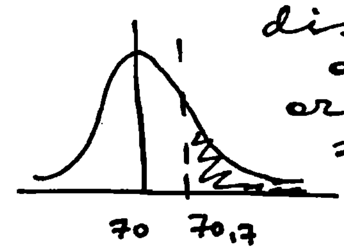
CollegeBoard
Advanced Placement Program

AP® Statistics
2004 Sample Student Responses
Form B

A

3. Trains carry bauxite ore from a mine in Canada to an aluminum processing plant in northern New York state in hopper cars. Filling equipment is used to load ore into the hopper cars. When functioning properly, the actual weights of ore loaded into each car by the filling equipment at the mine are approximately normally distributed with a mean of 70 tons and a standard deviation of 0.9 ton. If the mean is greater than 70 tons, the loading mechanism is overfilling.

(a) If the filling equipment is functioning properly, what is the probability that the weight of the ore in a randomly selected car will be 70.7 tons or more? Show your work.

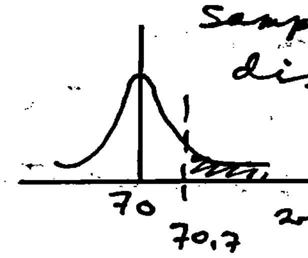


$Pr(\text{"weight of ore"} \geq 70{,}7) =$
$= Pr\left(Z \geq \frac{70{,}7-70}{0{,}9}\right) = Pr(Z > 0{,}78) \approx$
$\approx \underline{21{,}77\ \%}$;

(b) Suppose that the weight of ore in a randomly selected car is 70.7 tons. Would that fact make you suspect that the loading mechanism is overfilling the cars? Justify your answer. ⇒ no evidence to suspec

The probability of obtaining such result (weight = 70,7 tons) is 21,77 % (see part a)) ⇒ this is not an unusual result as it lies within 1σ away from the mean

(c) If the filling equipment is functioning properly, what is the probability that a random sample of 10 cars will have a mean ore weight of 70.7 tons or more? Show your work.



~~$X \sim N(70; 0{,}9^2) \Rightarrow Y = 10X \Rightarrow \mu_Y = 70$~~

$n = 10 \Rightarrow \mu_{\bar{x}} = 70;\ s_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0{,}9}{\sqrt{10}} \approx 0{,}285;$

$\Rightarrow \underline{Pr(\text{"mean weight"} \geq 70{,}7)} = Pr(\bar{X} \geq 70{,}7) =$
$= Pr\left(Z > \frac{70{,}7-70}{0{,}9/\sqrt{10}}\right) = Pr(Z > 2{,}46) \approx \underline{0{,}69\ \%}$;

(d) Based on your answer in part (c), if a random sample of 10 cars had a mean ore weight of 70.7 tons, would you suspect that the loading mechanism was overfilling the cars? Justify your answer.

The probability of obtaining such result (mean ore weight in a random sample of 10 cars > 70,7 tons) is only 0,69 % (approx.) ⇒ this is a rather unlikely result as it lies more than 2σ away from the mean in the sampling distr of means of ore weights ⇒ we would suspect that the loading mechanism was overfilling the cars.

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B

3. Trains carry bauxite ore from a mine in Canada to an aluminum processing plant in northern New York state in hopper cars. Filling equipment is used to load ore into the hopper cars. When functioning properly, the actual weights of ore loaded into each car by the filling equipment at the mine are approximately normally distributed with a mean of 70 tons and a standard deviation of 0.9 ton. If the mean is greater than 70 tons, the loading mechanism is overfilling.

(a) If the filling equipment is functioning properly, what is the probability that the weight of the ore in a randomly selected car will be 70.7 tons or more? Show your work.

$z = \frac{70.7 - 70}{.9} = \frac{7}{9} \approx .78$

$1 - .7823 = .2177$

The probability that the weight of the ore in a randomly selected car will be 70.7 tons or more is about .2177.

(b) Suppose that the weight of ore in a randomly selected car is 70.7 tons. Would that fact make you suspect that the loading mechanism is overfilling the cars? Justify your answer.

$H_0: \mu = 70$
$H_a: \mu > 70$

Assumptions:
randomly selected ✓
normally distributed ✓

$z = \frac{70.7 - 70}{.9} = \frac{7}{9} \approx .78$ p-value: .2177

Since the p-value is .2177, we don't have strong evidence to reject the null hypothesis, therefore the fact will not make me suspect that the loading mechanism is overfilling the cars

(c) If the filling equipment is functioning properly, what is the probability that a random sample of 10 cars will have a mean ore weight of 70.7 tons or more? Show your work.

$z = \frac{70.7 - 70}{\left(\frac{.9}{\sqrt{10}}\right)} \approx 2.46$

$1 - .9931 = .0069$

The probability that a random sample of 10 cars will have a mean ore weight of 70.7 tons or more is about .0069.

(d) Based on your answer in part (c), if a random sample of 10 cars had a mean ore weight of 70.7 tons, would you suspect that the loading mechanism was overfilling the cars? Justify your answer.

$H_0: \mu = 70$
$H_a: \mu > 70$

Assumptions
Randomly selected ✓
Normally distributed ✓

$z = \frac{70.7 - 70}{\left(\frac{.9}{\sqrt{10}}\right)} \approx 2.46$

$1 - .9931 = .0069$

p-value: .0069.

since the p-value is pretty small (.0069), there is a strong evidence to reject the null hypothesis, therefore I will suspect that the loading mechanism was overfilling the cars

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C

3. Trains carry bauxite ore from a mine in Canada to an aluminum processing plant in northern New York state in hopper cars. Filling equipment is used to load ore into the hopper cars. When functioning properly, the actual weights of ore loaded into each car by the filling equipment at the mine are approximately normally distributed with a mean of 70 tons and a standard deviation of 0.9 ton. If the mean is greater than 70 tons, the loading mechanism is overfilling.

(a) If the filling equipment is functioning properly, what is the probability that the weight of the ore in a randomly selected car will be 70.7 tons or more? Show your work.

the probability of this is $P(x \geq 70.7)$, where x is the weight of the ore in the car.

$$z_0 = \frac{70.7 - 70}{0.9} = \frac{x - \mu}{\sigma}$$

$z_0 \approx 0.778$

$P(x \geq 70.7) = P(z \geq z_0) \approx 0.218$

(b) Suppose that the weight of ore in a randomly selected car is 70.7 tons. Would that fact make you suspect that the loading mechanism is overfilling the cars? Justify your answer.

~~$H_0: \mu = 70$~~
~~$H_a: \mu > 70$~~

That would not provide sufficient evidence that the cars are overfilled (because the probability that a single car weighs more than 70.7 is rather large (22%), and, moreover, it is natural that the weight of ore in one car deviates from the sample mean ($70.7 - 70 = 0.7$, which is even less than the standard deviation)

(c) If the filling equipment is functioning properly, what is the probability that a random sample of 10 cars will have a mean ore weight of 70.7 tons or more? Show your work.

~~$n = 10$~~
~~$H_0: \mu = 70$~~
~~$H_a: \mu > 70$~~
~~$\bar{x} = 70.7$~~

$n = 10$

$$z = \frac{70.7 - 70}{0.9/\sqrt{10}} = 2.46$$

$P(z \geq z_0) = 0.0069$ - probability that a sample of 10 cars will have a mean weight of 70.7 or more tons.

(d) Based on your answer in part (c), if a random sample of 10 cars had a mean ore weight of 70.7 tons, would you suspect that the loading mechanism was overfilling the cars? Justify your answer.

$n = 10$
$H_0: \mu = 70$ (no overfilling)
$H_a: \mu > 70$ (overfilling)
$\bar{x} = 70.7$
To check ~~the~~ H_0, I will use a 1-sample T-test (small sample)
The p-value for the null hypothesis is 0.018

$$t\text{-statistic} = \frac{70.7 - 70}{0.9/\sqrt{10}}$$

At a significance level over 1.8% ~~[illegible]~~ H_0 may be rejected (p-value < error level), so at those significance levels we can state that the true mean exceeds 70 (the mechanism is overfilling the cars).

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