The materials included in these files are intended for noncommercial use by AP teachers for course and exam preparation; permission for any other use must be sought from the Advanced Placement Program®. Teachers may reproduce them, in whole or in part, in limited quantities, for face-to-face teaching purposes but may not mass distribute the materials, electronically or otherwise. This permission does not apply to any third-party copyrights contained herein. These materials and any copies made of them may not be resold, and the copyright notices must be retained as they appear here.
Question 1

Let $R$ be the region enclosed by the graph of $y = \sqrt{x - 1}$, the vertical line $x = 10$, and the $x$-axis.

(a) Find the area of $R$.

(b) Find the volume of the solid generated when $R$ is revolved about the horizontal line $y = 3$.

(c) Find the volume of the solid generated when $R$ is revolved about the vertical line $x = 10$.

| (a) Area | \[
\int_{1}^{10} \sqrt{x - 1} \, dx = 18
\]
| (b) Volume | \[
\pi \int_{1}^{10} \left(9 - (3 - \sqrt{x - 1})^2\right) \, dx = 212.057 \text{ or } 212.058
\]
| (c) Volume | \[
\pi \int_{0}^{3} \left(10 - (y^2 + 1)^2\right) \, dy = 407.150
\]
Question 2

For 0 ≤ t ≤ 31, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by

\[ R(t) = 5\sqrt{t} \cos\left(\frac{t}{5}\right) \]

mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time t = 0.

(a) Show that the number of mosquitoes is increasing at time t = 6.

(b) At time t = 6, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

(c) According to the model, how many mosquitoes will be on the island at time t = 31? Round your answer to the nearest whole number.

(d) To the nearest whole number, what is the maximum number of mosquitoes for 0 ≤ t ≤ 31? Show the analysis that leads to your conclusion.

(a) Since \( R(6) = 4.438 > 0 \), the number of mosquitoes is increasing at t = 6.

(b) \( R'(6) = -1.913 \)

Since \( R'(6) < 0 \), the number of mosquitoes is increasing at a decreasing rate at t = 6.

(c) \( 1000 + \int_{0}^{31} R(t) \, dt = 964.335 \)

To the nearest whole number, there are 964 mosquitoes.

(d) \( R(t) = 0 \) when \( t = 0, t = 2.5\pi, \text{ or } t = 7.5\pi \)

\[ R(t) > 0 \text{ on } 0 < t < 2.5\pi \]
\[ R(t) < 0 \text{ on } 2.5\pi < t < 7.5\pi \]
\[ R(t) > 0 \text{ on } 7.5\pi < t < 31 \]

The absolute maximum number of mosquitoes occurs at \( t = 2.5\pi \) or at \( t = 31 \).

\[ 1000 + \int_{0}^{2.5\pi} R(t) \, dt = 1039.357, \]

There are 964 mosquitoes at \( t = 31 \), so the maximum number of mosquitoes is 1039, to the nearest whole number.
A test plane flies in a straight line with positive velocity \( v(t) \), in miles per minute at time \( t \) minutes, where \( v \) is a differentiable function of \( t \). Selected values of \( v(t) \) for \( 0 \leq t \leq 40 \) are shown in the table above.

(a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate \( \int_{0}^{40} v(t) \, dt \). Show the computations that lead to your answer. Using correct units, explain the meaning of \( \int_{0}^{40} v(t) \, dt \) in terms of the plane’s flight.

(b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval \( 0 < t < 40 \)? Justify your answer.

(c) The function \( f(t) \), defined by \( f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right) \), is used to model the velocity of the plane, in miles per minute, for \( 0 \leq t \leq 40 \). According to this model, what is the acceleration of the plane at \( t = 23 \)? Indicates units of measure.

(d) According to the model \( f \), given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval \( 0 \leq t \leq 40 \)?

\[
\begin{array}{|c|c|c|c|c|c|c|c|}
\hline
\text{t (min)} & 0 & 5 & 10 & 15 & 20 & 25 & 30 & 35 & 40 \\
\hline
\text{v(t) (mpm)} & 7.0 & 9.2 & 9.5 & 7.0 & 4.5 & 2.4 & 2.4 & 4.3 & 7.3 \\
\hline
\end{array}
\]

(a) Midpoint Riemann sum is
\[
10 \cdot \left[ v(5) + v(15) + v(25) + v(35) \right]
= 10 \cdot \left[ 9.2 + 7.0 + 2.4 + 4.3 \right] = 229
\]
The integral gives the total distance in miles that the plane flies during the 40 minutes.

(b) By the Mean Value Theorem, \( v'(t) = 0 \) somewhere in the interval \((0, 15)\) and somewhere in the interval \((25, 30)\). Therefore the acceleration will equal 0 for at least two values of \( t \).

(c) \( f'(23) = -0.407 \) or \(-0.408\) miles per minute^2

(d) Average velocity = \[ \frac{1}{40} \int_{0}^{40} f(t) \, dt \]
\[
= 5.916 \text{ miles per minute}
\]

Copyright © 2004 by College Entrance Examination Board. All rights reserved. Visit apcentral.collegeboard.com (for AP professionals) and www.collegeboard.com/apstudents (for AP students and parents).
The figure above shows the graph of $f'$, the derivative of the function $f$, on the closed interval $-1 \leq x \leq 5$. The graph of $f'$ has horizontal tangent lines at $x = 1$ and $x = 3$. The function $f$ is twice differentiable with $f(2) = 6$.

(a) Find the $x$-coordinate of each of the points of inflection of the graph of $f$. Give a reason for your answer.

(b) At what value of $x$ does $f$ attain its absolute minimum value on the closed interval $-1 \leq x \leq 5$? At what value of $x$ does $f$ attain its absolute maximum value on the closed interval $-1 \leq x \leq 5$? Show the analysis that leads to your answers.

(c) Let $g$ be the function defined by $g(x) = xf(x)$. Find an equation for the line tangent to the graph of $g$ at $x = 2$.

---

(a) $x = 1$ and $x = 3$ because the graph of $f'$ changes from increasing to decreasing at $x = 1$, and changes from decreasing to increasing at $x = 3$.

(b) The function $f$ decreases from $x = -1$ to $x = 4$, then increases from $x = 4$ to $x = 5$.

Therefore, the absolute minimum value for $f$ is at $x = 4$.

The absolute maximum value must occur at $x = -1$ or at $x = 5$.

$$f(5) - f(-1) = \int_{-1}^{5} f'(t) \, dt < 0$$

Since $f(5) < f(-1)$, the absolute maximum value occurs at $x = -1$.

(c) $g'(x) = f(x) + xf'(x)$

$g'(2) = f(2) + 2f'(2) = 6 + 2(-1) = 4$

$g(2) = 2f(2) = 12$

Tangent line is $y = 4(x - 2) + 12$
Consider the differential equation $\frac{dy}{dx} = x^4(y - 2)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

(Note: Use the axes provided in the test booklet.)

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the $xy$-plane. Describe all points in the $xy$-plane for which the slopes are negative.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 0$.

### (a)

Slopes are negative at points $(x, y)$ where $x \neq 0$ and $y < 2$.

### (c)

\[
\frac{1}{y - 2} \, dy = x^4 \, dx
\]

\[
\ln|y - 2| = \frac{1}{5}x^5 + C
\]

\[
|y - 2| = e^{C} e^{\frac{1}{5}x^5}
\]

\[
y - 2 = Ke^{\frac{1}{5}x^5}, \quad K = \pm e^{C}
\]

\[
y = 2 - Ke^{\frac{1}{5}x^5}
\]

\[
-2 = Ke^{0} = K
\]

\[
y = 2 - 2e^{\frac{1}{5}x^5}
\]
Let \( \ell \) be the line tangent to the graph of \( y = x^n \) at the point \((1, 1)\), where \( n > 1 \), as shown above.

(a) Find \( \int_0^1 x^n \, dx \) in terms of \( n \).

(b) Let \( T \) be the triangular region bounded by \( \ell \), the \( x \)-axis, and the line \( x = 1 \). Show that the area of \( T \) is \( \frac{1}{2n} \).

(c) Let \( S \) be the region bounded by the graph of \( y = x^n \), the line \( \ell \), and the \( x \)-axis. Express the area of \( S \) in terms of \( n \) and determine the value of \( n \) that maximizes the area of \( S \).

\[
\int_0^1 x^n \, dx = \frac{x^{n+1}}{n+1} \bigg|_0^1 = \frac{1}{n+1}
\]

(b) Let \( b \) be the length of the base of triangle \( T \).

\[
\frac{1}{b} \text{ is the slope of line } \ell, \text{ which is } n
\]

\[
\text{Area}(T) = \frac{1}{2} b(1) = \frac{1}{2n}
\]

(c) \[
\text{Area}(S) = \int_0^1 x^n \, dx - \text{Area}(T)
\]

\[
\frac{1}{n+1} - \frac{1}{2n}
\]

\[
\frac{\text{d}}{\text{dn}} \text{Area}(S) = -\frac{1}{(n+1)^2} + \frac{1}{2n^2} = 0
\]

\[
2n^2 = (n+1)^2
\]

\[
\sqrt{2} n = (n+1)
\]

\[
n = \frac{1}{\sqrt{2} - 1} = 1 + \sqrt{2}
\]