



**AP[®] Calculus AB
2004 Sample Student Responses
Form B**

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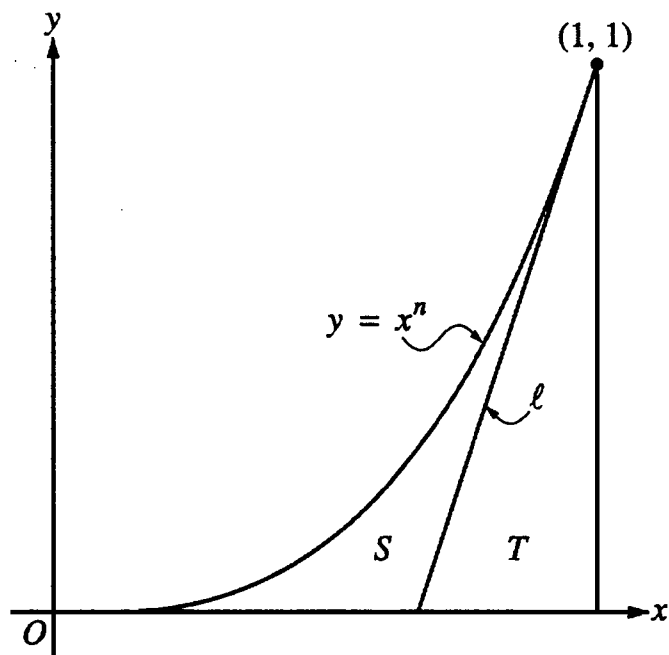
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NO CALCULATOR ALLOWED



Work for problem 6(a)

$$\int_0^1 x^n dx = \left(\frac{x^{n+1}}{n+1} \right) \Big|_0^1 = \frac{1^{n+1}}{n+1} - \frac{0^{n+1}}{n+1} = \frac{1}{n+1}$$

Work for problem 6(b)

$$y = x^n$$

$$y' = nx^{n-1}$$

$$y'(1) = n \cdot 1^{n-1}$$

$$= n$$

i) the equation of the tangent line will be

$$y - 1 = n(x - 1)$$

$$y = nx - n + 1$$

in order to get base of T,

$$y = nx - n + 1$$

$$0 = nx - n + 1$$

$$\frac{n-1}{n} = x$$



$$A_{\text{of } T} = \frac{1 \times \left(1 - \frac{n-1}{n}\right)}{2}$$

$$= \frac{n - n + 1}{n}$$

$$= \frac{1}{n}$$

Continue problem 6 on page 15.

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Work for problem 6(c)

$$A_S = \int_0^1 x^n dx - A_T$$

from (a) & (b) we know $\int_0^1 x^n dx = A_T$

$$\therefore A_S = \frac{1}{n+1} - \frac{1}{2n}$$

$$= \frac{2n - n - 1}{2n(n+1)} = \frac{n-1}{2n(n+1)}$$

$$A_S = \frac{(n-1)}{2n^2 + 2n}$$

$$A'_S = \frac{2n^2 + 2n - (n-1)(4n+2)}{(2n^2 + 2n)^2} = 0$$

$$2n^2 + 2n - (4n^2 - 2n - 2) = 0$$

$$n^2 + n - 2n^2 + n + 1 = 0$$

$$-n^2 + 2n + 1 = 0$$

by quadratic formula.

$$\frac{-2 \pm \sqrt{4+4}}{-2} = \frac{-2 \pm 2\sqrt{2}}{-2} = 1 \pm \sqrt{2}$$

$$A_S(1+\sqrt{2}) = \frac{\sqrt{2}}{2(1+2\sqrt{2}+2) + 2+2\sqrt{2}}$$

$$= \frac{\sqrt{2}}{6 + 2 + 2\sqrt{2} + 4\sqrt{2}}$$

$$= \frac{\sqrt{2}}{8 + 6\sqrt{2}}$$

$$A_S(1-\sqrt{2}) = \frac{-\sqrt{2}}{2(1-2\sqrt{2}+2) + 2(1-\sqrt{2})}$$

$$= \frac{-\sqrt{2}}{6 - 2\sqrt{2} + 2 - 2\sqrt{2}}$$

$$= \frac{-\sqrt{2}}{8 - 2\sqrt{2}}$$

$$\text{ANSWER}$$

$$n = 1 + \sqrt{2}$$

END OF EXAMINATION

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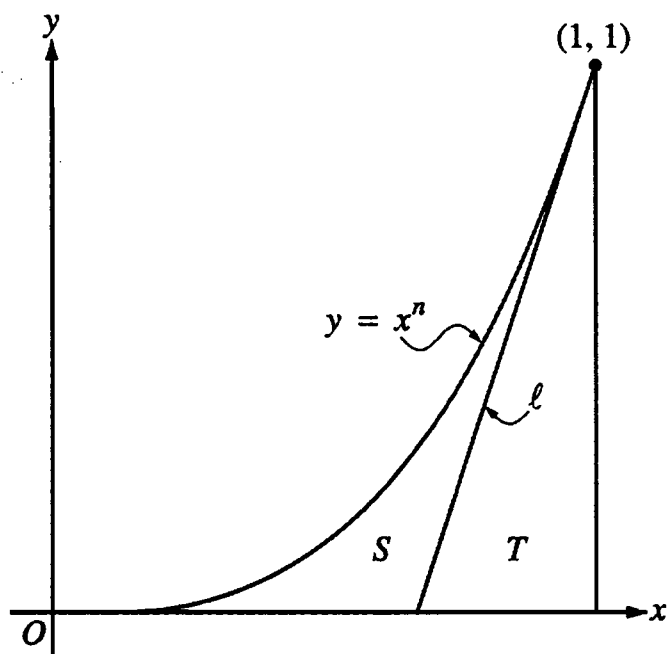
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Work for problem 6(a)

$$\int_0^1 x^n dx = \left[\frac{x^{n+1}}{n+1} \right]_0^1 = \left(\frac{(1)^{n+1}}{n+1} \right) - \left(\frac{(0)^{n+1}}{n+1} \right)$$

$$= \frac{1}{n+1} - \frac{0}{n+1} = \frac{1}{n+1} \text{ units}^2$$

Work for problem 6(b)

$$y = x^n$$

$$\frac{dy}{dx} = nx^{n-1}$$

$$\left. \frac{dy}{dx} \right|_1 = n$$

eqn. of line l :

$$y - y_0 = m(x - x_0)$$

$$y - 1 = n(x - 1)$$

$$y - 1 = nx - n$$

$$y = nx - n + 1$$

$$nx - n + 1 = 0$$

$$nx = n - 1$$

$$x = \frac{n-1}{n}$$

x-intercept \Rightarrow

~~$$y = 0 = nx - n + 1 = 0$$~~
~~$$y = 0 \Rightarrow n(x - 1) + 1 = 0$$~~

$$\text{Area} = \frac{1}{2} (1) \left(1 - \frac{n-1}{n} \right)$$

$$\text{Area} = \frac{1}{2} (1) \left(1 - \frac{n-1}{n} \right) = \frac{1}{2} (1) \left(\frac{1}{n} \right) = \frac{1}{2n} \text{ units}^2$$

Continue problem 6 on page 15.

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Work for problem 6(c)

$$\begin{aligned} \text{Area of } S &= \int_0^1 x^n dx - \text{Area of } T \\ &= \frac{1}{n+1} - \frac{1}{2n} \\ &= \frac{(2n) - (n+1)}{2n(n+1)} \\ &= \frac{n-1}{2n^2+2n} \text{ units}^2 \end{aligned}$$

Maximum : $S'(n) = 0$

$$\begin{aligned} \Rightarrow (1)(2n^2+2n) - (4n+2)(n-1) &= 0 \\ 2n^2+2n - (4n^2-4n+2n-2) &= 0 \\ 2n^2+n - 4n^2+4n-n-2 &= 0 \\ -2n^2+4n-2 &= 0 \\ n^2-2n+1 &= 0 \\ (n-1)^2 = 0 \Rightarrow n=1 &\text{ will maximize the area.} \end{aligned}$$

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