



AP[®] Calculus AB (Operational) 2004 Sample Student Responses

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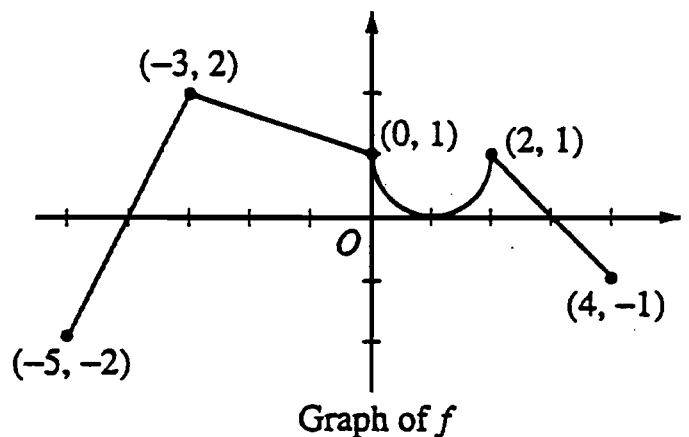
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NO CALCULATOR ALLOWED.



Work for problem 5(a)

$$g(0) = \int_{-3}^0 f(t) dt = \frac{1}{2}(2+1)3 = \frac{1}{2}3^2 = \frac{9}{2} = \boxed{4.5}$$

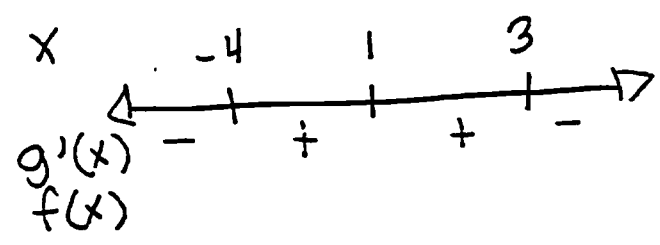
$$g'(0) = f(0) = \boxed{1}$$

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Work for problem 5(b)

$$g'(x) = f(x)$$



relative maximum at $x=3$

At $x=3$, the slope of $g(x)$ changes from positive to negative

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Work for problem 5(c)

x	g(x)
-5	0
-4	-1
4	+

$$\int_{-3}^{-5} f(t) dt = 0$$

$$\int_{-3}^{-4} f(t) dt = -1$$

$$\int_{-3}^4 f(t) dt = +$$

$$g(-4) = \boxed{-1}$$

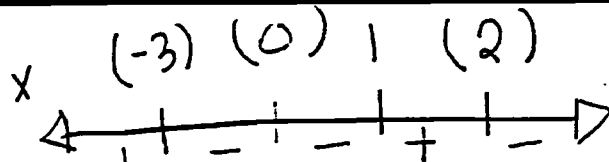
At $x=4$, the slope of $g(x)$ changes from negative to positive.
The value of $g(-4)$ is less than $g(-5)$ or $g(4)$.

Work for problem 5(d)

$$g'(x) = f(x)$$

$$g''(x) = f'(x)$$

$$g''(x) = f'(x)$$



$$\boxed{x = -3, 1, 2}$$

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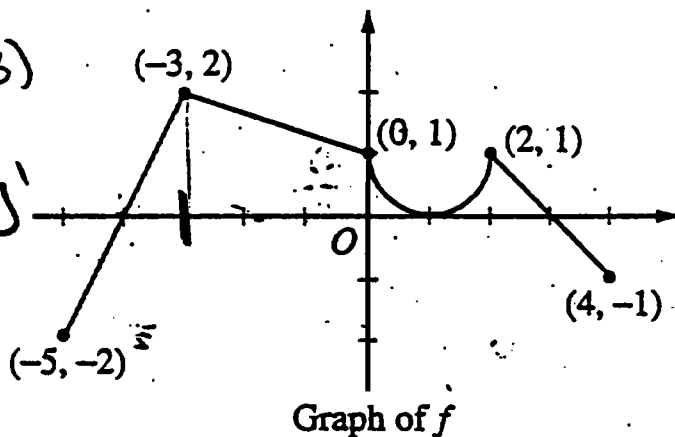
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C

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$$\begin{aligned}
 g(x) &= F(x) - F(-3) \\
 g'(x) &= [F(x) - F(-3)]' \\
 &= F'(x) - [F(-3)]' \\
 &= F'(x) - 0 \\
 &= f(x) \\
 \hline
 g'(x) &= f(x)
 \end{aligned}$$



Work for problem 5(a)

$$g(x) = \int_{-3}^x f(t) dt$$

$$g(0) = \int_{-3}^0 f(t) dt = \text{area of trapezoid} = \left(\frac{1}{2}\right)(2+1)(3) = \frac{1}{2}(3)(3) = \frac{9}{2}$$

$$g(0) = \frac{9}{2}$$

$$g'(0) = f(0) = 1 \quad g'(0) = 1$$

Work for problem 5(b)

g is a relative max on $(-5, 4)$ $g'(x) = f(x)$

$g(x)$ has a local maximum where $g'(x)$ is zero or where $g'(x)$ is undefined, and where $g'(x)$ changes sign from positive to negative. This occurs only at $x=3$, where $g'(x)=0$ and changes from positive to negative. [at $x=1$, $g'(x)$ does not experience a sign change; at $x=-4$, $g'(x)$ changes from negative to positive, a local minimum]

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Continue problem 5 on page 13.

Work for problem 5(c) abs min on $[-5, 4]$

There is a local min at $x = -4$ because $g'(x)$ is equal to zero on $g(x)$ and changes from negative to positive.

$$g(-4) = \int_{-3}^{-4} f(t) dt = -\int_{-4}^{-3} f(t) dt = -\left[\frac{1}{2}(2+1)(3) + \frac{1}{2}(1)(2)\right] = -\left[\frac{9}{2} + 1\right] = -\frac{11}{2} = -5.5$$

$$g(-4) = -\frac{11}{2}$$

The absolute minimum of g on $[-5, 4]$ is at $x = -4$ because $g'(x) = 0$ at $x = -4$ & changes from negative to positive. Also, the endpoints' values of g are bigger than the value of $g(-4)$.

ENDPOINT CHECK

$$g(-5) = \int_3^{-5} f(t) dt = -\int_{-5}^{-3} f(t) dt = -\left[\frac{1}{2}(2+1)(3) + \frac{1}{2}(1)(2) - 1\left(2 \times \frac{1}{2}\right)\right] = -\left[\frac{11}{2} - 1\right] = -\frac{9}{2} = -4.5$$

$$g(-5) = -\frac{9}{2} > -\frac{11}{2}$$

$$g(4) = \int_{-3}^4 f(t) dt = \frac{1}{2}(2+1)(3) + [(1)(2) - (\pi)(1^2 \times \frac{1}{2})] + (1)(1)\left(\frac{1}{2}\right) - 1(1)\left(\frac{1}{2}\right) = \frac{9}{2} + \left[2 - \frac{\pi}{2}\right] + 0 = \frac{9}{2} + 2 - \frac{\pi}{2} = \frac{13}{2} - \frac{\pi}{2} \approx 6.5 - 1.57 \rightarrow \text{positive}$$

Work for problem 5(d)

PI's ^{on $g(x)$} at $x = -3, 2,$ and $x = 1$.

g has a point of inflection (PI) where $g'(x)$ is zero or undefined & changes signs. ~~where $g''(x)$ equals zero~~ $g''(x)$ equals zero and changes signs at $x = 1$, where $g'(x)$ is a local min. $g''(x)$ is undefined and changes signs at $x = -3$ and $x = 2$ (even though $g''(x)$ is undefined at $x = 0$, $g'(x)$ doesn't change signs there) — These are local max's.

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