AP® Calculus BC

2003 Sample Student Responses

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NO CALCULATOR ALLOWED

Work for problem 6(a)

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!} = 1 = \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \dots$$

$$f'(x) = -\frac{2x}{3!} + \frac{4x^3}{5!} - \dots$$

$$f''(x) = -\frac{2}{3!} + \frac{4 \times 3 x^2}{5!} - \dots$$

$f'(0) = 0$

$f''(0) = -\frac{2}{3!} = -\frac{1}{3}$ Second derivative test

f has a local maximum at $x=0$ because $x=0$ is a critical point and f is concave down at that point

Work for problem 6(b)

Alternating Series

Error must be less than next term.

$$\text{Error} \le \frac{1}{5!} = \frac{1}{5 \times 4 \times 3 \times 2} = \frac{1}{20 \times 6} = \frac{1}{120}$$

$$\text{Error} \le \frac{1}{120} < \frac{1}{100}$$

Thus $1 - \frac{1}{3!}$ approximates $f(1)$ with an error less than $\frac{1}{100}$

Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

Work for problem 6(c)

$$f(x)=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n+1)!}$$

$$f'(x)=\sum_{n=0}^{\infty}\frac{2n(-1)^n x^{2n-1}}{(2n+1)!}$$

$$\cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\ldots=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}$$

$$xy'+y = x\sum_{n=0}^{\infty}\frac{2n(-1)^n x^{2n-1}}{(2n+1)!}+\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n+1)!}$$

$$=\sum_{n=0}^{\infty}\frac{2n(-1)^n x^{2n}}{(2n+1)!}+\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n+1)!}$$

$$=\sum_{n=0}^{\infty}\frac{2n(-1)^n x^{2n}+(-1)^n x^{2n}}{(2n+1)!}=\sum_{n=0}^{\infty}\frac{(2n+1)(-1)^n x^{2n}}{(2n+1)(2n)!}$$

$$=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}=\cos x$$

Thus $y=f(x)$ is a solution to the differential equation

$$xy'+y=\cos x$$

END OF EXAMINATION

THE FOLLOWING INSTRUCTIONS APPLY TO THE BACK COVER OF THIS SECTION II BOOKLET.

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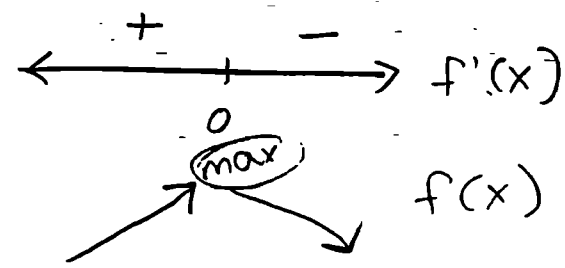
Work for problem 6(a)

$$f(x) = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \dots$$

$$f'(x) = -\frac{2x}{3!} + \frac{4x^3}{5!} - \frac{6x^5}{7!} + \dots$$

$$f'(0) = 0$$



$$f''(x) = -\frac{2}{3!} + \frac{12x^2}{5!} - \frac{30x^4}{7!} + \dots$$

$$f''(0) = -\frac{2}{3!} = -\frac{1}{3}$$

local max @ $x = 0$

$$f''(0) = -\frac{1}{3}$$

Work for problem 6(b)

Series is alternating so

error < first neglected term

first neglected term = $\frac{x^4}{5!}$

$$@1 = \frac{1}{5!} = \frac{1}{120}$$

$$\frac{1}{120} < \frac{1}{100}$$

Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

Work for problem 6(c)

$$xy' + y = \cos x$$

$$xy' = \cos x - y$$

$$y' = \frac{\cos x - y}{x}$$

$$\frac{dy}{dx} = \frac{\cos x - y}{x}$$

$$-\frac{\cos x}{x} + \frac{dy}{dx} = -\frac{y}{x}$$

$$\int -\frac{\cos x}{x} + \frac{dy}{y} = \int -\frac{1}{x}\,dx$$

$$-\frac{\cos x}{x} + \int \frac{dy}{y} = \int -\frac{dx}{x}$$

$$+ \ln|y| + C_1 = -\ln|x| + C_2$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$$

$$x \cdot \left(-\frac{2x}{3!} + \frac{4x^3}{5!} - \frac{6x^5}{7!} + \dots\right) + \left(1 - \frac{x^2}{3!} + \frac{x^4}{5!}\right)$$

$$-\frac{2x^2}{3!} + \frac{4x^4}{5!} - \frac{6x^6}{7!} + \dots + 1 - \frac{x^2}{3!} + \frac{x^4}{5!} -$$

$$1 - \left(\frac{3x^2}{3!}\right) + \left(\frac{5x^4}{5!}\right) - \dots$$

$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots = \cos x$$

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