AP® Calculus BC
2004 Sample Student Responses
Form B

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B1

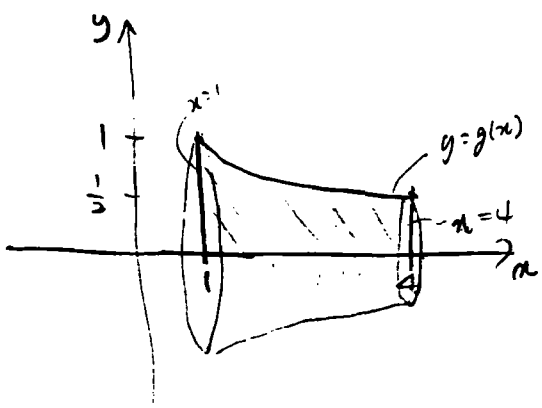
NO CALCULATOR ALLOWED

Work for problem 5(a)

average value of g : $\dfrac{\int_1^4 g(x)\,dx}{3} = \dfrac{\int_1^4 x^{-\frac{1}{2}}\,dx}{3} = \dfrac{\left[2x^{\frac{1}{2}}\right]_1^4}{3} = \dfrac{4-2}{3}$

$= \boxed{\dfrac{2}{3}}$

Work for problem 5(b)



$\int_1^4 (g(x))^2 \cdot \pi\,dx$

$= \int_1^4 \frac{1}{x} \cdot \pi\,dx$

$= \left[\pi \ln x\right]_1^4 = \pi \ln 4 - \pi \ln 1 = \boxed{\pi \ln 4}$

Continue problem 5 on page 13.

B2

NO CALCULATOR ALLOWED

Work for problem 5(c)

average value of the areas $= \dfrac{\text{volume for } x=1 \text{ to } x=4}{3}$

$= \boxed{\dfrac{\pi \ln 4}{3}}$

Work for problem 5(d)

$\int_4^\infty g(x)\,dx = \lim_{n\to\infty} \int_4^n \frac{1}{\sqrt{x}}\,dx = \lim_{n\to\infty} \int_4^n x^{-\frac{1}{2}}\,dx = \lim_{n\to\infty} \left[2x^{\frac{1}{2}}\right]_4^n$

$= \lim_{n\to\infty} 2n^{\frac{1}{2}} - 4 = \infty$ → divergent.

average value of g on the interval $[4,\infty)$

$: \lim_{b\to\infty} \left[\frac{\int_4^b g(x)\,dx}{b-4}\right] = \lim_{b\to\infty} \frac{2b^{\frac{1}{2}} - 4}{b-4} \overset{\infty}{=} \lim_{b\to\infty} \frac{2\cdot\frac{1}{2}b^{-\frac{1}{2}}}{1} = \lim_{b\to\infty} \frac{1}{\sqrt{b}} = 0$ → finite

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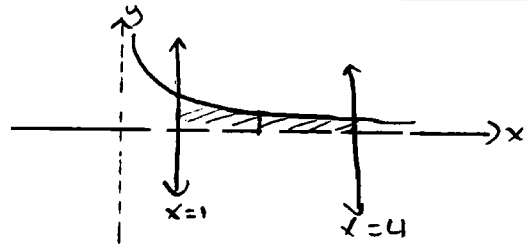
NO CALCULATOR ALLOWED

Work for problem 5(a)

$$\text{Avg } g(x) = \frac{1}{4-1}\int_1^4 \frac{1}{\sqrt{x}}dx = \frac{1}{3}\int_1^4 x^{-1/2}dx = \frac{1}{3}\left[2\sqrt{x}\right]_1^4 = \frac{1}{3}\left[2\sqrt{4} - 2\sqrt{1}\right]$$

$$= \frac{1}{3}\left[2(2) - 2\right] = \frac{2}{3}$$

Work for problem 5(b)



$$V = \pi\int_1^4 R^2(x) - r^2(x)\,dx$$

$$R(x) = \frac{1}{\sqrt{x}} \Rightarrow R^2(x) = \frac{1}{x}$$

$$r(x) = 0$$

$$= \pi\int_1^4 \frac{1}{x}\,dx$$

$$= \pi\left[\ln x\right]_1^4 = \pi \ln 4 \text{ units}^3$$

Continue problem 5 on page 13.

NO CALCULATOR ALLOWED

Work for problem 5(c)

$\overline{z}$ length of cross sections perperdicular to x-axis $= \frac{1}{\sqrt{x}}$ $\quad A = \left(\frac{1}{\sqrt{x}}\right)^2 = \frac{1}{x}$

$$\text{avg} = \frac{1}{4-1}\int_1^4 \left(\frac{1}{\sqrt{x}}\right)^2 dx = \frac{1}{3}\int_1^4 \frac{1}{x}\,dx = \frac{1}{3}\ln x\Big]_1^4 = \frac{1}{3}\ln 4$$

Work for problem 5(d)

$$\int_4^\infty g(x)\,dx = \lim_{a\to\infty}\int_4^a \frac{1}{\sqrt{x}}\,dx = \lim_{a\to\infty} 2\sqrt{x}\Big]_4^a = \lim_{a\to\infty} 2\sqrt{a} - 2\sqrt{4} = \infty$$

$\Rightarrow$ the improper integral $\int_4^\infty g(x)\,dx$ diverges.

$\lim_{b\to\infty} \frac{\int_a^b f(x)\,dx}{b-a} \Rightarrow$ average value of $g = \lim_{b\to\infty} \frac{\int_a^b g(x)\,dx}{b-a}$

$$= \lim_{b\to\infty} \frac{\int_4^b g(x)\,dx}{b-4}$$

$$= \lim_{b\to\infty} g(x) = 0$$

$\Rightarrow$ avg value of g on $[4,\infty)$ is finite.

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