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Solution

Part (a):

There is a strong nonlinear relationship between impact rate and age. Impact rate declines rapidly with age over the age range from 0.4 to about 0.7 billion years, and then seems to level out.

Part (b):

89.4% of the variability in ln(impact rate) can be explained by a linear, or straight line, relationship between ln(impact rate) and ln(age).

OR

For this data set, there is an 89.4% reduction in sum of squared errors when the least squares line is used to describe the relationship between ln(impact rate) and ln(age) when compared to using the line $\hat{y} = \bar{y}$.

Part (c):

There is a noticeable curved pattern in the residual plot, which indicates that the linear model is not the best choice for describing the relationship between ln(impact rate) and ln(age).

Scoring

Each part is scored as essentially correct, partially correct, or incorrect.

Part (a) is essentially correct if

1. the description includes an indication that impact rate declines with age
2. the relationship is described as nonlinear
3. the strength of the relationship is described*

Part (a) is partially correct if either component 1 or component 2 is present.

*Note: While it is important to describe the strength of the relationship, failure to comment on the strength of the relationship is treated as a minor error because it is difficult to judge the strength of the relationship in this nonlinear situation with so few data points. The overall quality of the responses to this question can compensate for this minor error, so this minor error does not necessarily drop the part (a) score from essentially correct to partially correct.

Part (b) is essentially correct if

1. the value 0.894 or 89.4% is used in the interpretation
2. the response gives a correct interpretation as either proportion of variability explained or as reduction of sum of squared error compared to the $\hat{y} = \bar{y}$ line
3. the interpretation is in context
Question 1 (cont’d.)

Part (b) is partially correct if:

- the student gives a generic definition of $r^2$ that is not in the context of the problem,
- the student interprets the value of $r^2$ as if it were the correlation coefficient (for example, says there is a strong linear relationship between $\ln(\text{impact rate})$ and $\ln(\text{age})$),
- impact rate and age are used in the interpretation rather than $\ln(\text{impact rate})$ and $\ln(\text{age})$,
- the student interprets $r^2$ as the square of a negative correlation, $r = -\sqrt{0.894} = -0.9455$, between $\ln(\text{impact rate})$ and $\ln(\text{age})$.

Part (c) is essentially correct if the student

1. comments on the pattern in the residual plot
2. concludes that the linear model is not the best model for describing the relationship between $\ln(\text{impact rate})$ and $\ln(\text{age})$.

Part (c) is partially correct if the student

- states that the model is not appropriate or not the best model, but fails to relate this conclusion to the residual plot.

Part (c) is incorrect if the student says the model is appropriate because of the high $r^2$ value OR the student claims there is no pattern in the residual plot.

4 Complete Response
   All three parts essentially correct

3 Substantial Response
   Two parts essentially correct and 1 part partially correct

2 Developing Response
   2 parts essentially correct and no parts partially correct
   OR
   One part essentially correct and 2 parts partially correct
   OR
   3 parts partially correct

1 Minimal Response
   One part essentially correct and either 0 or 1 parts partially correct
   OR
   No parts essentially correct and 2 parts partially correct
Question 2

Solution

Part (a):

Since the manager used a convenience sample (the first 100 students entering the cafeteria), bias may have been introduced. Students who arrive at the cafeteria early may have opinions of food quality that differ in some important way from other students who live in the dormitories.

This bias could be avoided by selecting a random sample of 100 dormitory residents instead of just asking the first 100 students entering the cafeteria.

Part (b):

The way the question is worded may be leading. The first part of the question included a statement that many students think the food needs improvement. This may lead people to support this view by responding that the food does need improvement.

The inclusion of the phrase "even though that would increase the cost of the meal plan" may lead students to say the food is OK only because they do not want to pay more.

A better wording might be to simply ask "Do you think that the quality of the food served in the cafeteria needs improvement?"

Scoring

Each part is scored as either essentially correct, partially correct, or incorrect.

Part (a) is essentially correct if the response

1. indicates that selecting the first 100 students to arrive at the cafeteria could introduce bias because opinions of the first 100 students might differ from the opinions of other students who live in the dormitories
2. proposes a reasonable alternative that involves random selection—a simple random sample of dorm residents, or some sort of stratified random sample, or a systematic sample with a random starting point.

Part (a) is partially correct if the response

indicates why selecting the first 100 students to arrive is not reasonable, but proposes an alternative that does not involve random selection

OR

proposes a reasonable alternative that involves random selection, but does not explain how selecting the first 100 students could introduce bias.
Question 2 (cont’d.)

Part (b) is essentially correct if the response

1. points out at least one of the two possible problems with the question wording
2. proposes reasonable alternate wording that addresses the concern(s) raised.

Note: The student only needs to identify one problem and take care of it. If only one of the two wording concerns is raised, the alternate wording need only address the one wording problem. However, if the student identifies both problems, appropriate alternate wording must be provided for both problems.

Part (b) is partially correct if the response

identifies one or both of the potential wording problems, but does not propose a new wording that adequately addresses an identified problem.

Part (b) is incorrect if the response

points out both wording problems, but then argues that the question wording is OK as is because the biases are in opposing directions and so will balance each other.

4 Complete Response
   Both parts essentially correct

3 Substantial Response
   One part essentially correct and the other part partially correct

2 Developing Response
   One part essentially correct and the other part incorrect
   OR
   Both parts partially correct

1 Minimal Response
   One part partially correct
Solution

Let $X =$ weight of ore in a randomly selected car.

Part (a):

$$P(X > 70.7) = P \left( Z > \frac{70.7 - 70}{0.9} \right) = P(Z > 0.78) = 0.2177$$

Part (b):

No. Approximately 22% of the cars will have ore weights of 70.7 or greater when the filling equipment is working properly, so a car that was filled with 70.7 tons of ore would not be an unusual occurrence.

Part (c):

$$P(\bar{X} > 70.7) = P \left( Z > \frac{70.7 - 70}{0.9} \right) = P \left( Z > \frac{0.7}{0.285} \right) = P(Z > 2.46) = 0.0069$$

Part (d):

Yes, we would suspect that the filling mechanism is overfilling. If it is working properly, the probability that the mean weight of the ore in 10 randomly selected cars is 70.7 or greater is 0.0069, which is very small.

Note: To receive complete credit for part (a) or part (c) students must show how the probability is computed. Since part (a) and part (c) involve different normal distributions, it is important to identify which normal distribution is used in each part. As shown above, this could be done by displaying a probability statement containing the mean and standard deviation for the appropriate normal distribution. It could be done in other ways, such as listing the mean and standard deviation and displaying an appropriate graph.
Scoring

Parts (a) and (b) are scored together as section 1 and parts (c) and (d) are scored together as section 2.

Section 1 and 2 are each scored as either essentially correct, partially correct, or incorrect.

**Section 1** is essentially correct if

1. the probability in part (a) is correctly computed (except for minor arithmetic errors as long as the given answer is a number between 0 and 0.5)
2. the response in part (b) says it would not necessarily indicate that the filling equipment was malfunctioning and the justification is based on the probability computed in part (a).

Section 1 is partially correct if

the probability computed in part (a) is not correct, but the conclusion in (b) is reasonable relative to the computed probability and the justification is based on the computed probability,

OR

the probability in part (a) is correctly computed, except for minor arithmetic errors, but the justification in part (b) is not linked to the computed probability,

OR

the probability in part (a) is correctly computed, except for minor arithmetic errors, and the conclusion in part (b) is not consistent with the computed probability.

**Section 2** is essentially correct if

1. the probability in part (c) is correctly computed (except for minor arithmetic errors as long as the given answer is a number between 0 and 0.5) using the mean and standard deviation of the \( \overline{X} \) distribution
2. the response in part (d) says we would suspect that the filling mechanism was overfilling
3. the justification is based on the probability computed in part (c).

Section 2 is partially correct if

the probability computed in part (c) is not correct, but the conclusion in (d) is reasonable relative to the computed probability and the justification is based on the computed probability

OR

the probability in part (c) is correctly computed except for minor arithmetic errors, but the justification in part (d) is not linked to the computed probability.

Note: The response in part (b) could be justified by indicating that the 70.7 tons is less than one standard deviation away from desired mean of 70 tons. The response in part (d) could be justified by indicating that the 70.7 tons is more than two standard deviations above the desired mean of 70 tons.
4  Complete Response
    Both sections are essentially correct

3  Substantial Response
    One section is essentially correct and the other section is partially correct

2  Developing Response
    One section is essentially correct and the other section is incorrect
    OR
    Both sections are partially correct

1  Minimal Response
    One section is partially correct
Solution

Part (a):

Step 1: Identifies appropriate confidence interval by name or by formula and checks appropriate assumptions.

Two-sample confidence interval for the difference in means, \((\mu_1 - \mu_2)\) is

\[
(\bar{X}_1 - \bar{X}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}
\]

where

\[
\mu_1 = \text{the mean time spent on homework by all 6th graders at Crest Middle School}
\]

\[
\mu_2 = \text{the mean time spent on homework by all 7th graders at Crest Middle School}
\]

Assumptions:
1. Independent random samples,
2. Large samples or normal population distributions

The problem states that samples were independent random samples and that dotplots of the data showed it was not unreasonable to assume that the population distributions are approximately normal. So, it is okay to proceed.

Step 2: Correct mechanics

A 95% confidence interval for \((\mu_1 - \mu_2)\) is computed as

\[
(47.0 - 27.3) \pm t^* \sqrt{\frac{12.4^2}{20} + \frac{10.8^2}{20}}
\]

which gives

\[
19.7 \pm t^* (3.68)
\]

Most calculators use \(t^* = 2.026\) based on 37.297 degree of freedom. Then the confidence interval is

\((12.25, 27.15)\).

Step 3: Interpretation

(For 95% confidence level) Based on these samples, we can be 95% confident that the difference in mean times spent studying for seventh graders and sixth graders is between 12.25 minutes and 27.15 minutes.

Part (b):

Matching students in this way, using the measured responses, is inappropriate because it creates artificial association between responses from independent samples. (1) Pairs should be created on the basis of one or more variables that might be related to the response, not on the response itself. (2) The matching plan must be established before the data are collected.
Scoring

Part (a) is scored according to the number of correct steps:

Steps 1, 2, and 3 are each scored as either essentially correct or incorrect.

Note for Step 1: If a student uses the pooled confidence interval, then he or she must address the assumption that the two variances are equal. This assumption can be checked by looking at the ratio of the two sample variances. Since \(12.4^2 / 10.8^2 = 1.32\) is close to 1 (smaller than 2.53, the upper 0.975 percentile of an F-distribution with (19,19) df), it is okay to proceed with the pooled confidence interval.

Note for Step 2: An identifiable minor arithmetic error in step 2 will not necessarily change a score from essentially correct to incorrect.

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Part (b) is scored as either essentially correct or incorrect.

Part (b) is essentially correct if it

1. says that it is not reasonable to pair in the proposed way
   AND
2. gives an explanation that says either that pairing must be done prior to collecting the data or that pairing must be done using variables other than the response, or proposes an acceptable scheme of matching using some variable other than the response.

Note: The assistant principal’s suggestion to match sixth and seventh graders based on their responses would create an artificial positive correlation between the responses from two independent random samples and result in an interval estimate of the difference in the means that tends to be too narrow to provide the required level of confidence.
Question 4 (cont’d.)

4  Complete Response

All three steps of the confidence interval in part (a) are essentially correct and part (b) is essentially correct

3  Substantial Response

All three steps of the confidence interval in part (a) are essentially correct and part (b) is incorrect
OR
Two steps of the confidence interval in part (a) are essentially correct and part (b) is essentially correct

2  Developing Response

Two steps of the confidence interval in part (a) are essentially correct and part (b) is incorrect
OR
One step of the confidence interval in part (a) is essentially correct and part (b) is essentially correct

1  Minimal Response

One step of the confidence interval in part (a) is essentially correct and part (b) is incorrect
OR
Part (b) is essentially correct
Solution

Part (a):

Modern Thai Dogs
n = 16
IQR = 128 - 121 = 7
Outlier boundaries:
121 – 1.5(7) = 110.5
128 + 1.5(7) = 138.5
no outliers for Modern Thai Dogs

Golden Jackals
n = 16
IQR = 112 - 107 = 5
Outlier boundaries:
107 – 1.5(5) = 99.5
112 + 1.5(5) = 119.5
outliers on the high end: 122, 124, 125
The distributions of mandible lengths for Modern Thai Dogs and Golden Jackals are not similar. The distribution of mandible lengths for Modern Thai Dogs is approximately symmetric and a typical value is about 125, whereas the distribution of mandible lengths for Golden Jackals has a typical value that is much smaller, around 108, and the distribution appears to be skewed to the right with outliers (relative to likely samples from a normal distribution) on the high end. The variability in the lengths is roughly the same for both types of dogs.

Part (b):
Yes, the boxplot for this random sample is roughly symmetric and there are no outliers. It is reasonable to assume that the distribution of mandible lengths for Modern Thai Dogs is approximately normal, and it is okay to construct a t-confidence interval for the mean mandible length for Modern Thai Dogs.

Part (c):
No, it would not be reasonable to use this data to perform a two-sample t-test. The boxplot for mandible lengths of Golden Jackals is not symmetric, and there are three outliers in a sample of size 16. This indicates that the distribution of mandible lengths for Golden Jackals is not approximately normal, and so it would not be appropriate to use the data to perform a two-sample t-test.

Scoring

Part (a) is divided into two sections—(a1) the construction of the plots and (a2) the comparison of the two mandible length distributions.

Each part is scored as either essentially correct, partially correct, or incorrect.

Part (a1) is essentially correct if parallel boxplots are correctly drawn and the display includes labels. The labels for the dogs (Modern Thai or Golden jackal) are required, but if the label for the x-axis is the only item missing, the part (a1) is still scored essentially correct.

Part (a1) is partially correct if:
- labels for Modern Thai Dogs and Golden Jackals are omitted, OR
- there is an error in the construction of the boxplots, OR
- no outliers are shown, but otherwise the boxplots are correct.

Part (a1) is incorrect if the numbers provided in the stem of the problem are used as a data set to form the boxplots below.
Part (a2) is essentially correct if a comparison of the two distributions on the basis of at least two of center, shape and spread is given.

Part (a2) is partially correct if the distributions are compared on only one of center, shape, or spread.

Part (b) is essentially correct if the student says it is reasonable to construct a confidence interval and bases the justification on the approximate symmetry of the boxplot from part (a). Simply reporting that there are no outliers is not sufficient.

Part (c) is essentially correct if the student says that it is unreasonable to perform the hypothesis test and bases the justification on skewness of the Golden Jackal mandible length distribution, suggested by the boxplot in part (a) or the presence of outliers.
Question 5 (cont’d.)

FOR PARTS (a1), (a2), (b), and (c), ESSENTIALLY CORRECT RESPONSES COUNT AS 1 PART AND FOR PARTS (a1) and (a2) PARTIALLY CORRECT RESPONSES COUNT AS ½ PART.

4  Complete Response
    Four parts correct

3  Substantial Response
    Three parts correct

2  Developing Response
    Two parts correct

1  Minimal Response
    One part correct

IF A PAPER IS BETWEEN TWO SCORES (FOR EXAMPLE, 2 ½ PARTS) USE A HOLISTIC APPROACH TO DETERMINE WHETHER TO SCORE UP OR DOWN DEPENDING ON THE STRENGTH OF THE RESPONSE AND COMMUNICATION.
Solution

Part (a):

Step 1: States a correct pair of hypotheses

Let \( p_A \) = proportion of banded birds on island A
\( p_B \) = proportion of banded birds on island B

\[
\begin{align*}
H_0: p_A - p_B &= 0 \\
H_a: p_A - p_B &\neq 0
\end{align*}
\]

OR

\[
\begin{align*}
H_0: p_A &= p_B \\
H_a: p_A &\neq p_B
\end{align*}
\]

Step 2: Identifies a correct test (by name or by formula) and checks appropriate assumptions.

Two-sample test for proportions

\[
z = \frac{\hat{p}_A - \hat{p}_B}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_A} + \frac{1}{n_B}\right)}}
\]

where

\[
\hat{p} = \frac{n_A \hat{p}_A + n_B \hat{p}_B}{n_A + n_B} = \frac{12 + 25}{180 + 220} = 0.1175
\]

Assumptions: independent random samples and large sample sizes

The problem states that it is reasonable to regard the samples as random samples. Since the samples are taken on different islands it may be reasonable to assume that they are independent. Since the expected counts are greater than 10 (or 5), the sample sizes are large enough to accurately use a \( z \)-test or a chi-square test.

\[
\begin{align*}
n_A \hat{p}_A &= 12 & n_A (1 - \hat{p}_A) &= 168 \\
n_B \hat{p}_B &= 35 & n_B (1 - \hat{p}_B) &= 185
\end{align*}
\]

These estimates of expected counts are used to check the accuracy of normal approximations to two different binomial distributions when the standard error is not estimated from the pooled estimate of the banding probability.

\[
\begin{align*}
n_A \hat{p} &= 21.15 & n_A (1 - \hat{p}) &= 158.85 \\
n_B \hat{p} &= 25.85 & n_B (1 - \hat{p}) &= 194.15
\end{align*}
\]

These estimates of expected counts are used to assess the accuracy if a chi-square approximation or a normal approximation to the \( z \)-statistic when the standard deviation is estimated using the pooled estimate of the banding probability.
Step 3: Correct mechanics, including the value of the test statistic and P-value (or rejection region)

\[ \hat{p}_A = \frac{12}{180} = 0.067 \quad \text{and} \quad \hat{p}_B = \frac{35}{220} = 0.159 \]

\[ \hat{p} = \frac{12 + 35}{180 + 220} = \frac{47}{400} = 0.1175 \]

\[ z = \frac{0.067 - 0.159}{\sqrt{\frac{(0.1175)(0.8825)}{180} + \frac{(0.1175)(0.8825)}{220}}} \]

\[ = \frac{-0.092}{\sqrt{0.00105}} = \frac{-0.092}{0.032} = -2.875 \]

p-value = 0.00429

Step 4: Stating a correct conclusion in the context of the problem, using the result of the statistical test.

Since p-value=0.00429 is smaller than \( \alpha = 0.05 \), reject the null hypothesis. There is convincing evidence that the proportions of banded birds on the two islands are not the same.

Part (b):

For island A,

\[ n_I = 200 \quad \text{where} \quad n_I \quad \text{is the number of birds banded in the initial sample,} \]

\[ \hat{p}_S = \frac{12}{180} = 0.067 \quad \text{where} \quad \hat{p}_s \quad \text{is the proportion banded in the subsequent sample.} \]

We expect that the proportion of banded birds in the subsequent sample is approximately equal to the proportion of the population that is banded. Then

\[ \hat{p}_S \approx \frac{n_I}{\text{population size}} \]

and the population size can be estimated by

\[ \text{estimated population size} = \frac{n_I}{\hat{p}_S} = \frac{200}{0.06667} = 3000 \]
Part (c):

Possible concerns are:

1. Are some birds more likely to be captured than others? If, for example, slower birds are more likely to be captured in both the initial and subsequent samples, we would tend to underestimate the population size, thinking that a larger proportion of birds has been banded than was actually the case (because the birds that were caught and banded in the initial sample were also more likely to be the ones caught in the subsequent sample).

2. It may be the case that birds that are caught and banded in the initial sample learn from the experience and are less likely to be caught as part of the second sample. This would cause us to overestimate the population size.

3. There must be enough time between the samples so that there is adequate mixing of the banded and unbanded birds.

4. By banding the birds the researchers might make them more susceptible to their predators. In order to have a reasonable estimate, we must assume that the death rate of the banded birds is the same as the death rate of the unbanded birds so the banding procedure should not harm the birds or make them more conspicuous. For example, using large fluorescent bands is not a good idea.

5. Differentiable catchability. For example, birds that spend most of their time on the nest may be much less likely to be captured than other birds, and young birds may be more likely to be captured.

6. If the time between samples is too long, births could occur in the populations. Obviously, the new arrivals will not be banded.

Scoring

Each part is scored as either essentially correct, partially correct, or incorrect.

Part (a) is essentially correct if
all four parts of the hypothesis test are correct.

Part (a) is partially correct if
2 or 3 parts of the hypothesis test are correct.

Notes on part (a):

- If the hypotheses are reversed in step 1 (i.e. $H_0: \ p_A \neq p_B$ and $H_a: \ p_A = p_B$), then no credit can be received for the context of the conclusion in step 4.
- If both an $\alpha$ and a p-value are given, the linkage in step 4 is implied. If no $\alpha$ is given, the solution must be explicit about the linkage by giving a correct interpretation of the p-value or explaining how the conclusion follows from the p-value.
- If the p-value in step 3 is incorrect but the conclusion is consistent with the computed p-value, step 4 can be considered correct.
• While not recommended, a confidence interval can be used (if the pooled variance estimate is used in the test, the CI and the test for unequal population proportions are not exactly equivalent). If a confidence interval is used, the four parts for the scoring are
  1. Identifies appropriate confidence interval by name or by formula
  2. Checks appropriate assumptions
  3. Correct mechanics
  4. Interpretation.

A 95% confidence interval for the difference in the proportions of banded birds on each island is

\[ (0.067 - 0.159) \pm (1.96)(0.03088) \Rightarrow (-0.153, -0.032) \]

**Part (b)** is essentially correct if
  1. the response develops a reasonable approach to estimating population size using the initial sample size and the proportion banded in the subsequent sample
  2. correctly computes the value of the estimate.

Note: The estimate will depend on the value used for 12/180. The estimate should not be rounded down, so values between 200/0.067=2986 and 200/0.0666667=3000 are OK.

Part (b) is partially correct if
  1. the response develops an estimator that does not combine the initial sample size and the proportion banded in the subsequent sample in a reasonable way, but the response shows some statistical reasoning leading to the proposed estimator
  2. the value of the proposed estimator is computed.

Note: An interval estimate of the population size could be presented instead of a point estimate. First make a 95% confidence interval for the proportion of banded birds on Island A:

\[ 0.066667 \pm (1.96)(0.0185924) \Rightarrow (0.0302255, 0.1031078) \]

Then an approximate 95% confidence interval for the population size is

\[ \left( \frac{200}{0.1031078}, \frac{200}{0.0302255} \right) \Rightarrow (1939, 6627) \]

**Part (c)** is essentially correct if
  Two reasonable concerns are articulated that could affect the estimated proportions.

Note: If the student does not describe the impact of the concern, that is OK.

Part (c) is partially correct if
  only one reasonable concern is articulated.

Note: If the student only provides two examples of the same concern, then the solution is scored as partially correct.
Question 6 (cont’d.)

4 Complete Response:

All three parts are essentially correct.

3 Substantial Response

The hypothesis test in part (a) is essentially correct and one of the other two parts is essentially correct

OR

The hypothesis test in part (a) is essentially correct and both of the other two parts are partially correct

OR

The hypothesis test in part (a) is partially correct and both of the other two parts are essentially correct

2 Developing Response

The hypothesis test in part (a) is essentially correct

OR

The hypothesis test in part (a) is partially correct and one of the other two parts is essentially correct

OR

The hypothesis test in part (a) is partially correct and both of the other two parts are partially correct

1 Minimal Response

The hypothesis test in part (a) is partially correct

OR

Either part (b) or part (c) is essentially correct

OR

Both parts (b) and (c) are partially correct