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Work for problem 3(a)

\[
\frac{5}{3} y = \sqrt{1 + y^2} \Rightarrow y = \frac{7}{5}.
\]

\[
x = \frac{5}{3} \left(\frac{7}{5}\right) = 1.25
\]

\[
P \equiv (1.25, 0.75)
\]

\[
dx/dy = \left(\frac{1 + y^2}{2}\right)^{1/2} \cdot 2y
\]

\[
= \frac{y}{\sqrt{1 + y^2}}
\]

\[
dx/dy \bigg|_{y=0.75} = 0.6
\]

Work for problem 3(b)

\[
A = -\int_{0}^{0.75} \frac{5}{3} y \, dy + \int_{0}^{0.75} \sqrt{1 + y^2} \, dy
\]

\[
= \int_{0}^{0.75} \left(\sqrt{1 + y^2} - \frac{5}{3} y\right) \, dy
\]

\[
\approx 0.347
\]

Continue problem 3 on page 9.
Work for problem 3(c)

\[ x = r \cos \theta \quad y = r \sin \theta \]
\[ x^2 - y^2 = 1 \]
\[ (r \cos \theta)^2 - (r \sin \theta)^2 = 1 \]
\[ r^2 \cos^2 \theta - r^2 \sin^2 \theta = 1 \]
\[ r^2 (\cos^2 \theta - \sin^2 \theta) = 1 \]
\[ r^2 = \frac{1}{\cos^2 \theta - \sin^2 \theta} \]

---

Work for problem 3(d)

\[ \tan \theta = \frac{y}{x} \]
\[ \tan \theta = \frac{1.25}{1.25} \]
\[ \theta \approx 0.540 \]
\[ A = \frac{1}{2} \int_0^{540} \frac{d\theta}{\cos^2 \theta - \sin^2 \theta} \]

---

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.
Work for problem 3(a)

\[
\frac{5}{3} y = \sqrt{1 + y^2}
\]

when \( y = 0.75 \)

\[
\left(\frac{5}{3}\right)(0.75) = 1.25
\]

\[ P = (1.25, 0.75) \]

\[
x = (1 + y^2)^{\frac{1}{2}}
\]

\[
\frac{dx}{dx} = \frac{1}{2} \left(1 + y^2\right)^{-\frac{1}{2}} \left(2y \frac{dy}{dx}\right)
\]

\[
1 = \frac{dy}{dx} \frac{dy}{dx} \left(1 + y^2\right)
\]

\[
\frac{\sqrt{1 + y^2}}{y} = \frac{dy}{dx} \frac{\sqrt{1 - (0.75)^2}}{0.75} = \frac{5}{3}
\]

---

Work for problem 3(b)

\[
\int_{0}^{0.75} \left(\left(1 + y^2\right) - \left(\frac{5}{3} y\right)\right) dy = S
\]

\[ S = 0.3466 \]

---

Continue problem 3 on page 9.
Work for problem 3(c)

\[
\begin{align*}
  y &= r \sin \theta \\
  x &= r \cos \theta \\
  y^2 &= r^2 \sin^2 \theta \\
  x^2 &= r^2 \cos^2 \theta
\end{align*}
\]

\[
\begin{align*}
  x^2 - y^2 &= 1 \\
  r^2 \cos^2 \theta - r^2 \sin^2 \theta &= 1 \\
  \frac{r^2 (\cos^2 \theta - \sin^2 \theta)}{\cos^2 \theta - \sin^2 \theta} &= 1 \\
  r^2 &= \frac{1}{\cos^2 \theta - \sin^2 \theta}
\end{align*}
\]

Work for problem 3(d)

\[
\frac{1}{2} \int_{0}^{0.545} \frac{1}{\cos^2 \theta - \sin^2 \theta} \, d\theta
\]

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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