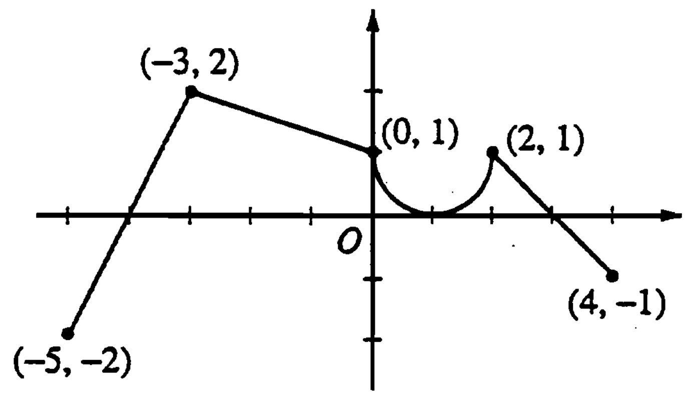
CollegeBoard
Advanced Placement Program

AP® Calculus AB (Operational)
2004 Sample Student Responses

NO CALCULATOR ALLOWED.



Graph of f

Work for problem 5(a)

$g(0) = \int_{-3}^{0} f(t)\,dt = \frac{1}{2}(2+1)3 = \frac{1}{2}33 = \frac{9}{2} = \boxed{4.5}$

$g'(0) = f(0) = \boxed{1}$

Work for problem 5(b)

$g'(x) = f(x)$

x: −4, 1, 3

g'(x) / f(x): − | + | + | −

relative maximum at $x=3$

At $x=3$, the slope of $g(x)$ changes from positive to negative

Work for problem 5(c)

x	g(x)
-5	0
-4	-1
4	+

$\int_{-3}^{-5} f(t)\,dt = 0$

$\int_{-3}^{-4} f(t)\,dt = -1$

$\int_{-3}^{4} f(t)\,dt = +$

$g(-4) = \boxed{-1}$

At x=4, the slope of g(x) changes from negative to positive.
The value of g(-4) is less than g(-5) or g(4).

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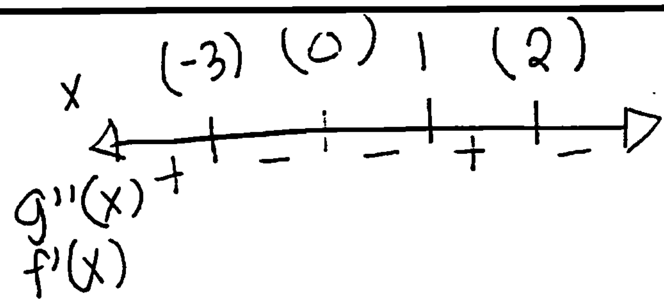
Work for problem 5(d)

$g'(x) = f(x)$

$g''(x) = f'(x)$



$\boxed{x = -3, 1, 2}$

NO CALCULATOR ALLOWED

$g(x) = F(x) - F(-3)$

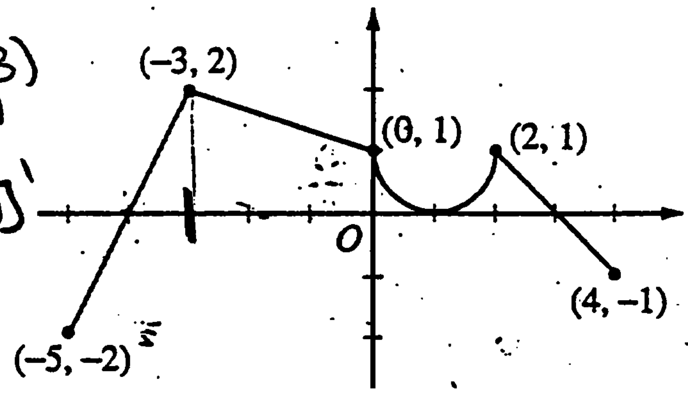
$g'(x) = [F(x) - F(-3)]'$

$= F'(x) - [F(-3)]'$

$= F'(x) - 0$

$= f(x)$

$g'(x) = f(x)$



Graph of f

Work for problem 5(a)

$g(x) = \int_{-3}^{x} f(t)\,dt$

$g(0) = \int_{-3}^{0} f(t)\,dt$ = area of trapezoid $= \left(\frac{1}{2}\right)(2+1)(3) = \frac{1}{2}(3)(3) = \frac{9}{2}$

$g(0) = \frac{9}{2}$

$g'(0) = f(0) = 1$ $g'(0) = 1$

Work for problem 5(b)

g is a relative max on $(-5, 4)$ $g'(x) = f(x)$

$g(x)$ has a local maximum where $g'(x)$ is zero or where $g'(x)$ is undefined, and where $g'(x)$ changes sign from positive to negative. This occurs only at $x = 3$, where $g'(x) = 0$ and changes from positive to negative. [at $x=1$, $g'(x)$ does not experience a sign change; at $x = -4$, $g'(x)$ changes from negative to positive, a local minimum]

Continue problem 5 on page 13.

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C_2

Work for problem 5(c) abs min on [-5,4]

There is a local min at x = -4 [on g(x)] because g'(x) is equal to zero and changes from negative to positive.

$g(-4) = \int_{-3}^{-4} f(t)dt = -\int_{-4}^{-3} f(t)dt = -[\frac{1}{2}(2+1)(3) + \frac{1}{2}(1)(2)]$
$= -[\frac{9}{2}+1] = -\frac{11}{2} = -5.5$

$g(-4) = -\frac{11}{2}$

The absolute minimum of g on [-5,4] is at x = -4 because g'(x)=0 at x=-4 & changes from negative to positive. Also, the endpoints' values on g are bigger than the value of g(-4).

ENDPOINT CHECK

$g(-5) = \int_{-3}^{-5} f(t)dt = -\int_{-5}^{-3} f(t)dt = -[\frac{1}{2}(2+1)(3) + \frac{1}{2}(1)(2) - 1(2)(\frac{1}{2})]$
$= -[\frac{11}{2}-1] = -\frac{9}{2} = -4.5$

$g(-5) = -\frac{9}{2} > -\frac{11}{2}$

$g(4) = \int_{-3}^{4} f(t)dt = \frac{1}{2}(2+1)(3) + [(1)(2) - (\pi)(1^2)(\frac{1}{2})] + (1)(1)(\frac{1}{2}) - 1(1)(\frac{1}{2})$
$= \frac{9}{2} + [2 - \frac{\pi}{2}] + 0$
$= \frac{9}{2} + 2 - \frac{\pi}{2} = \frac{13}{2} - \frac{\pi}{2} \approx 6.5 - 1.57 \rightarrow$ positive #

Work for problem 5(d) PI's on g(x) at x = -3, 2, and x = 1.

g has a point of inflection (PI) where g''(x) is zero or undefined & changes signs. g''(x) equals zero and changes signs at x=1, where g'(x) is a local min. g''(x) is undefined and changes signs at x=-3 and x=2 (even though g''(x) is undefined at x=0, g''(x) doesn't change signs there)—These are local max's.

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