



AP[®] Calculus AB (Operational) 2004 Sample Student Responses

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4 4 4 4 4 4 4 4 A

NO CALCULATOR ALLOWED

CALCULUS BC
SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

$$x^2 + 4y^2 = 7 + 3xy$$

$$2x + 4 \cdot 2y \cdot y' = 0 + 3xy' + 3y$$

$$8y \cdot y' - 3x \cdot y' = 3y - 2x$$

$$y'(8y - 3x) = \frac{3y - 2x}{8y - 3x}$$

$$y' = \frac{3y - 2x}{8y - 3x} \quad \checkmark$$

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NO CALCULATOR ALLOWED

Work for problem 4(b)

$$x^2 + 4y^2 = 7 + 3xy \quad x=3$$

$$9 + 4y^2 = 7 + 9y$$

$$4y^2 - 9y + 2 = 0 \quad y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{9 \pm \sqrt{81 - 4 \cdot 2 \cdot 4}}{2 \cdot 4} = \frac{9 \pm \sqrt{81 - 32}}{8} = \frac{9 \pm \sqrt{49}}{8} = \frac{9 \pm 7}{8} = \frac{16}{8}, \frac{2}{8}$$

$$y = 2, \frac{1}{4}$$

$$\frac{3y - 2x}{8y - 3x} = 0$$

$$3y - 2x = 0$$

$$3y = 2x$$

$$3y = 2 \cdot 3$$

$$y = 2 \quad \checkmark$$

P(3, 2)

$$\frac{3 \cdot 2 - 2 \cdot 3}{8 \cdot 2 - 3 \cdot 3} = \frac{0}{16 - 9} = 0 \quad \checkmark = 0$$

Work for problem 4(c)

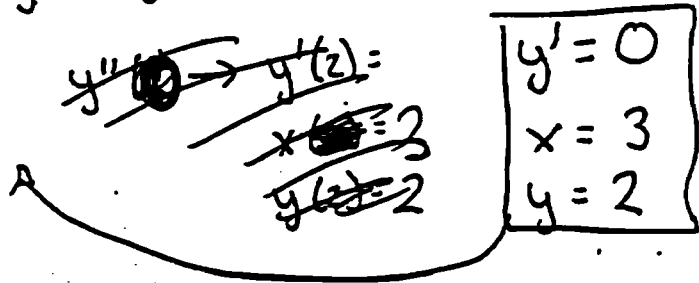
$$\frac{3y - 2x}{8y - 3x} = y'$$

$$8y \cdot y' - 3x \cdot y' = 3y - 2x$$

$$8y \cdot y'' + 8y' \cdot y' - 3x \cdot y'' - 3y' = 3y' - 2$$

$$y''(8y - 3x) = 3y' - 2 - 8y'^2 + 3y' =$$

$$y'' = \frac{3y' - 2 - 8y'^2 + 3y'}{8y - 3x}$$



$$y'' = \frac{0 - 2 - 0 + 0}{8 \cdot 2 - 3 \cdot 3} = \frac{-2}{16 - 9} = -\frac{2}{7} < 0$$

$y'' < 0$
concave down

maximum

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4



4



4



4



4

C₁

NO CALCULATOR ALLOWED

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

$$x^2 + 4y^2 = 7 + 3xy$$

$$\frac{dy}{dx} = \frac{3y - 2x}{8y - 3x} \quad \underline{\text{SHOW}}$$

$$2x \cdot \frac{dx}{dx} + 8y \frac{dy}{dx} = 0 + 3(x \cdot \frac{dx}{dx} + y)$$

$$2x + 8y \frac{dy}{dx} = 3x \frac{dy}{dx} + 3y$$

$$8y \frac{dy}{dx} - 3x \frac{dy}{dx} = 3y - 2x$$

$$\frac{dy}{dx} (8y - 3x) = 3y - 2x$$

$$\frac{dy}{dx} = \boxed{\frac{3y - 2x}{8y - 3x}} \quad \checkmark$$

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Continue problem 4 on page 11.

4



4



4



4



4

C₂

NO CALCULATOR ALLOWED

Work for problem 4(b)

$$P(3, \underline{2})$$

$$m = 0$$

$$\frac{dy}{dx} = m = \frac{3y - 2x}{8y - 3x}$$

$$\frac{3y - 2 \cdot 3}{8y - 3 \cdot 3} = 0$$

$$\frac{3y - 6}{8y - 9} = 0$$

$$3y - 6 = 0$$

$$\boxed{y = 2}$$

Work for problem 4(c)

$$P(3, 2)$$

$$\frac{dy}{dx} = \frac{3y - 2x}{8y - 3x}$$

$$\frac{d^2y}{dx^2} = \frac{(8y - 3x)(3 \frac{dy}{dx} - 2) - (3y - 2x)(8 \frac{dy}{dx} - 3)}{(8y - 3x)^2}$$

$$\frac{d^2y}{dx^2} = \frac{(8 \cdot 2 - 3 \cdot 3)(-2) - (3 \cdot 2 - 2 \cdot 3)(-3)}{(8 \cdot 2 - 3 \cdot 3)^2}$$

$$\frac{d^2y}{dx^2} = \frac{-14 - 0}{49} = \frac{-14}{9}$$

$$\frac{3y - 2x}{8y - 3x}$$

The curve has a local maximum because at pt. $P(3, 2)$, the 2nd derivative is negative. This makes the curve concave down.

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