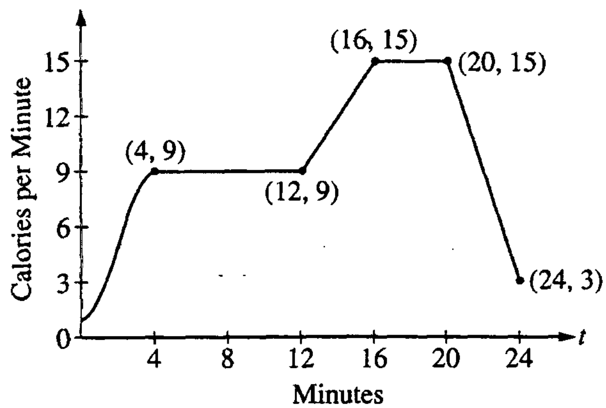

Work for problem 4(a)

$$f'(22) = \frac{f(24)-f(20)}{24-20} = \frac{3-15}{4} = -\frac{12}{4} = -3$$

$\therefore\ -3\,\text{cal}/\text{min}^2$

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Work for problem 4(b)

i) $0 \leq t \leq 4$, $f'(t) = -\frac{3}{4}t^2 + 3t$

$= -\frac{3}{4}(t^2 - 4t + 4) + 3$

$= -\frac{3}{4}(t-2)^2 + 3$

ii) $f'(t) = 0$ for $4 \leq t < 12$

iii) $f'(t) = \frac{15-9}{16-12} = \frac{6}{4} = \frac{3}{2}$ for $12 \leq t < 16$

iv) $f'(t) = 0$ for $16 \leq t < 20$

v) $f'(t) < 0$ for $20 \leq t < 24$

$\therefore$ $f'(t)$ is the greatest when $t=2$, which also means that $f(t)$ is increasing at its greatest rate

$\therefore t = 2$.

Work for problem 4(c)

i) $6 \leq t < 12$ $f(t) = 9$ $6 \times 9 = 54$

ii) $12 \leq t < 16$ $\frac{1}{2} \times (9 + 15) \times 4 = 48$

iii) $16 \leq t \leq 18$ $f(t) = 15$ $2 \times 15 = 30$

$\therefore 54 + 48 + 30 = 132$ $\therefore$ 132 calories

Work for problem 4(d)

$132 + c \times (18 - 6) = 132 + 12c$

Since $\frac{132 + 12c}{12} = 15$, $c = \frac{180 - 132}{12} = 4$

$\therefore c = 4$

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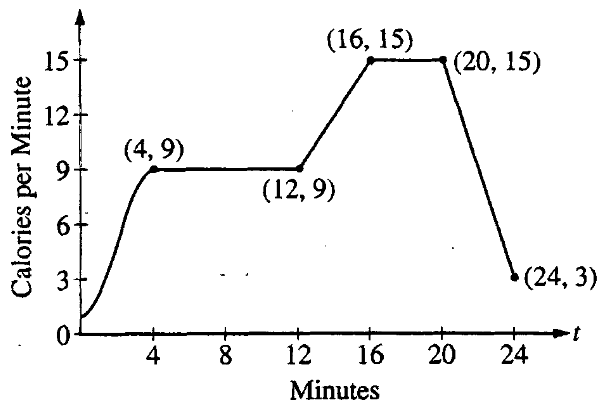
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Work for problem 4(a)

function for $20 \leq x \leq 24$

$m = \frac{3-15}{24-20}$ $\quad y - 3 = -3(x-24)$

$\frac{-12}{4}$ $\quad y = -3x + 75$

$m = -3$

$f(x) = -3x + 75$

$f'(22) = -3$ calories/minutes^2

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4C

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Work for problem 4(b)

function increases: at $0 \le x < 4$, $12 \le x \le 16$

$f(t) = -\frac{1}{4}t^3 + \frac{3}{2}t^2 + 1$

$f'(t) = -\frac{3}{4}t^2 + 3t$

$f''(t) = -\frac{3}{2}t + 3 = 0$

$t = 2$

maximum for $f'(t)$ at $t = 2$

$f'(2) = -\frac{3}{4}(4) + 6$

$= 3$

$m = \frac{15-9}{16-12} = \frac{3}{2}$

$f(x) = y - 15 = \frac{3}{2}(x-16)$

$f(x) = \frac{3}{2}x - 9$

$f'(x) = \frac{3}{2}$

Work for problem 4(c)

Calories total $= \int_6^{12} 9\,dx + \int_{12}^{16} (\frac{3}{2}x - 9)\,dx + \int_{16}^{18} 15\,dx$

$9x\Big]_6^{12} + \frac{3}{4}x^2 - 9\Big]_{12}^{16} + 15x\Big]_{16}^{18}$

$108 - 54 + 183 - 99 + 270 - 240$

$54 + 30 + 84$

164 calories burned

Work for problem 4(d)

$15 = \frac{\int_6^{18} (f(x) + c)\,dx}{12}$

$180 = \int_6^{18} f(x)\,dx + \int_6^{18} c\,dx$

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AP® CALCULUS AB
2006 SCORING COMMENTARY (Form B)

Question 4

Overview

This problem presented students with a piecewise-defined function f that modeled the rate at which a person using an exercise machine burns calories. The graph of f consisted of a cubic part and a part that was piecewise linear. In part (a) students were asked to find $f'(22)$, which required them to recognize the relationship between this value and the slope of one of the line segments in the graph of f. It was also important to use correct units. For part (b) students had to consider the two parts of the graph where f was increasing and determine the time when f was increasing at its greatest rate. In part (c) students had to use a definite integral to find the total number of calories burned over a given time interval. The evaluation of the definite integral could be done using geometry since the graph over the given time interval consisted of two line segments, one of which was horizontal. For part (d) students were expected to use the value of their integral from part (c) to work with the average value of the function f shifted up by c calories per minute.

Sample: 4A
Score: 9

The student earned all 9 points.

Sample: 4B
Score: 6

The student earned 6 points: 1 point in part (a), 1 point in part (b), 2 points in part (c), and 2 points in part (d). The student shows correct work for parts (a), (c), and (d). In part (b) the first derivative is correct and the student earned the first point. The student neither explains why $f'(t)$ has a maximum at 2 nor states the value of $f'(2)$. A proper reasoning for the final answer is not given, and the student did not earn any more points.

Sample: 4C
Score: 4

The student earned 4 points: 1 point in part (a), 1 point in part (b), 1 point in part (c), and 1 point in part (d). The work in part (a) is correct. In part (b) the first derivative is correct, so the student earned the first point. The student neither explains why $f'(t)$ has a maximum at 2 in the interval $0 \le t \le 4$ nor shows a comparison among the values of $f'(t)$. The student does not provide a final answer and did not earn any more points. In part (c) the setup is correct. The antiderivative of the second integral is incorrect, so the student did not earn the answer point. In part (d) the setup is correct, but the student does not finish the problem and could not earn the second point.

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