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Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part, NOT in the green insert.

E&M. 1.

A spherical cloud of charge of radius \( R \) contains a total charge \( +Q \) with a nonuniform volume charge density that varies according to the equation

\[ \rho(r) = \rho_0 \left(1 - \frac{r}{R}\right) \quad \text{for} \quad r \leq R \quad \text{and} \quad \rho = 0 \quad \text{for} \quad r > R, \]

where \( r \) is the distance from the center of the cloud. Express all algebraic answers in terms of \( Q, R, \) and fundamental constants.

(a) Determine the following as a function of \( r \) for \( r > R \).

i. The magnitude \( E \) of the electric field

\[ \oint E \cdot dA = \frac{Q}{\varepsilon_0} \quad \Rightarrow \quad E(4\pi r^2) = \frac{Q}{\varepsilon_0} \quad \Rightarrow \quad E = \frac{Q}{4\pi \varepsilon_0 r^2} \]

ii. The electric potential \( V \)

A spherical distribution acts as a point charge so

\[ V = \frac{Q}{4\pi \varepsilon_0 r} \]

(b) A proton is placed at point \( P \) shown above and released. Describe its motion for a long time after its release.

The proton experiences a force to the right from the electric field. The field strength decreases with distance, so the acceleration of the proton decreases with time until it reaches a final constant speed.
(c) An electron of charge magnitude $e$ is now placed at point $P$, which is a distance $r$ from the center of the sphere, and released. Determine the kinetic energy of the electron as a function of $r$ as it strikes the cloud.

\[
\begin{align*}
\text{Energy conserved} \\
\varepsilon t + \varepsilon v_i &= \varepsilon f + \varepsilon f \\
\varepsilon t + \varepsilon v_i &= \varepsilon f + \varepsilon f \\
\varepsilon t - \varepsilon v_2 &= \varepsilon f \\
\varepsilon t - \varepsilon v_2 &= \varepsilon f \\
\frac{eQ}{4\pi\varepsilon_0 r} + \frac{eQ}{4\pi\varepsilon_0 R} &= k(r) = \frac{eQ}{4\pi\varepsilon_0} \left( \frac{1}{R} - \frac{1}{r} \right)
\end{align*}
\]

(d) Derive an expression for $\rho_0$.

\[
\begin{align*}
d\rho &= \rho(r) dV = 4\pi r^2 dr \\
dV &= 4\pi r^2 dr \\
\int d\rho &= \int_0^R \frac{1}{4\pi} \left( 1 - \frac{r^2}{R^2} \right) 4\pi r^2 dr \\
\rho &= \frac{1}{4\pi} \int_0^R r^2 \left( 1 - \frac{r^2}{R^2} \right) dr \\
\rho &= \frac{1}{4\pi} \int_0^R r^3 \left( 1 - \frac{1}{R^2} \right) dr \\
\rho_0 &= \frac{3Q}{\pi R^3} \left( \frac{1}{3} - \frac{1}{4R} \right)
\end{align*}
\]

(e) Determine the magnitude $E$ of the electric field as a function of $r$ for $r \leq R$.

\[
\begin{align*}
\mathbf{E} \cdot dA &= \frac{\varepsilon_0}{\varepsilon_0} \\
\varepsilon_0 &= \frac{4\pi r^2}{1 - \frac{r^2}{R^2}} \\
\mathbf{E} &= \frac{4\pi \rho_0 r \left( \frac{1}{3} - \frac{1}{4R} \right)}{\varepsilon_0} \\
E &= \frac{3\varepsilon_0 r R^3}{\pi \varepsilon_0} \left( \frac{1}{3} - \frac{1}{4R} \right)
\end{align*}
\]

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PHYSICS C
Section II, ELECTRICITY AND MAGNETISM
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part, NOT in the green insert.

E&M. 1.

A spherical cloud of charge of radius $R$ contains a total charge $+Q$ with a nonuniform volume charge density that varies according to the equation

$$\rho(r) = \rho_0 \left(1 - \frac{r}{R}\right) \text{ for } r \leq R \text{ and } \rho = 0 \text{ for } r > R,$$

where $r$ is the distance from the center of the cloud. Express all algebraic answers in terms of $Q$, $R$, and fundamental constants.

(a) Determine the following as a function of $r$ for $r > R$.

i. The magnitude $E$ of the electric field

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

ii. The electric potential $V$

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

(b) A proton is placed at point $P$ shown above and released. Describe its motion for a long time after its release.

The proton will accelerate away from the cloud along a straight line from the center of the cloud through point $P$. The acceleration decreases as the proton gets farther away, but the velocity increases.

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(c) An electron of charge magnitude $e$ is now placed at point $P$, which is a distance $r$ from the center of the sphere, and released. Determine the kinetic energy of the electron as a function of $r$ as it strikes the cloud.

$U_{E_i} = qV = \frac{eQ}{4\pi\varepsilon_0 r}$

$U_{Ef} = \frac{eQ}{4\pi\varepsilon_0 R}$

$U_{Ei} = U_{Ef} + K$

$K = U_{Ei} - U_{Ef} = \frac{eQ}{4\pi\varepsilon_0} \left( \frac{1}{r} - \frac{1}{R} \right) = K(r)$

(d) Derive an expression for $\rho_0$.

$Q = \frac{1}{3} \pi R^3 \rho_0$

$Q = \int_0^R \rho(r) 4\pi r^2 dr$

$Q = \rho_0 \int_0^R (1 - \frac{r}{R}) 4\pi r^2 dr$

$= \rho_0 4\pi \int_0^R (R^2 - \frac{r^3}{R}) dr$

$= \rho_0 4\pi \left[ R^2 r - \frac{r^4}{4} \right]_0^R$

$\rho_0 = 3\frac{Q}{\pi R^3}$

(e) Determine the magnitude $E$ of the electric field as a function of $r$ for $r \leq R$.

$E = \frac{Q_{net}}{\varepsilon_0}$

$E = \frac{9Q}{4\pi \varepsilon_0 r^2}$

$E = \frac{3Qr^2}{4R^3} \left(\frac{R}{3} - \frac{r}{4R} \right)$

$E = 3Q \frac{r}{16 \pi R^3} \left(\frac{1}{3} - \frac{r}{4R} \right)$

$q \geq \frac{3Qr^3}{4R^3} \left(\frac{1}{3} - \frac{r}{4R} \right)$

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