AP® Calculus BC
2003 Sample Student Responses
Form B

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NO CALCULATOR ALLOWED

CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

Velocity vector $= \left(\frac{dx}{dt}, \frac{dy}{dt}\right) = \left((2e^{3t} + e^{-7t})', (3e^{3t} - e^{-2t})'\right)$

$= \left(6e^{3t} - 7e^{-7t}, 9e^{3t} + 2e^{-2t}\right)$

Speed $= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{(6e^{3t} - 7e^{-7t})^2 + (9e^{3t} + 2e^{-2t})^2}$

$t = 0$

$= \sqrt{(6-7)^2 + (9+2)^2} = \sqrt{1 + 121} = \boxed{\sqrt{122}}$

Work for problem 4(b)

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \boxed{\frac{9e^{3t} + 2e^{-2t}}{6e^{3t} - 7e^{-7t}}}$

$\lim_{t \to \infty} \frac{dy}{dx} = \frac{9e^{3t}}{6e^{3t}}$ $\left(\because \lim_{t \to \infty} 7e^{-7t} \text{ and } 2e^{-2t} = 0\right)$

$= \boxed{\frac{3}{2}} = \boxed{1.5}$

Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

Work for problem 4(c)

b. horizontal tangent is $f'(x)=0 = \frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{9e^{3t}+2e^{-2t}}{6e^{3t}-7e^{-7t}} \Rightarrow 0$$

but $9e^{3t} > 0 \quad (t \in \mathbb{R})$

$2e^{-2t} > 0 \quad (t \in \mathbb{R})$

so $9e^{3t}+2e^{-2t} > 0 \quad (t \in \mathbb{R})$

therefore there is no horizontal tangent.

Work for problem 4(d)

$$\frac{dy}{dt} = \frac{9e^{3t}+2e^{-2t}}{6e^{3t}-7e^{-7t}}$$

→ when this part value goes 0, the value of $\frac{dy}{dx}$ increases to ∞ or $-\infty$ which is vertical tangent.

$\therefore \quad 6e^{3t}-7e^{-7t}=0$

$6e^{3t}=7e^{-7t}$

$6e^{10t}=7 \qquad e^{10t}=\frac{7}{6}$

$10t = \ln\frac{7}{6} \qquad t = \frac{1}{10}\ln\frac{7}{6}$

$t = 0.1(\ln 7 - \ln 6)$

at this point there must be

C1

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Work for problem 4(a)

$y'(t) = 3 \times e^{3t} \times 3 - e^{-2t} \times (-2) = 9e^{3t} + 2e^{-2t}$

$x'(t) = 6e^{3t} - 7e^{-7t}$

velocity vector for the particle $(6e^{3t} - 7e^{-7t},\ 9e^{3t} + 2e^{-2t})$

speed = |velocity| = $\left| \frac{9e^{3t} + 2e^{-2t}}{6e^{3t} - 7e^{-7t}} \right| = \left| \frac{9+2}{6-7} \right| = 11$

(at $t = 0$)

Work for problem 4(b)

$$\frac{dy}{dx} = \frac{9e^{3t} + 2e^{-2t}}{6e^{3t} - 7e^{-7t}}$$

$$\lim_{t \to \infty} \frac{dy}{dx} = \lim_{t \to \infty} \frac{9e^{3t}}{6e^{3t}} = \lim_{t \to \infty} \frac{3}{2} = \frac{3}{2}$$

Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

Work for problem 4(c)

The line tangent to the path of the particle that is horizontal must have a slope of 0.

Find values of t where $\frac{dy}{dx} = 0$

$9e^{3t} + 2e^{-2t} > 0$. for all values of t

because e^{3t} is always a positive value

and e^{-2t} is also always a positive value

Therefore, none exists.

Work for problem 4(d)

For the line tangent to the path of the particle to be vertical, $\frac{dy}{dx}$ must be infinite.

$\frac{dy}{dx}$ is infinite when $6e^{3t} - 7e^{-7t} = 0$

$6e^{3t} = 7e^{-7t}$