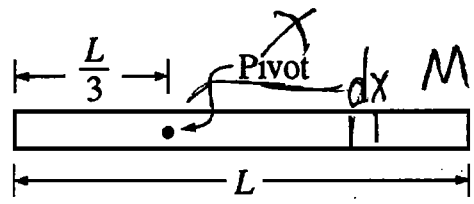
AP® Physics C: Mechanics
2004 Sample Student Responses

A1



Mech. 3.

A uniform rod of mass M and length L is attached to a pivot of negligible friction as shown above. The pivot is located at a distance $\frac{L}{3}$ from the left end of the rod. Express all answers in terms of the given quantities and fundamental constants.

(a) Calculate the rotational inertia of the rod about the pivot.

$\lambda = \frac{M}{L} \quad dm = \lambda dx = \frac{M}{L} dx$

$I = \sum mr^2 = \int_0^{\frac{2}{3}L} \frac{M}{L} dx \cdot x^2 + \int_0^{\frac{L}{3}} \frac{M}{L} dx \cdot x^2$

$= \frac{M}{L}\int_0^{\frac{2}{3}L} x^2 dx + \frac{M}{L}\int_0^{\frac{L}{3}} x^2 dx$

$= \frac{M}{L}\left(\frac{x^3}{3}\Big|_0^{\frac{2}{3}L}\right) + \frac{M}{L}\left(\frac{x^3}{3}\Big|_0^{\frac{L}{3}}\right)$

$= \frac{M}{L}\left(\frac{\frac{8}{27}L^3}{3} + \frac{\frac{L^3}{27}}{3}\right)$

$= \frac{ML^2}{9}$

(b) The rod is then released from rest from the horizontal position shown above. Calculate the linear speed of the bottom end of the rod when the rod passes through the vertical.

$h = \frac{2}{3}L \quad h_1 = \frac{1}{2}L \quad \omega = \frac{v}{\frac{2}{3}L} \Rightarrow v = \frac{2}{3}L \cdot \omega$

$U_i + K_i = U_f + K_f$

$Mgh + 0 = Mgh_1 + \frac{1}{2}I\omega^2$

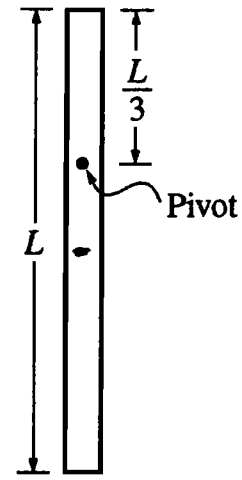
$Mg \cdot \frac{2L}{3} = Mg \cdot \frac{L}{2} + \frac{1}{2} \cdot \frac{ML^2}{9} \cdot \omega^2$

$4MgL = 3MgL + \frac{ML^2}{3}\omega^2$

$\Rightarrow \frac{ML^2\omega^2}{3} = MgL \Rightarrow \omega^2 = \frac{3Mg}{ML} \Rightarrow \omega = \sqrt{\frac{3g}{L}}$

$\Rightarrow v = \frac{2L}{3} \cdot \sqrt{\frac{3g}{L}} = \sqrt{\frac{4L^2}{9} \cdot \frac{3g}{L}} = \sqrt{\frac{4Lg}{3}}$

A2



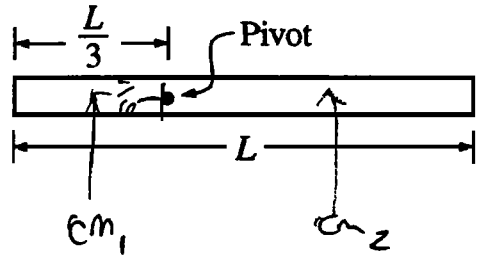
(c) The rod is brought to rest in the vertical position shown above and hangs freely. It is then displaced slightly from this position. Calculate the period of oscillation as it swings.

$$\omega = \sqrt{\frac{Mgd}{I}} \Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{Mgd}}$$

$$d = \frac{L}{2} - \frac{L}{3} = \frac{L}{6}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{\frac{ML^2}{9}}{Mg\frac{L}{6}}} = 2\pi\sqrt{\frac{ML^2}{3} \cdot \frac{6}{MgL}} = 2\pi\sqrt{\frac{2L}{3g}}$$

B1



Mech. 3.

A uniform rod of mass M and length L is attached to a pivot of negligible friction as shown above. The pivot is located at a distance $\frac{L}{3}$ from the left end of the rod. Express all answers in terms of the given quantities and fundamental constants.

(a) Calculate the rotational inertia of the rod about the pivot.

Rod's inertia around center of mass: $\frac{1}{12}mL^2$

$\frac{1}{12}mL^2 + m\left(\frac{L}{6}\right)^2 = \boxed{\frac{1}{9}mL^2}$

(b) The rod is then released from rest from the horizontal position shown above. Calculate the linear speed of the bottom end of the rod when the rod passes through the vertical.

mechanical energy conserved

$U_g = K_R + U_g$

$0 = mgh = \frac{1}{2}I\omega^2 - \left(\frac{m}{3}\right)(g)\left(\frac{L}{6}\right) + \left(\frac{2m}{3}\right)(g)\left(\frac{L}{3}\right)$

$0 = \frac{1}{2}\left(\frac{1}{9}mL^2\right)(\omega^2) - \frac{mgL}{18} + \frac{2mgL}{9} = \frac{mL^2\omega^2}{18} + \frac{mgL}{6}$

$\frac{L^2\omega^2}{18} = \frac{gL}{6}$

$L\omega^2 = 3g$

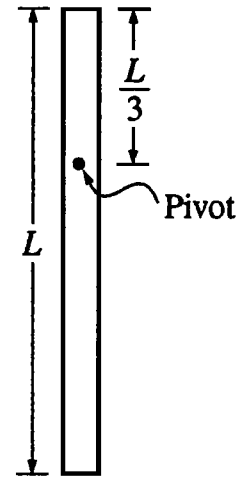
$\omega^2 = 3g/L$

$\omega = \sqrt{3g/L}$

$v = \omega r$

$v = \sqrt{\frac{3g}{L}}\left(\frac{2L}{3}\right)$

$v = \frac{2\sqrt{3gL}}{3}$



(c) The rod is brought to rest in the vertical position shown above and hangs freely. It is then displaced slightly from this position. Calculate the period of oscillation as it swings.

$$\omega = \sqrt{\frac{mgd}{I}} = \sqrt{\frac{mg\frac{L}{3}}{\frac{1}{9}mL^2}}$$

$$= \sqrt{\frac{9mgL}{3mL^2}} = \sqrt{\frac{3g}{L}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{3g}}$$