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Work for problem 2(a)

\[ \int_{0}^{1} \left( 2x(1-x) - (3(x-1)\sqrt{x}) \right) \, dx = \boxed{1.133} \]
Work for problem 2(b)

\[ \pi \int_{0}^{1} \left(2 - 3(x - 1)\sqrt{x}\right)^2 - (2 - 2x(1-x))^2 \, dx = 14.179 \]

Work for problem 2(c)

\[ h(x) = kx(1-x) \quad 0 \leq x \leq 1 \]

\[ \int_{0}^{1} \left[ kx(1-x) - 3(x-1)\sqrt{x} \right]^2 \, dx = 15 \]
Work for problem 2(a)

\[ f(x) = 2x(1-x), \quad 0 \leq x \leq 1 \]
\[ g(x) = 3(x-1)\sqrt{x} \]

Area under \( f(x) \) = \( \int_{0}^{1} 2x(1-x) \, dx \) \approx 0.333 = \frac{1}{3} \)

Area under \( g(x) \) = \( \int_{0}^{1} 3(x-1)\sqrt{x} \, dx \) \approx -0.8

To make area under \( g(x) \) positive for total area

\[ \left| -0.8 \right| = 0.8 \]

Area enclosed by \( f(x) \) & \( g(x) \) = \( \frac{1}{3} + 0.8 = \frac{1}{3} + \frac{8}{10} = \frac{1}{3} + \frac{4}{5} = \frac{5}{15} + \frac{12}{15} = \frac{17}{15} \approx 1.1333 \)
Work for problem 2(b)

Volume of solid

\[ \pi \int_{0}^{1} (R^2(x) - r^2(x)) \, dx \]
\[ \pi \int_{0}^{1} (2 - 3(x-1) - \sqrt{x})^2 - (2 - 2x(1-x))^2 \, dx \]
\[ = \frac{103\pi}{20} \approx 16.179 \]

Work for problem 2(c)

\[ h(x) = k \times (1-x) \]

\[ A_0 = 8^2 \]

\[ S = h(x) - g(x) \]

\[ A_{new} = \pi \int_{0}^{1} kx(1-x) - 3(x-1)\sqrt{x} \, dx \]

Use to find a 'k' value.