Mech. 2.

An ideal spring is hung from the ceiling and a pan of mass $M$ is suspended from the end of the spring, stretching it a distance $D$ as shown above. A piece of clay, also of mass $M$, is then dropped from a height $H$ onto the pan and sticks to it. Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) Determine the speed of the clay at the instant it hits the pan.

Taking level of pan to have $V_{\text{gravitational}} = 0$.

Apply law of Conservation of Mechanical Energy:

\[ KE_i + U_i = KE_f + U_f \]

\[ 0 + mgh = \frac{1}{2}mv^2 + 0 \]

\[ v^2 = 2gh \]

\[ \Rightarrow v = \sqrt{2gh} \]

(b) Determine the speed of the pan just after the clay strikes it.

Applying law of Conservation of Linear momentum:

\[ m_i u_i + m_f u_f = m_f v_f + m_v v_v \]

\[ \Rightarrow M\sqrt{2gh} + 0 = (M+M) \frac{v}{2} \]

\[ \Rightarrow M\sqrt{2gh} = 2M \frac{v}{2} \]

\[ \Rightarrow v_{\text{pan}} = \frac{\sqrt{2gh}}{2} \]
(c) Determine the period of the simple harmonic motion that ensues.

\[
T_{\text{spring}} = 2\pi \sqrt{\frac{m}{k}}
\]

where \( k \) is the spring constant.

\[
F = k \Delta l \Rightarrow k = \frac{F}{\Delta l} = \frac{mg}{\Delta l} = \frac{Mg}{D}
\]

\[
\Rightarrow T_{\text{spring}} = 2\pi \sqrt{\frac{M}{\frac{D}{g}}} = 2\pi \sqrt{\frac{2D}{g}}
\]

(d) Determine the distance the spring is stretched (from its initial unstretched length) at the moment the speed of the pan is a maximum. Justify your answer.

Speed of the pan is maximum when it is passing through equilibrium position.

Equilibrium position: \( F = k \Delta l \Rightarrow mg = k \Delta l \)

\[
2Mg = \frac{Mg}{D} \Delta l \Rightarrow \Delta l = 2D
\]

\( \Rightarrow \) The spring is stretched a length \( 2D \) from the original length.

(e) The clay is now removed from the pan and the pan is returned to equilibrium at the end of the spring. A rubber ball, also of mass \( M \), is dropped from the same height \( H \) onto the pan, and after the collision is caught in midair before hitting anything else.

Indicate below whether the period of the resulting simple harmonic motion of the pan is greater than, less than, or the same as it was in part (c).

\( \bigcirc \) Greater than \( \bigcirc \) Less than \( \bigcirc \) The same as

Justify your answer.

Since \( T_{\text{spring}} = 2\pi \sqrt{\frac{m}{k}} \)

since \( k \) is constant

since mass on spring is smaller because ball bounces off the pan whereas the clay sticks;

\( T_{\text{spring}} \) will be smaller.
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An ideal spring is hung from the ceiling and a pan of mass $M$ is suspended from the end of the spring, stretching it a distance $D$ as shown above. A piece of clay, also of mass $M$, is then dropped from a height $H$ onto the pan and sticks to it. Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) Determine the speed of the clay at the instant it hits the pan.

\[ ME_1 = ME_f \]
\[ M g H = \frac{1}{2} n M v^2 \]
\[ v = \sqrt{2 g H} \text{ m/s} \]

(b) Determine the speed of the pan just after the clay strikes it.

\[ M A (\sqrt{2 g H}) = 2 M A (V_p) \]
\[ v_{pan} = \frac{\sqrt{2 g H}}{2} \text{ m/s} \]
(c) Determine the period of the simple harmonic motion that ensues.

\[ F = kx \]
\[ mg = kd \]
\[ k = \frac{mg}{d} \]
\[ T = 2\pi \sqrt{\frac{m}{k}} \]
\[ T = 2\pi \sqrt{\frac{2M}{mg}} = \frac{2\pi \sqrt{\frac{2d}{g}}}{s} \]

(d) Determine the distance the spring is stretched (from its initial unstretched length) at the moment the speed of the pan is a maximum. Justify your answer.

\[ F = kx \]
\[ W = \int_{0}^{d} kx \, dx \]
\[ W = \frac{1}{2} kd^2 \]

\[ W = F_{\text{distance}} \]
\[ \frac{1}{2} kd^2 = kd \, (\text{distance}) \]

\[ \text{distance} = \frac{1}{2} \, d \]

(e) The clay is now removed from the pan and the pan is returned to equilibrium at the end of the spring. A rubber ball, also of mass \( m \), is dropped from the same height \( H \) onto the pan, and after the collision is caught in midair before hitting anything else.

Indicate below whether the period of the resulting simple harmonic motion of the pan is greater than, less than, or the same as it was in part (c).

- Greater than
- Less than
- The same as

Justify your answer.

The new period will be less than it was in part c because the mass of the system is decreased.

\[ T = 2\pi \sqrt{\frac{m}{k}} \]

Because \( m \) decreased, \( T \) decreases.

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