3. (15 points)
A rail gun is a device that propels a projectile using a magnetic force. A simplified diagram of this device is shown above. The projectile in the picture is a bar of mass $M$ and length $D$, which has a constant current $I$ flowing through it in the +y-direction, as shown. The space between the thin frictionless rails contains a uniform magnetic field $B$, perpendicular to the plane of the page. The magnetic field and rails extend for a distance $L$. The magnetic field exerts a constant force $F$ on the projectile, as shown.

Express all algebraic answers to the following parts in terms of the magnitude $F$ of the constant magnetic force, other quantities given above, and fundamental constants.

(a) Determine the position $x$ of the projectile as a function of time $t$ while it is on the rail if the projectile starts from rest at $x = 0$ when $t = 0$.

$$a = \frac{F}{m}$$

$$x = \frac{F}{2m} t^2$$

(b) Determine the speed of the projectile as it leaves the right-hand end of the track.

At the end of the track:

$$L = \frac{Ft^2}{2m} \Rightarrow \frac{2Lt}{F} = t^2 \Rightarrow t = \sqrt{\frac{2Lm}{F}}$$

Speed ($v_{(x,y)}$) = $a + \frac{F}{m}$

$$v = \frac{F}{m} \sqrt{\frac{2Lm}{F}} \quad \text{or} \quad v = \sqrt{\frac{2Lf}{m}}$$

GO ON TO THE NEXT PAGE.
(c) Determine the energy supplied to the projectile by the rail gun.

\[ E = \frac{1}{2} mv^2 - \text{original energy} \]

\[ E = \frac{1}{2} \left( \frac{2LF}{m} \right) \Rightarrow E = \frac{2LF}{m} \]

\[ E = LF \]

(d) In what direction must the magnetic field \( B \) point in order to create the force \( F \)? Explain your reasoning.

\( B \) must point out of the page because of the second right hand rule. For the current to go up, and the force to go right, your fingers point out of the page.

(e) Calculate the speed of the bar when it reaches the end of the rail given the following values.

\( B = 5 \, T \quad L = 10 \, \text{m} \quad I = 200 \, \text{A} \quad M = 0.5 \, \text{kg} \quad D = 10 \, \text{cm} \)

\[ V = \sqrt{\frac{2(10)F}{(1.5)}} \]

\[ V = \sqrt{40 \, F} \]

\[ V = \sqrt{40(100)} \]

\[ V = 63.2 \, \text{m/s} \]
3. (15 points)
A rail gun is a device that propels a projectile using a magnetic force. A simplified diagram of this device is shown above. The projectile in the picture is a bar of mass \( M \) and length \( D \), which has a constant current \( I \) flowing through it in the \( +y \)-direction, as shown. The space between the thin frictionless rails contains a uniform magnetic field \( B \), perpendicular to the plane of the page. The magnetic field and rails extend for a distance \( L \). The magnetic field exerts a constant force \( \mathbf{F} \) on the projectile, as shown.

Express all algebraic answers to the following parts in terms of the magnitude \( F \) of the constant magnetic force, other quantities given above, and fundamental constants.

(a) Determine the position \( x \) of the projectile as a function of time \( t \) while it is on the rail if the projectile starts from rest at \( x = 0 \) when \( t = 0 \).

\[
F = ma \\
\frac{\Delta x}{\Delta t} = \frac{a}{t} \\
x = \frac{1}{2} a t^2 \\
x = \frac{F}{\frac{a}{m}} t^2
\]

(b) Determine the speed of the projectile as it leaves the right-hand end of the track.

\[
\begin{align*}
\sqrt{v^2} &= 2 \left( \frac{\mathbf{F} \cdot \mathbf{A}}{m} \right) \left( x - x_0 \right) \\
\sqrt{v^2} &= 2 \left( \frac{F}{m} \right) (L) \\
v &= \sqrt{2 \frac{F L}{m}}
\end{align*}
\]
(c) Determine the energy supplied to the projectile by the rail gun.

\[ kE = \frac{1}{2} m v^2 \]
\[ \frac{1}{\mu} \cdot \frac{BL}{m} \]
\[ E = FL \]

(d) In what direction must the magnetic field \( B \) point in order to create the force \( F \)? Explain your reasoning.

The field \( B \) must point out of the page. This is because using the right hand rule, the only way to satisfy the direction of current and the force is if the magnetic field is out of the page.

(e) Calculate the speed of the bar when it reaches the end of the rail given the following values.

\( B = 5 \text{ T} \quad L = 10 \text{ m} \quad I = 200 \text{ A} \quad M = 0.5 \text{ kg} \quad D = 10 \text{ cm} \)

\[ V = \sqrt{\frac{2FL}{M}} \]
\[ V = \sqrt{\frac{2(5 \text{ T})(10 \text{ m})(200 \text{ A})(\sin 90)}{5 \text{ kg}}} \]
\[ V = \sqrt{2(10000)(10 \text{ m})} \]
\[ V = \sqrt{15 \text{ kg}} \]
\[ V = 632.5 \text{ m/s} \]

\[ F = BIL \sin \alpha \]
\[ F = (5 \text{ T})(10 \text{ m})(200 \text{ A})(\sin 90) \]
\[ F = 10000 \text{ N} \]