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4. Two antibiotics are available as treatment for a common ear infection in children.

- Antibiotic A is known to effectively cure the infection 60 percent of the time. Treatment with antibiotic A costs $50.
- Antibiotic B is known to effectively cure the infection 90 percent of the time. Treatment with antibiotic B costs $80.

The antibiotics work independently of one another. Both antibiotics can be safely administered to children. A health insurance company intends to recommend one of the following two plans of treatment for children with this ear infection.

- Plan I: Treat with antibiotic A first. If it is not effective, then treat with antibiotic B.
- Plan II: Treat with antibiotic B first. If it is not effective, then treat with antibiotic A.

(a) If a doctor treats a child with an ear infection using plan I, what is the probability that the child will be cured?

\[ p(\text{cure}) = p(\text{A cures it}) + p(\text{B cures it}) \cdot p(\text{child didn't cure it}) \]

\[ p(\text{cure}) = .6 + (.9)(.4) \]

\[ = .6 + .36 \]

\[ p(\text{cure}) = .96 \]

If a doctor treats a child with an ear infection using plan II, what is the probability that the child will be cured?

\[ p(\text{cure}) = p(\text{B cures it}) + p(\text{A cures it}) \cdot p(\text{B didn't cure it}) \]

\[ = .9 + (.6)(.1) \]

\[ = .9 + .06 \]

\[ p(\text{cure}) = .96 \]
(b) Compute the expected cost per child when plan I is used for treatment.

\[
\text{expected cost} = \text{cost A} + \text{cost B} \times p(A \text{ doesn't cure})
\]

\[
= 50 + 50 \times 0.4
\]

\[
= 50 + 20
\]

\[
= 70
\]

Compute the expected cost per child when plan II is used for treatment.

\[
\text{expected cost} = \text{cost B} + \text{cost A} \times p(B \text{ doesn't cure})
\]

\[
= 80 + 50 \times 0.1
\]

\[
= 80 + 5
\]

\[
= 85
\]

(c) Based on the results in parts (a) and (b), which plan would you recommend? Explain your recommendation.

I would recommend plan I. The probability that the child will be cured is the same, so that does not affect the choice. However, on average, plan I will cost less than plan II. Each child will cost an average of $70 on plan I, while costing $85 on plan II. Since the insurance companies normally end up paying for the medication, they would want to recommend the plan that would cost them the least per child on average.
4. Two antibiotics are available as treatment for a common ear infection in children.

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The antibiotics work independently of one another. Both antibiotics can be safely administered to children. A health insurance company intends to recommend one of the following two plans of treatment for children with this ear infection.

- Plan I: Treat with antibiotic A first. If it is not effective, then treat with antibiotic B.

- Plan II: Treat with antibiotic B first. If it is not effective, then treat with antibiotic A.

(a) If a doctor treats a child with an ear infection using plan I, what is the probability that the child will be cured?

\[
P(\text{cured}) = p(\text{success with } A) + p(\text{success with } B) - p(\text{success with } A \text{ and } B)
\]

\[
P(\text{cured}) = .6 + .9 - .54 = .96
\]

The probability the child is cured is .96

If a doctor treats a child with an ear infection using plan II, what is the probability that the child will be cured?

\[
P(\text{cured}) = p(\text{success with } B) + p(\text{success with } A) - p(\text{success with both})
\]

\[
P(\text{cured}) = .9 + .6 - .54 = .96
\]
(b) Compute the expected cost per child when plan I is used for treatment.

\[ \begin{align*}
\Pr(\text{treatment}) & : 0.6 \quad (A) & 0.4 \quad (A \text{ then } B) \\
\text{Price} & : $50 & $130 \\
\text{Expected Cost} & : 0.6 \times 50 + 0.4 \times 130 = $82
\end{align*} \]

Compute the expected cost per child when plan II is used for treatment.

\[ \begin{align*}
\Pr(\text{treatment}) & : 0.9 \quad (B) & 0.1 \quad (B \text{ then } A) \\
\text{Price} & : $80 & $130 \\
\text{Expected Cost} & : 0.9 \times 80 + 0.1 \times 130 = $85
\end{align*} \]

(c) Based on the results in parts (a) and (b), which plan would you recommend? Explain your recommendation.

I would recommend Plan I. Both treatments have the same probability of curing the kids, because it is the same procedure in reverse order. However, the expected cost for II is $85 per child, while the expected cost for plan I is $82 per child. Since Plan I is less costly, I would recommend the insurance company to go with Plan I.