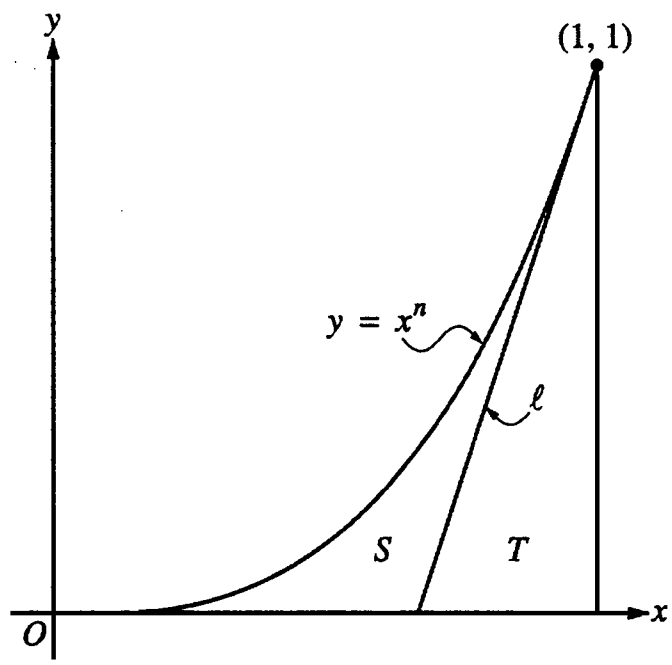
AP® Calculus AB
2004 Sample Student Responses
Form B

B

NO CALCULATOR ALLOWED



Work for problem 6(a)

$$\int_0^1 x^n\,dx = \left(\frac{x^{n+1}}{n+1}\right)_0^1 = \frac{1^{n+1}}{n+1} - \frac{0^{n+1}}{n+1}$$

$$= \frac{1}{n+1}$$

Work for problem 6(b)

$y = x^n$

$y' = nx^{n-1}$

$y'(1) = n\,1^{n-1}$

$= n$

i) the equation of the tangent line will be

$y - 1 = n(x-1)$

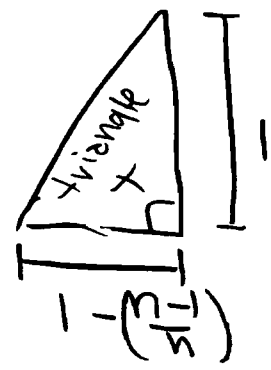
$\boxed{y = nx - n + 1}$

$\Rightarrow$ in order to get base of T,

$y = nx - n + 1$

$0 = nx - n + 1$

$\frac{n-1}{n} = x$



$$A_{\triangle T} = \frac{1 \times \left(1 - \frac{n-1}{n}\right)}{2}$$

$$= \frac{\frac{n-n+1}{n}}{2}$$

$$= \frac{1}{2n}$$

Continue problem 6 on page 15.

B2

NO CALCULATOR ALLOWED

Work for problem 6(c)

$A_S = \int_0^1 x^n\,dx - A_T$

from (a) & (b) we know $\int_0^1 x^n\,dx$ & A_T

$\therefore A_S = \frac{1}{n+1} - \frac{1}{2n}$

$= \frac{2n-n-1}{2n(n+1)}$ = ~~Answer~~ $\frac{n-1}{2n(n+1)}$

$A_S = \frac{(n-1)}{2n^2+2n}$

$A'_S = \frac{2n^2+2n-(n-1)(4n+2)}{(2n^2+2n)^2} = 0$

$2n^2+2n-(4n^2-2n-2) = 0$

$n^2+n-2n^2+n+1 = 0$

$-n^2+2n+1 = 0.$

by quadratic formula.

$\frac{-2\pm\sqrt{4+4}}{-2} = \frac{-2\pm 2\sqrt{2}}{-2} = 1\pm\sqrt{2}$

$A_S(1+\sqrt{2}) = \frac{\sqrt{2}}{2(1+2\sqrt{2}+2)+2+2\sqrt{2}}$

$= \frac{\sqrt{2}}{8+6\sqrt{2}}$

$A_S(1-\sqrt{2}) = \frac{-\sqrt{2}}{2(1-2\sqrt{2}+2)+2(1-\sqrt{2})}$

$= \frac{-\sqrt{2}}{8-2\sqrt{2}}$

~~Answer~~

$n = 1+\sqrt{2}$

END OF EXAMINATION

THE FOLLOWING INSTRUCTIONS APPLY TO THE BACK COVER OF THIS SECTION II BOOKLET.

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D

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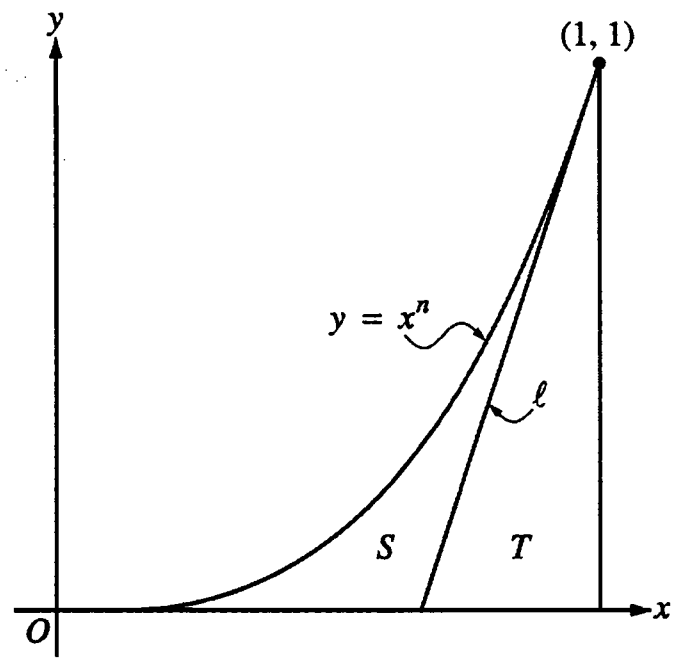

Work for problem 6(a)

$$\int_0^1 x^n\,dx = \frac{x^{n+1}}{n+1}\Big]_0^1 = \left(\frac{(1)^{n+1}}{n+1}\right) - \left(\frac{(0)^{n+1}}{n+1}\right)$$

$$= \frac{1}{(n+1)} - \frac{0}{n+1} = \frac{1}{n+1} \text{ units}^2$$

Work for problem 6(b)

$y = x^n$

$\frac{dy}{dx} = nx^{n-1}$

$\left.\frac{dy}{dx}\right|_1 = n$

eqn. of line ℓ:

$y - y_0 = m(x - x_0)$

$y - 1 = n(x - 1)$

$y - 1 = nx - n$

$y = nx - n + 1$

$nx - n + 1 = 0$

$nx = n - 1$

$x = \frac{n-1}{n}$

x-intercept $\Rightarrow$ $y = 0$

Area $= \frac{1}{2}(1)\left(1 - \frac{n-1}{n}\right)$

Area $= \frac{1}{2}(1)\left(\left|\frac{n-n-1}{n}\right|\right) = \frac{1}{2}(1)\left(\left|-\frac{1}{n}\right|\right) = \frac{1}{2}(1)\left(\frac{1}{n}\right) = \frac{1}{2n}$ units^2

Continue problem 6 on page 15.

D2

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Work for problem 6(c)

Area of S $= \int_0^1 x^n \, dx$ − Area of T

$= \frac{1}{n+1} - \frac{1}{2n}$

$= \frac{(2n) - (n+1)}{2n(n+1)}$

$= \frac{n-1}{2n^2+2n}$ units^2

maximum : $S'(n) = 0$

$\Rightarrow (1)(2n^2+2n) - (4n+2)(n-1) = 0$

$2n^2+2n - (4n^2 - 4n + 2n - 2) = 0$

$2n^2+2n - 4n^2 + 4n - 2n - 2 = 0$

$-2n^2 + 4n - 2 = 0$

$n^2 - 2n + 1 = 0$

$(n-1)^2 = 0 \Rightarrow n = 1$ will maximize the area.

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