

AP[®] Calculus AB 2004 Scoring Commentary

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Question 1

This question gave students a function that defined the rate of traffic flow in terms of cars per minute at a given time *t*. In part (a), students had to use the definite integral to find the total number of cars that pass through the intersection in a given time period. In part (b), students had to use the derivative to determine whether the traffic flow was increasing or decreasing at time t = 7. Parts (c) and (d) tested whether students understood the difference between the average value of a function and the average rate of change of a function. It was important to use the correct units: cars per minute in part (c) and cars per minute per minute (or cars/min²) in part (d).

Sample A (Score 9)

The student earned all 9 points. In parts (c) and (d), rounding to the nearest whole number was acceptable since some students applied the instructions from part (a) throughout the question.

Sample D (Score 7)

The student earned 7 points: 3 points in part (a), 3 points in part (c), and 1 point in part (d). In part (b), although the student considered t = 7, the student computed F(7) instead of F'(7). The units are incorrect in both parts (c) and (d).

Question 2

This question gave two functions whose graphs intersected at x = 0 and x = 1. In part (a), students were asked to find the area bounded by these graphs. In part (b), students had to calculate the volume of the solid formed by revolving this region about the horizontal line with equation y = 2, a line that lies above the given region. Part (c) tested the students' ability to set up an integral of a solid with square cross sections that lies over a specified region. The upper bounding curve of the region for this part was given as the function h with h(x) = kx(1-x), where k was an unspecified positive parameter. Students were asked to set up an equation that could be used to find the value of k for which the resulting solid would have volume equal to 15. Students were not asked to find the value of k.

Sample A (Score 9)

The student earned all 9 points.

Sample D (Score 7)

The student earned 7 points: both points in part (a), all 4 points in part (b), and 1 point in part (c). In part (c), the student earned one point for using the side of the cross sectional square in the integrand. The student did not square the side, losing the second integrand point. The student did not earn the answer point as the integral is multiplied by p and is set equal to an incorrect value.

Question 3

This question considered particle motion along a straight line. Students were given a velocity function and an initial position. Part (a) asked for the acceleration at time t = 2. Part (b) tested the students' understanding of the distinction between speed and velocity and their ability to use the values of velocity and acceleration to determine whether the speed was increasing or decreasing at a given time. Part (c) asked students to find and justify a maximum value of the position. For part (d), students needed to know how to use a definite integral to find the position of the particle at a given time and how to use position and velocity to determine whether the particle was moving toward or away from the origin at a given time.

Sample A (Score 9)

The student earned all 9 points.

Sample C (Score 7)

The student earned 7 points: all 3 points in part (c) and all 4 points in part (d). The student did not earn the point in part (a) due to a chain rule error in the derivative. The student did not earn the point in part (b) because the student has drawn an incorrect conclusion based on his or her value of a(2), and the student has made no reference to the value of v(2). In part (c), the third point was earned when the student's verbal justification is paired with the statement that there are no other critical points.

Question 4

Students were given an equation relating x and y. In part (a), they needed to know how to use implicit differentiation and the product rule to find an expression for $\frac{dy}{dx}$. In part (b), students needed to know that the tangent line is horizontal when the derivative is zero, and they needed to use this fact to solve for y when x = 3. They also needed to check that the point they found lies on the curve defined by the equation. Part (c) required students either to apply implicit differentiation a second time to the original equation or to use the quotient rule with implicit differentiation to find the second derivatives to determine whether there was a maximum, minimum, or neither at a specified point on the curve and to justify their answers.

Sample A (Score 9)

The student earned all 9 points. In part (b), the student determined that at x = 3, y = 2 or $\frac{1}{4}$. The student then

used the derivative to determine that at P = (3, 2), the slope was 0 and that the line tangent to the curve at *P* was horizontal. In part (c), the student used implicit differentiation to find the expression for the second derivative and then evaluated it at point *P*. The student used the Second Derivative Test to determine that the curve had a local maximum at *P*.

Sample C (Score 7)

The student earned 7 points: both points in part (a), 2 points in part (b), and 3 points in part (c). In part (b), the student determined that the slope at point P was 0, but did not verify that P = (3, 2) was on the given curve. In part (c), the student earned 2 points by finding the expression for the second derivative of y with respect to x using the quotient rule, but made an error when simplifying the numerical value, losing the third point. The student earned the fourth point by determining that point P was a local maximum on the curve, using the facts that the first derivative was 0 at point P, and that the second derivative was negative at point P.

Question 5

This was a Fundamental Theorem of Calculus question in which students were given the graph of the function f and asked about the function g defined as the definite integral of f from -3 to x. It was necessary to interpret the graph to answer questions about g, g', and g''. Part (a) asked for the values of g(0) and g'(0). Parts (b) and (c) asked for the locations of the relative maxima and for the absolute minimum value of g and also asked for justifications, testing knowledge of how to use the derivative to find these values and the ability to get this information about the derivative from the graph of f. When justifying the absolute minimum value, it was important for students to explain why each of the other critical values was rejected. Part (d) asked to find the points of inflection.

Sample B (Score 9)

The student earned all 9 points.

Sample C (Score 7)

The student earned 7 points: both points in part (a), both points in part (b), the location of the minimum in part (c), and both points in part (d). In part (b), the justification point was earned because the student has explicitly identified the derivative of g as being f, earlier in part (a). In part (c), the student calculated an incorrect value of g(-4), losing the point for the minimum value. The value of g(-5) is also incorrect, which lost the justification point.

Question 6

This question presented students with a differential equation. Part (a) asked the students to sketch a slope field, identifying the slopes at twelve specified points. It was not necessary to draw the slopes precisely, but it was necessary to distinguish between horizontal, increasing, and decreasing tangent line segments. Part (b) probed student understanding of the fact that a slope field only exhibits a small sample of the possible tangent line segments. Students had to use knowledge of the first derivative to determine all points at which the tangent line segments had positive slope. For part (c), students needed to use separation of variables to solve a differential equation with initial value.

Sample A (Score 9)

The student earned all 9 points.

Sample C (Score 7)

The student earned 7 points: 2 points in part (a) and 5 points in part (c). The student lost the point in part (b) for omitting the condition $x \neq 0$. The student lost one point in part (c) for incorrectly integrating the dy side; the student should have had $\ln|y-1|$.