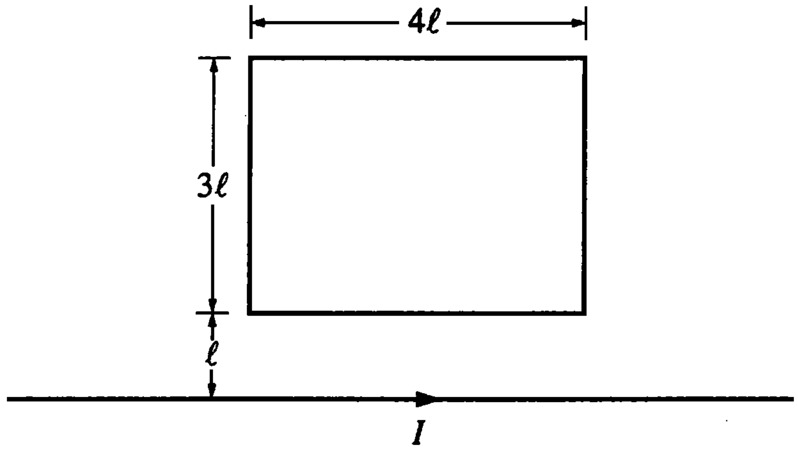
CollegeBoard
Advanced Placement Program

AP® Physics C: Electricity and Magnetism
2004 Sample Student Responses



E&M. 3.

A rectangular loop of dimensions 3 ℓ and 4 ℓ lies in the plane of the page as shown above. A long straight wire also in the plane of the page carries a current I.

(a) Calculate the magnetic flux through the rectangular loop in terms of I, ℓ, and fundamental constants.

$\Phi_B = \int B \cdot dA$ $\quad \oint B \cdot ds = \mu_0 I$

$B 2\pi r = \mu_0 I$

$\Phi_B = \int B \cdot dA$

$\oint B \cdot ds = \mu_0 I$

$B 2\pi r = \mu_0 I$

$B = \frac{\mu_0 I}{2\pi r}$

$\Phi_B = \int \frac{\mu_0 I}{2\pi r} \cdot dA$ $\quad A = 4\ell \cdot r$, $dA = 4\ell dr$

$\Phi_B = \int_{\ell}^{4\ell} \frac{\mu_0 I}{2\pi r} 4\ell dr$

~~$\Phi_B =$~~

$\Phi_B = \frac{2\mu_0 I \ell}{\pi} \ln(r) \Big|_{\ell}^{4\ell}$

$\Phi_B = \frac{2\mu_0 I \ell}{\pi}(\ln(4\ell) - \ln \ell)$

$\Phi_B = \frac{2\mu_0 I \ell}{\pi} \ln(4)$

$\boxed{\Phi_B = \frac{4\mu_0 I \ell}{\pi} \ln(2)}$

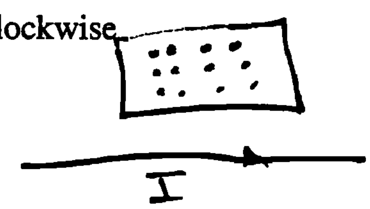
Starting at time $t = 0$, the current in the long straight wire is given as a function of time t by

$I(t) = I_0 e^{-kt}$, where I_0 and k are constants.

(b) The current induced in the loop is in which direction?

____ Clockwise $\quad$ _X_ Counterclockwise

Justify your answer.



$I(t) = I_0 e^{-kt}$

function starts at I_0 and decreases from there ∴ B field is decreasing out of the loop. ⇒ according to Lenz's law: B field created by induced current must be in same direction as decreasing B field thus..

GO ON TO THE NEXT PAGE.

The loop has a resistance R. Calculate each of the following in terms of R, I_0, k, ℓ, and fundamental constants.

(c) The current in the loop as a function of time t

$$\Phi_B = \frac{4\ell\mu_0 I_0 e^{-kt}}{\pi}\ln(2)$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$I = -\frac{1}{R}\frac{d\Phi_B}{dt}$$

$$I = -\frac{1}{R}\cdot\left(\frac{4\ell\mu_0 I_0 \ln(2) e^{-kt}}{\pi}\right)$$

$$\boxed{I = \frac{4\ell\mu_0 I_0 \ln(2) k}{R\pi} e^{-kt}}$$

(d) The total energy dissipated in the loop from $t = 0$ to $t = \infty$

$$P = \frac{dW}{dt}$$

$$\int_0^\infty P\,dt = W$$

$$P = IV$$

$$V = IR$$

$$P = I^2 R$$

$$\int_0^\infty I^2 R\,dt = W$$

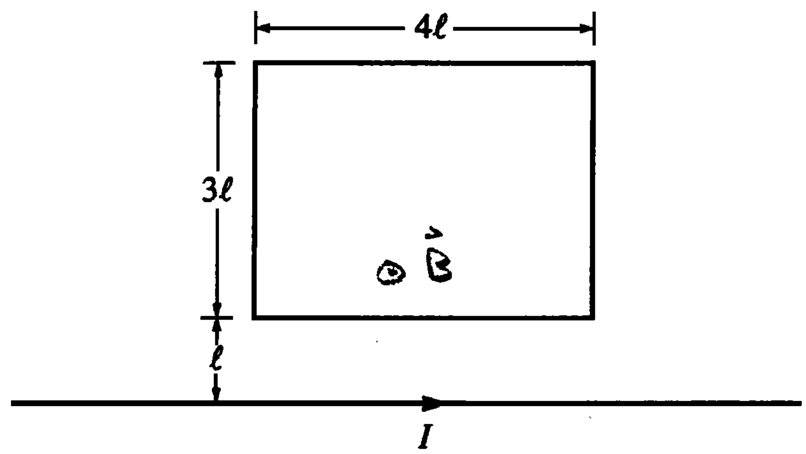
$$\int_0^\infty \frac{16\ell^2\mu_0^2 I_0^2 \ln^2(2) k^2}{R^{\cancel{2}}\pi^2} e^{-2kt}\,\cancel{R} = W$$

$$\frac{16\ell^2\mu_0^2 I_0^2 \ln^2(2) k^2}{R\pi^2}\left(\frac{-e^{-2kt}}{2k}\right)\Bigg|_0^\infty$$

$$\left(\frac{-1}{2ke^{2k\infty}} - \frac{-1}{2ke^0}\right)$$

$$0 + \frac{1}{2k}$$

$$\boxed{\frac{8\ell^2\mu_0^2 I_0^2 \ln^2(2)\, k}{R\pi^2} = W}$$



E&M. 3.

A rectangular loop of dimensions 3ℓ and 4ℓ lies in the plane of the page as shown above. A long straight wire also in the plane of the page carries a current I.

(a) Calculate the magnetic flux through the rectangular loop in terms of I, ℓ, and fundamental constants.

$$\phi = \int B \cdot dA \Rightarrow B \cdot A = B(12x^2) = \phi = \underline{\mu_0 I}$$

$$\oint_{2\pi l}^{2\pi 3l} B \cdot dl = \mu_0 I \Rightarrow Bl\Big|_{2\pi l}^{2\pi 3l} = 6\pi Bl - 2\pi Bl \Rightarrow 4\pi Bl$$

$$4\pi Bl = \mu_0 I$$

$$B = \frac{\mu_0 I}{4\pi l}$$

$$\phi = \frac{\mu_0 I}{4\pi \cancel{l}} \cdot 12 l^2 \Rightarrow \frac{12\mu_0 I l}{4\pi} \Rightarrow \frac{3\mu_0 I l}{\pi} \Rightarrow \phi$$

Starting at time $t = 0$, the current in the long straight wire is given as a function of time t by

$I(t) = I_0 e^{-kt}$, where I_0 and k are constants.

(b) The current induced in the loop is in which direction?

____ Clockwise   _✓_ Counterclockwise

Justify your answer.

Since the current is decreasing, the current is induced in such a way that the B-field is added to the one produced by the current. this would be a B-field coming out of ~~the~~ page, and according to the RH rule, this would be a counterclockwise current

GO ON TO THE NEXT PAGE.

The loop has a resistance R. Calculate each of the following in terms of R, I_0, k, ℓ, and fundamental constants.

(c) The current in the loop as a function of time t

$$\phi = BA = \frac{3\mu_0 I \ell}{\pi} \Rightarrow \frac{3\mu_0 \ell (I_0 e^{-kt})}{\pi} = \phi$$

$$\mathcal{E} = \frac{\partial \phi}{\partial t} \Rightarrow \frac{3 I_0 k \ell (e^{-kt}) \mu_0}{\pi}$$

$$I = \frac{\mathcal{E}}{R} \Rightarrow \frac{3 I_0 k \ell \mu_0 (e^{-kt})}{\pi R}$$

(d) The total energy dissipated in the loop from $t = 0$ to $t = \infty$

$$P = I^2 R \Rightarrow P = R \int_0^{\infty} \frac{3 I_0 k \ell \mu_0 (e^{-kt})}{\pi R} \Rightarrow$$

$$\frac{3 I_0 K \ell \mu_0}{\pi} \int_0^{\infty} e^{-kt} \Rightarrow \frac{3 I_0 K \ell \mu_0}{\pi} \left(\frac{e^{-kt}}{K} \right|_0^{\infty} \Rightarrow$$

$$\frac{3 I_0 k \ell \mu_0}{\pi} \left(\infty - \right.$$