AP® Calculus AB
2003 Scoring Commentary

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Question 1

This problem presented a region bounded by two graphs and a vertical line. Students needed to use integration to find an area and two volumes. A graphing calculator was required to find the point of intersection of the two graphs. Part (a) required the use of a definite integral to find the area of the region. Part (b) required another application of integration to find the volume generated by revolving this region about a horizontal line. The resulting solid had cross sections in the shape of washers. Part (c) again required integration, this time to find the volume of a solid whose base was the region and whose cross sections were rectangles.

The mean score was 3.90.

<u>Sample C</u> (Score 9)

The student earned all 9 points.

<u>Sample F</u> (Score 7)

The student earned 7 points: 2 points in part (a), 3 points in part (b), 1 point in part (c), and the region point. In part (c), the student failed to earn the integrand point since there is an extraneous factor of π. The student also failed to earn the answer point since the stated answer is not correct for the integral.

Question 2

This problem presented the velocity function for a particle moving along the x-axis and asked questions about acceleration, change of direction, distance traveled, and greatest distance from the origin — questions that involved interpretation of velocity and knowledge of its relationship to the position of the particle. Both differentiation at a point and integration over a specified interval were used in solving the problem. Part (a) required students to differentiate the velocity function at a specific time in order to find the acceleration. Students needed to understand the difference between velocity and speed. One way to decide whether or not the speed was increasing was to recognize that positive acceleration and negative velocity imply decreasing speed. Part (b) required students to use the change in the sign of the velocity function to determine where the particle changes direction. Part (c) asked students to set up and compute a definite integral that gave the distance traveled over a time interval. This could be done either by using the turning point from part (b) or by integrating the speed. Part (d) brought everything together, asking for the furthest distance from the origin. Because there was no closed formula for the position, students needed to know how to use the definite integral of the velocity to determine position. The answer to part (b) provided one candidate for a time at which the particle could be furthest from the origin. It was necessary to find the position of the particle at this time. Students also needed to compare this distance to the distance from the origin at times $t = 0$ and $t = 3$.

The mean score was 3.01.

<u>Sample VV</u> (Score 9)

The student earned all 9 points.

<u>Sample LL</u> (Score 7)

The student earned 7 points: 1 point in part (a), 2 points in part (b), 3 points in part (c), and 1 point in part (d). In part (a), the student failed to make a conclusion about the speed of the particle, losing a point. In part (d), the student failed to earn the answer point since no conclusion about the maximum distance from the origin is reached.

Question 3

This problem presented the rate of fuel consumption, $R(t)$, of an airplane. The rate was given in both graphical and tabular form. Students were asked questions that addressed the values and meanings of the derivative, the second derivative, and the integral of the function describing the rate of consumption. Units of measure were important in parts (a) and (d). In part (a), students were required to know how to use the values in the table to find an approximation to the derivative of R at time $t = 45$. Students should have used those values closest to $t = 45$. Part (b) required students to recognize that when the rate of increase of R is maximized, the second derivative of R will be zero. For part (c), students needed to calculate a left Riemann sum approximation to a definite integral and to know that for an increasing function, this always underestimates the true value of the integral. Part (d) asked for the meaning of the definite integral of R. It inverted the usual question about the average value of a function by presenting the definite integral divided by the length of the time interval and asking the student for its meaning in terms of the fuel consumption of the airplane. Reference to the specific time interval was an important part of the explanation.

The mean score was 2.43.

Sample O (Score 9)

The student earned all 9 points.

Sample I (Score 7)

The student earned 7 points: 1 point in each of parts (a) and (b), 2 points in part (c), and 3 points in part (d). In part (a), the student did not state the units associated with the approximation and lost a point. In part (b), the student correctly stated that the second derivative is zero, but the supporting reason failed to earn the point.

Question 4

This problem presented students with the graph of the derivative of f. It explored the students' understanding of the relationship between the behavior of the derivative and of the function f. In part (a), students needed to know how to use the sign of the derivative to determine when the function is increasing. Part (b) could be answered by knowing how to determine the concavity of f by observing where the graph of f' is increasing or decreasing. Part (c) required knowing the relationship between the values of f' and the slope of the tangent line of f. Part (d) required an application of the Fundamental Theorem of Calculus, recognizing that $f(b) - f(a)$ is the integral of f' over the closed interval $[a, b]$, and then using this and the fact that $f(0) = 3$ to calculate the values of f at $x = -3$ and $x = 4$. The value of f at $x = 4$ is most easily computed by finding the area between the graph of f' and the x-axis (a rectangle minus a semicircle) and subtracting this area from the value of f at $x = 0$.

The mean score was 2.68.

<u>Sample C</u> (Score 9)

The student earned all 9 points.

<u>Sample E</u> (Score 7)

The student earned 7 points: 2 points in each of parts (a), (b), and (d), and 1 point in part (c). In part (d), the student handled the initial condition incorrectly in finding $f(-3)$ and lost a point. Furthermore, in finding $f(4)$, the student evaluated the definite integral incorrectly, losing another point.

Question 5

This problem presented a related rates setting, as well as a separable differential equation. The volume of coffee in a cylindrical pot was changing at a rate that was given as a function of the height. Part (a) asked students to find the rate of change of height as a function of the height. This required translating the information on the rate of change of the volume V into mathematical notation and showing how to use the chain rule to find the desired result. The solution was given so that all students could start part (b) on an equal footing. Part (b) asked students to solve the separable differential equation that was the solution to part (a). Part (c) used the answer to part (b) and required recognition that the coffeepot was empty precisely when the height was zero.

The mean score was 3.26.

Sample B (Score 9)

The student earned all 9 points.

Sample Y (Score 7)

The student earned 7 points: 3 points in part (a), 4 points in part (b), and 0 points in part (c). In part (b), the student failed to earn full credit since there is a sign error in solving for h. Since the student's solution in part (c) was not consistent with the work in part (b), the student failed to earn the point in this part.

Question 6

This problem presented two piecewise-defined functions. It explored students' understanding of continuity, differentiability, and average value of a function. Part (a) required students to know that a function is continuous if and only if the value of the function agrees with both its limit from the left and its limit from the right. Part (b) was an average value problem. Student needed to know how to integrate a piecewise-defined function. Part (c) gave a piecewise-defined function with two free parameters which needed to be determined using the knowledge that the function was differentiable and hence also continuous at $x = 3$. These conditions produced two linear equations in the two unknowns.

The mean score was 2.68.

<u>Sample E</u> (Score 9)

The student earned all 9 points.

<u>Sample V</u> (Score 7)

The student earned 7 points: 2 points in part (a), 4 points in part (b), and 1 point in part (c). In part (c), the student failed to consider conditions that guarantee differentiability and lost 2 points.