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E&M. 2.

In the circuit shown above left, the switch S is initially in the open position and the capacitor C is initially uncharged. A voltage probe and a computer (not shown) are used to measure the potential difference across the capacitor as a function of time after the switch is closed. The graph produced by the computer is shown above right. The battery has an emf of 20 V and negligible internal resistance. Resistor \( R_1 \) has a resistance of 15 k\( \Omega \) and the capacitor C has a capacitance of 20 \( \mu \)F.

(a) Determine the voltage across resistor \( R_2 \) immediately after the switch is closed.

\[ \frac{20 \text{ V}}{20 \text{ V}}, \text{ since no current will go through } R_1 \]

(b) Determine the voltage across resistor \( R_2 \) a long time after the switch is closed.

\[ 20 - 12 = 8 \text{ V}, \text{ since voltage across } C \text{ is } 12 \text{ V, and voltage across } R_1 \text{ is } 12 \text{ V} \]

(c) Calculate the value of the resistor \( R_2 \).

\[
\frac{8}{20} = \frac{R_2}{15 + R_2} \quad \Rightarrow \quad 12R_2 = 120 \\
R_2 = 10 \text{ k}\Omega
\]

(d) Calculate the energy stored in the capacitor a long time after the switch is closed.

\[
E = \frac{1}{2} CV^2 \\
= \frac{1}{2} (20 \mu F)(12 \text{ V})^2 \\
= \frac{1}{2} (20 \times 10^{-6})(144) \\
= 1.44 \times 10^{-3} \text{ J}
\]
(e) On the axes below, graph the current in $R_2$ as a function of time from 0 to 15 s. Label the vertical axis with appropriate values.

\[ I_2 = \frac{V_2}{R_2} \]
\[ = 0.8 \times 10^{-3} \]
\[ = 0.8 \text{ mA} \]

\[ I_0 = \frac{20}{25 \times 10^3} \]
\[ = 2 \times 10^{-3} \]
\[ = 2 \text{ mA} \]

Resistor $R_2$ is removed and replaced with another resistor of lesser resistance. Switch $S$ remains closed for a long time.

(f) Indicate below whether the energy stored in the capacitor is greater than, less than, or the same as it was with resistor $R_2$ in the circuit.

\[ \checkmark \text{ Greater than } \quad \_\_\_ \text{ Less than } \quad \_\_\_ \text{ The same as} \]

Explain your reasoning.

If $R_2$ is replaced with a resistor of smaller resistance, the voltage drop across the new resistor will be smaller and thus the voltage drop across the capacitor will be greater, leading to a larger stored energy, since $E = \frac{1}{2} CV^2$. 

GO ON TO THE NEXT PAGE.
E&M. 2.

In the circuit shown above left, the switch $S$ is initially in the open position and the capacitor $C$ is initially uncharged. A voltage probe and a computer (not shown) are used to measure the potential difference across the capacitor as a function of time after the switch is closed. The graph produced by the computer is shown above right. The battery has an emf of 20 V and negligible internal resistance. Resistor $R_1$ has a resistance of 15 kΩ and the capacitor $C$ has a capacitance of 20 μF.

(a) Determine the voltage across resistor $R_2$ immediately after the switch is closed.

Immediately after the switch is closed, the capacitor acts as a wire, so $V_{R_2} = \varepsilon = 20 [\text{V}]$

(b) Determine the voltage across resistor $R_2$ a long time after the switch is closed.

A long time afterwards,

$V_{R_2} + V_C = \varepsilon \implies V_{R_2} = \varepsilon - V_C = 20 [\text{V}] - 12 [\text{V}] = 8 [\text{V}]$

(c) Calculate the value of the resistor $R_2$.

$V_{R_1} = V_C \text{ by the loop rule.}$

So, $V_{R_1} = \varepsilon - R_1 \cdot I$

$\implies \varepsilon - R_1 \cdot I = \frac{V_C}{R_1} = \frac{12 [\text{V}]}{15 \text{ k} \Omega} = 0.8 \times 10^{-4} [\text{A}]$

$I = 8.0 \times 10^{-4} [\text{A}]$

$R_2 = \frac{V_{R_2}}{I} = \frac{8 [\text{V}]}{0.8 \times 10^{-4} [\text{A}]} = 100 \text{ k} \Omega$

(d) Calculate the energy stored in the capacitor a long time after the switch is closed.

$E = \frac{1}{2} C V^2 = \frac{1}{2} \cdot 20 \times 10^{-6} [\text{F}] \cdot (12 [\text{V}])^2$

$= 1.44 \times 10^{-3} [\text{J}]$

GO ON TO THE NEXT PAGE.
(e) On the axes below, graph the current in $R_2$ as a function of time from 0 to 15 s. Label the vertical axis with appropriate values.

\[ i = \frac{\varepsilon - V}{R_2} \]

Resistor $R_2$ is removed and replaced with another resistor of lesser resistance. Switch $S$ remains closed for a long time.

(f) Indicate below whether the energy stored in the capacitor is greater than, less than, or the same as it was with resistor $R_2$ in the circuit.

___ Greater than ___ Less than ___ The same as

Explain your reasoning.

The energy stored by a capacitor is determined by its capacitance and the amount of charge it can hold. These do not change when $R_2$ changes, so the energy stored is the same.

\[ E = \frac{1}{2} CV^2 \]