AP® Calculus AB (Operational)
2004 Sample Student Responses

The materials included in these files are intended for noncommercial use by AP teachers for course and exam preparation; permission for any other use must be sought from the Advanced Placement Program®. Teachers may reproduce them, in whole or in part, in limited quantities, for face-to-face teaching purposes but may not mass distribute the materials, electronically or otherwise. This permission does not apply to any third-party copyrights contained herein. These materials and any copies made of them may not be resold, and the copyright notices must be retained as they appear here.

The College Board is a not-for-profit membership association whose mission is to connect students to college success and opportunity. Founded in 1900, the association is composed of more than 4,500 schools, colleges, universities, and other educational organizations. Each year, the College Board serves over three million students and their parents, 23,000 high schools, and 3,500 colleges through major programs and services in college admissions, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT®, the PSAT/NMSQT®, and the Advanced Placement Program® (AP®). The College Board is committed to the principles of excellence and equity, and that commitment is embodied in all of its programs, services, activities, and concerns.

For further information, visit www.collegeboard.com

Copyright © 2004 College Entrance Examination Board. All rights reserved. College Board, Advanced Placement Program, AP, AP Central, AP Vertical Teams, APCD, Pacesetter, Pre-AP, SAT, Student Search Service, and the acorn logo are registered trademarks of the College Entrance Examination Board. PSAT/NMSQT is a registered trademark of the College Entrance Examination Board and National Merit Scholarship Corporation. Educational Testing Service and ETS are registered trademarks of Educational Testing Service. Other products and services may be trademarks of their respective owners.

For the College Board's online home for AP professionals, visit AP Central at apcentral.collegeboard.com.

4 4 4 4 4 4 4 4 4 4 A

NO CALCULATOR ALLOWED

CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

$x^2 + 4y^2 = 7 + 3xy$

$2x + 4 \cdot 2y \cdot y' = 0 + 3xy' + 3y$

$8y \cdot y' - 3x \cdot y' = 3y - 2x$

$y'(8y - 3x) = \frac{3y - 2x}{8y - 3x}$

$y' = \frac{3y - 2x}{8y - 3x}$ ✓

Do not write beyond this border.

Copyright © 2004 by College Entrance Examination Board. All rights reserved.
Visit apcentral.collegeboard.com (for AP professionals) and www.collegeboard.com/apstudents (for AP students and parents).

NO CALCULATOR ALLOWED

Work for problem 4(b)

$x^2 + 4y^2 = 7 + 3xy \quad x = 3$

$9 + 4y^2 = 7 + 9y \quad 4y^2 - 9y + 2 = 0 \quad y = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$y = \frac{9 \pm \sqrt{81 - 4 \cdot 2 \cdot 4}}{2 \cdot 4} = \frac{9 \pm \sqrt{81-32}}{8} = \frac{9 \pm \sqrt{49}}{8} = \frac{9 \pm 7}{8} = \frac{16}{8}, \frac{2}{8}$

$y = 2, \frac{1}{4}$

$\frac{3y-2x}{8y-3x} = 0 \qquad 3y - 2x = 0 \qquad 3y = 2x$

$3y = 2 \cdot 3$

$y = 2$ ✓

$P(3,2)$

$\frac{3 \cdot 2 - 2 \cdot 3}{8 \cdot 2 - 3 \cdot 3} = \frac{0}{7}$ ✓ $= 0$

$16 - 9$

Work for problem 4(c)

$\frac{3y-2x}{8y-3x} = y'$

$8y \cdot y' - 3x \cdot y' = 3y - 2x$

$8y \cdot y'' + 8y' \cdot y' - 3x \cdot y'' - 3y' = 3y' - 2$

$y''(8y - 3x) = 3y' - 2 - 8y'^2 + 3y' =$

$y'' = \frac{3y' - 2 - 8y'^2 + 3y'}{8y - 3x}$

$y'(2) =$

$x = 3$

$y(2) = 2$

$y' = 0$

$x = 3$

$y = 2$

$y'' = \frac{0 - 2 - 0 + 0}{8 \cdot 2 - 3 \cdot 3} = \frac{-2}{16 - 9} = -\frac{2}{7} < 0$

$y'' < 0$

concave down

maximum

Do not write beyond this border.

Copyright © 2004 by College Entrance Examination Board. All rights reserved.
Visit apcentral.collegeboard.com (for AP professionals) and www.collegeboard.com/apstudents (for AP students and parents).

NO CALCULATOR ALLOWED

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

$x^2 + 4y^2 = 7 + 3xy$

$\frac{dy}{dx} = \frac{3y-2x}{8y-3x}$ show

$2x \cdot \frac{dx}{dx} + 8y\frac{dy}{dx} = 0 + 3\left(x \cdot \frac{dy}{dx} + y\right)$

$2x + 8y\frac{dy}{dx} = 3x\frac{dy}{dx} + 3y$

$8y\frac{dy}{dx} - 3x\frac{dy}{dx} = 3y - 2x$

$\frac{dy}{dx}(8y - 3x) = 3y - 2x$

$\frac{dy}{dx} = \boxed{\frac{3y-2x}{8y-3x}}$ ✓

Do not write beyond this border.

Continue problem 4 on page 11.

Copyright © 2004 by College Entrance Examination Board. All rights reserved.
Visit apcentral.collegeboard.com (for AP professionals) and www.collegeboard.com/apstudents (for AP students and parents).

NO CALCULATOR ALLOWED

Work for problem 4(b)

$P(3, \underline{2})$

$m = 0$

$\frac{dy}{dx} = m = \frac{3y - 2x}{8y - 3x}$

$\frac{3y - 2 \cdot 3}{8y - 3 \cdot 3} = 0$

$\frac{3y - 6}{8y - 9} = 0$ $\quad 3y - 6 = 0$

$y = 2$

Work for problem 4(c)

$P(3,2)$ $\quad \frac{dy}{dx} = \frac{3y - 2x}{8y - 3x}$

$$\frac{d^2y}{dx} = \frac{(8y - 3x)\left(3\frac{dy}{dx} - 2\right) - (3y - 2x)\left(8\frac{dy}{dx} - 3\right)}{(8y - 3x)^2}$$

$16 - 9 \qquad 0$

$$\frac{d^2y}{dx} = \frac{(8 \cdot 2 - 3 \cdot 3)(-2) - (3 \cdot 2 - 2 \cdot 3)(-3)}{(8 \cdot 2 - 3 \cdot 3)^2}$$

$\frac{d^2y}{dx} = \frac{-14 - 0}{49} = \frac{-14}{9}$

$\frac{3x - 2x}{8y - 3x}$ ZERO

The curve has a local maximum because at pt. $P(3,2)$, the 2nd derivative is negative. This makes the curve concave down.

Do not write beyond this border.

Do not write beyond this border.

Copyright © 2004 by College Entrance Examination Board. All rights reserved.
Visit apcentral.collegeboard.com (for AP professionals) and www.collegeboard.com/apstudents (for AP students and parents).