3. (15 points)
A thin convex lens $A$ of focal length $f_0 = 10$ cm is positioned on an $x$-axis as shown above. An object of height 5 cm, represented by the arrow, is positioned 15 cm to the left of lens $A$.

(a) On the figure above, draw necessary rays and sketch the image produced by lens $A$.

(b) Calculate the location of the image produced by lens $A$.

\[
\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f_0}
\]

\[
\frac{1}{s_i} = \frac{1}{f_0} - \frac{1}{s_o}
\]

\[
= \frac{1}{100} - \frac{1}{150}
\]

\[
s_i = 30\text{ cm}
\]

\[\therefore s_i = 30\text{ cm}\]

(c) Calculate the height of the image produced by lens $A$.

\[
M = \frac{h_i}{h_0}
\]

\[
M = \frac{s_i}{s_o}
\]

\[
\frac{h_i}{h_0} = \frac{s_i}{s_o}
\]

\[
h_i = \frac{-s_i \cdot h_0}{s_o}
\]

\[
= \frac{-30\text{ cm} \times 5\text{ cm}}{15\text{ cm}}
\]

\[
= -10\text{ cm}
\]

\[\therefore h_i = 10\text{ cm downwards}
\]

\[(-10\text{ cm})\]

GO ON TO THE NEXT PAGE.
A second thin convex lens $B$ of focal length $f_B = 10\, \text{cm}$ is now positioned 10 cm to the right of lens $A$, as shown above.

(d) Determine the location on the $x$-axis given above of the final image produced by the combination of lenses.

Let's assume $F_A$ is at 0 cm.

$\rightarrow F_B = -10\, \text{cm} \Rightarrow$ object is at $-25\, \text{cm}$

\[
\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f}
\]

\[
\frac{1}{s_i} = \frac{1}{f} - \frac{1}{s_o}
\]

\[
= \frac{1}{-10\, \text{cm}} - \left( \frac{1}{25\, \text{cm}} \right)
\]

\[
s_i = 17\, \text{cm}
\]

(e) Check the appropriate spaces below to indicate the characteristics of the final image produced by the combination of lenses.

\[\checkmark\] inverted \hspace{1cm} \[\ ]\] larger than the original object

\[\ ]\] upright \hspace{1cm} \[\checkmark\] smaller than the original object

Explain your answers.

**Inverted:** Rays start from the tip of the arrow (upright) and they meet at one point below $x$-axis, this indicates the tip of the arrow points downwards, thus inverted.

**Smaller:** According to the calculation below determining the length of the image, the final image is 3.4 cm, thus smaller than the real object.

\[
\frac{h_i}{h_o} = \frac{-s_i}{s_o}
\]

\[
h_i = \frac{-s_i h_o}{s_o}
\]

\[
h_i = \frac{- (17\, \text{cm})(5\, \text{cm})}{25\, \text{cm}}
\]

\[
h_i = -3.4\, \text{cm}
\]

GO ON TO THE NEXT PAGE.
3. (15 points)
A thin convex lens $A$ of focal length $f_A = 10$ cm is positioned on an $x$-axis as shown above. An object of height 5 cm, represented by the arrow, is positioned 15 cm to the left of lens $A$.

(a) On the figure above, draw necessary rays and sketch the image produced by lens $A$.

(b) Calculate the location of the image produced by lens $A$.
\[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}\\
\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_0}\\
\frac{1}{d_i} = \frac{1}{10} - \frac{1}{15} = \frac{1}{30}\\
\therefore d_i = 30 \text{ cm}.
\]
:. Image position = 30 cm behind the lens.

(c) Calculate the height of the image produced by lens $A$.
\[
M = \frac{-d_i}{d_0} = \frac{-30}{15} = -2\\
\therefore h_o = |M| d_i = 2 \times 5 \text{ cm} = 10 \text{ cm}.
\]
:. Height of the image = 10 cm.

GO ON TO THE NEXT PAGE.
A second thin convex lens \( B \) of focal length \( f_B = 10 \) cm is now positioned 10 cm to the right of lens \( A \), as shown above.

(d) Determine the location on the \( x \)-axis given above of the final image produced by the combination of lenses.

\[ d_0 = 20 \text{ cm from lens } B, \text{ virtual image} \]

\[ \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_0} \]
\[ = \frac{1}{10} + \frac{1}{20} = \frac{3}{20} \]
\[ d_i = 6.67 \text{ cm} \]

Image position = 6.67 cm to the right of lens \( B \).

(e) Check the appropriate spaces below to indicate the characteristics of the final image produced by the combination of lenses.

- [ ] inverted  - [ ] larger than the original object
- [x] upright  - [x] smaller than the original object

Explain your answers.