AP® Calculus BC (Operational)
2004 Sample Student Responses

NO CALCULATOR ALLOWED

Work for problem 6(a)

$f(x) = \sin(5x + \frac{\pi}{4})$ $\quad f(0) = \frac{\sqrt{2}}{2}$

$f'(x) = \cos(5x + \frac{\pi}{4}) \cdot 5$ $\quad f'(0) = \frac{5}{2}\sqrt{2}$

$f''(x) = -5\sin(5x + \frac{\pi}{4}) \cdot 5$ $\quad f''(0) = -\frac{25}{2}\sqrt{2}$

$f'''(x) = -25\cos(5x + \frac{\pi}{4}) \cdot 5$ $\quad f'''(0) = -\frac{125}{2}\sqrt{2}$

$$\therefore P(x) = \frac{\sqrt{2}}{2} + \frac{\frac{5}{2}\sqrt{2}}{1!}x - \frac{\frac{25}{2}\sqrt{2}}{2!}x^2 - \frac{\frac{125}{2}\sqrt{2}}{3!}x^3$$

$$= \frac{\sqrt{2}}{2} + \frac{5}{2}\sqrt{2}x - \frac{25}{4}\sqrt{2}x^2 - \frac{125}{12}\sqrt{2}x^3$$

Work for problem 6(b)

$\left|f^{(n)}(0)\right| = \frac{5^n}{2}\sqrt{2}$ ...

signs alternate every 2 terms

$\therefore \frac{22+1}{4} = 5$ ... 2 – remainder.

$\therefore$ sign negative

$\therefore$ coefficient $= -\frac{\frac{5^{22}}{2}\sqrt{2}}{22!} = -\frac{5^{22}\sqrt{2}}{2 \cdot 22!}$

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Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

Work for problem 6(c)

$|f(\frac{1}{10}) - P(\frac{1}{10})| \le \frac{f^4(c)}{4!}(\frac{1}{10})^4$ for $0 < c < \frac{1}{10}$

$f^4(c) = 125 \cdot \sin(5c + \frac{\pi}{4}) \cdot 5$ for $0 < c < \frac{1}{10}$

$\frac{\pi}{4} < 5c + \frac{\pi}{4} < \frac{1}{2} + \frac{\pi}{4} = \frac{2+\pi}{4} < \pi$

$\therefore \sin(5c + \frac{\pi}{4}) > 0$ and $\sin(5c + \frac{\pi}{4}) < 1$

$\therefore f^4(c) < 125 \cdot 5 = 625$

$\therefore \frac{f^4(c)}{4!}(\frac{1}{10})^4 < \frac{625}{24} \cdot \frac{1}{10000} = \frac{1}{24 \cdot 16} = \frac{1}{384} < \frac{1}{100}$

$\therefore |f(\frac{1}{10}) - P(\frac{1}{10})| < \frac{1}{100}$

Work for problem 6(d)

$G(x) = \int_0^x f(t)\,dt$ } fundamental theorem of calculus.

$\therefore G'(x) = f(x)$

$G''(x) = f'(x)$

Taylor polynomial for G about $x = 0$ is an antiderivative of $P(x)$.

$\therefore P(x) = \frac{\sqrt{2}}{2} + \frac{5}{2}\sqrt{2}x - \frac{25}{4}\sqrt{2}x^2 - \frac{125}{12}\sqrt{2}x^3$

let $k(x)$ be the Taylor Polynomial for G about $x = 0$.

$\therefore \boxed{k(x) = \frac{\sqrt{2}}{2}x + \frac{5}{4}\sqrt{2}x^2 - \frac{25}{12}\sqrt{2}x^3}$

END OF EXAMINATION

THE FOLLOWING INSTRUCTIONS APPLY TO THE BACK COVER OF THIS SECTION II BOOKLET.

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MM1

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NO CALCULATOR ALLOWED

Work for problem 6(a)

$f(x) = \sin(5x + \frac{\pi}{4})$

$f(0) = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$f'(c) = 5\cos(5c + \frac{\pi}{4})$ $\quad f'(0) = \frac{5}{\sqrt{2}}$

$f''(c) = -25\sin(5c + \frac{\pi}{4})$ $\quad f''(0) = \frac{-25}{\sqrt{2}}$

$f'''(c) = -125\cos(5c + \frac{\pi}{4})$ $\quad f'''(0) = \frac{-125}{\sqrt{2}}$

$P(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!}$

$P(x) = \frac{1}{\sqrt{2}} + \frac{5x}{\sqrt{2}} - \frac{25}{2\sqrt{2}}x^2 - \frac{125}{6\sqrt{2}}x^3$

Work for problem 6(b)

coefficient of $x^n = \frac{5^n}{n!\sqrt{2}}$

coefficient of $x^{22} = \frac{5^{22}}{22!\sqrt{2}}$

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Continue problem 6 on page 15.

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NO CALCULATOR ALLOWED

Work for problem 6(c)

$f^{iv}(z) = 625(\sin(5z + \frac{\pi}{4}))$

Lagange error $\frac{f^{(n+1)}(z)(x-c)^{n+1}}{(n+1)!}$

$|f(\frac{1}{10}) - P(\frac{1}{10})| \leq \frac{f^{iv}(z)(x)^4}{4!} < \frac{1}{100}$ $c < z < x$ $0 < z < \frac{1}{10}$

$\leq \frac{f^{iv}(z)(\frac{1}{10})^4}{4!} < \frac{1}{100}$

$\leq \frac{625(\sin(5z + \frac{\pi}{4}))\frac{1}{10000}}{24} < \frac{1}{100}$

$\leq \frac{625}{10000}\sin(\frac{2+\pi}{4}) < \frac{1}{100}$

$\leq \frac{625\sin(\frac{2+\pi}{4})}{240000} < \frac{1}{100}$

$|f(\frac{1}{10}) - P(\frac{1}{10})| \leq \frac{\sin(\frac{2+\pi}{4})}{384} < \frac{1}{100}$

$\sin(\frac{2+\pi}{4}) < 1$

$\frac{1}{384} < \frac{1}{100}$

Work for problem 6(d)

$G(x) = C + \frac{1}{\sqrt{2}}x + \frac{5x^2}{2\sqrt{2}} - \frac{25x^3}{6\sqrt{2}} + \frac{125x^4}{24\sqrt{2}}$

$G(0) = 0 \quad C = 0$

$G(x) = \frac{1}{\sqrt{2}}x + \frac{5x^2}{2\sqrt{2}} - \frac{25x^3}{6\sqrt{2}}$

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