AP® Calculus BC
2003 Sample Student Responses
Form B

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Work for problem 3(a)

\[ \text{Since radius} = \frac{1}{2} (\text{diameter}) \]

\[ \Rightarrow \text{Average radius} = \frac{1}{2} \left( \frac{360}{(360 - 0)} \int_{0}^{360} B(x) \, dx \right) = \frac{360}{720} \int_{0}^{360} B(x) \, dx \]

Work for problem 3(b)

\[ \int_{0}^{360} B(x) \, dx = \lim_{n \to \infty} \sum_{k=1}^{3} B(c_k) \Delta x \]

\[ c_1 = 60 \text{mm} \Rightarrow B(c_1) = 30 \text{mm} \]
\[ c_2 = 180 \text{mm} \Rightarrow B(c_2) = 30 \text{mm} \]
\[ c_3 = 300 \text{mm} \Rightarrow B(c_3) = 24 \text{mm} \]
\[ \Delta x = \frac{3 \times 360}{3} = \frac{360 \text{mm}}{3} = 120 \text{mm} \]

\[ \Rightarrow \sum_{k=1}^{3} B(c_k) \Delta x = 120 \text{mm} \left( B(c_1) + B(c_2) + B(c_3) \right) \]

\[ = 120 \text{mm} \left( 30 \text{mm} + 30 \text{mm} + 24 \text{mm} \right) = 10800 \text{mm}^2 \]

\[ \Rightarrow \text{Average radius} = \frac{1}{720} \int_{0}^{360} B(x) \, dx \approx \frac{1}{720} \left( 10800 \text{mm}^2 \right) = 14 \text{mm} \]

Continue problem 3 on page 9.
Work for problem 3(c)

It is the volume of blood in the blood vessel starting from a distance of 125mm from one end to a distance of 275mm from the same end. The units will be $(\text{mm})^3$.

Work for problem 3(d)

\[
B''(x) = 0 \implies \frac{B'(b) - B'(a)}{b - a} = 0
\]

\[
= \frac{B'(b) = B'(a)}{b - a}
\]

\[
= \frac{B(b) - B(a)}{d - e} = \frac{B(c) - B(f)}{c - f}
\]

Since for all $x$, $x$ is the same

\[
= \frac{B(d) - B(e)}{d - e} = \frac{B(c) - B(f)}{c - f}
\]

From the table there are values of $d$, $e$, $c$, $f$ such that

\[
B(300) - B(300) = B(300) - B(240) \implies 26 - 24 = 26 - 24 \implies 0 = 0
\]

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.
Work for problem 3(a)

\[ B(x)_{\text{avg}} = \frac{1}{360-0} \int_{0}^{360} \frac{B(x)}{2} \, dx \]

\[ = \frac{1}{360} \int_{0}^{360} \frac{B(x)}{2} \, dx \]

Work for problem 3(b)

\[ \frac{360}{3} = 120 \]

\[ B(x)_{\text{avg}} = \frac{1}{360} \left[ \frac{120f(60)}{2} + \frac{120f(150)}{2} + \frac{120f(300)}{2} \right] \]

\[ = \frac{120}{360} \left[ 15 + 15 + 12 \right] \]

\[ = \frac{27}{36} \times 3 \]

\[ = 14 \text{ mm} \]

Continue problem 3 on page 9.
Work for problem 3(c)

\[ \frac{B(x)}{2} = \text{radius of blood vessel} \]

\[ \pi \int_{125}^{275} \left( \frac{B^2}{2} \right)^2 \, dx \]

Volume of the blood vessel from

\[ x = 125 \text{ mm} \text{ to } x = 275 \text{ mm} \text{ in } (\text{mm})^3 \]

Work for problem 3(d)

At \( x \) where \( B''(x) = 0 \)

There is an inflection on the graph

The sign of \( B'(x) \) changes

\( B'(x) \), the change of diameter

From the table we know that when the diameter increases \( B'(x) > 0 \) when diameter decreases \( B'(x) < 0 \)

\( B'(x) \) changes signs

\[ B''(x) = 0 \]

END OF PART A OF SECTION II

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