



AP[®] Calculus AB 2003 Sample Student Responses

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NO CALCULATOR ALLOWED

Work for problem 6(a)

yes, $f(3)$ is 2, which is the same as the limits as x approaches 3 from either side.

Work for problem 6(b)

$$\frac{\int_0^3 \sqrt{x+1} dx + \int_3^5 (5-x) dx}{5}$$

$$\frac{\left[\frac{2}{3}(x+1)^{3/2}\right]_0^3 + \left[5x - \frac{1}{2}x^2\right]_3^5}{5}$$

$$\frac{\frac{2}{3}(4)^{3/2} - \frac{2}{3} + 25 - \frac{1}{2}25 - (15 + \frac{1}{2}9)}{5}$$

$$\frac{\frac{2}{3}8 - \frac{2}{3} + 10 - \frac{16}{2}}{5}$$

$$\frac{\frac{16}{3} - \frac{2}{3} + 10 - 8}{5} = \frac{14}{15} + 2 - \frac{8}{5} = \frac{14}{15} + \frac{30}{15} - \frac{24}{15} = \frac{20}{15} = \frac{4}{3}$$

Continue problem 6 on page 15.



NO CALCULATOR ALLOWED

Work for problem 6(c)

$$g' = k \frac{1}{2\sqrt{x+1}}$$

m

$$k \frac{1}{2\sqrt{3+1}} = m$$

$$\frac{1}{4}k = m$$

← slopes must be equal at 3
functions must be equal at 3

$$k(2) = m(3) + 2$$

$$m = \frac{2k-2}{3}$$

$$\frac{1}{4}\left(\frac{8}{5}\right) = \frac{8}{20} = \frac{2}{5}$$

$$k = \frac{8}{5} \quad m = \frac{2}{5}$$

$$\frac{1}{4}k = \frac{2k-2}{3}$$

$$k = \frac{8k-8}{3}$$

$$k = \frac{8}{3}k - \frac{8}{3}$$

$$-\frac{5}{3}k = -\frac{8}{3}$$

$$k = \frac{24}{15} = \frac{8}{5}$$

END OF EXAMINATION

THE FOLLOWING INSTRUCTIONS APPLY TO THE BACK COVER OF THIS SECTION II BOOKLET.

- MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE BACK OF THIS SECTION II BOOKLET.
- CHECK TO SEE THAT YOUR AP NUMBER APPEARS IN THE BOX(ES) ON THE BACK COVER.
- MAKE SURE THAT YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMINATIONS YOU HAVE TAKEN THIS YEAR.

Work for problem 6(a)

$$f(x) = \begin{cases} \sqrt{x+1} & 0 \leq x \leq 3 \\ 5-x & 3 < x \leq 5 \end{cases}$$

$$f(3) = \begin{cases} \sqrt{3+1} = \sqrt{4} = 2 \\ 5-3 = 2 = 2 \end{cases}$$

f is continuous at $x=3$ because the $\lim_{x \rightarrow 3} f(x) = 2$
 and $f(3) = 2$.

Work for problem 6(b)

$$\text{Average value} = \frac{1}{5-0} \int_0^5 f(x) dx$$

$$\begin{aligned} u &= x+1 \\ du &= 1 \\ u(3) &= 4 \\ u(0) &= 1 \end{aligned}$$

The average value is $\frac{4}{3}$ on $0 \leq x \leq 5$.

$$\begin{aligned} \text{The average value} &= \frac{1}{5} \left(\int_0^3 \sqrt{x+1} dx + \int_3^5 (5-x) dx \right) \\ &= \frac{1}{5} \left(\int_1^4 \sqrt{u} du + \int_3^5 (5-x) dx \right) \end{aligned}$$

$$= \frac{1}{5} \left(\left. \frac{2}{3} u^{3/2} \right|_1^4 + \left. 5x - \frac{1}{2} x^2 \right|_3^5 \right)$$

$$= \frac{1}{5} \left(\left(\frac{2}{3} (4)^{3/2} - \frac{2}{3} (1)^{3/2} \right) + \left(5(5) - \frac{1}{2} (5)^2 - 5(3) - \frac{1}{2} (3)^2 \right) \right)$$

$$= \frac{1}{5} \left(\left(\frac{2}{3} (8) - \frac{2}{3} \right) + \left(25 - \frac{25}{2} - 15 - \frac{9}{2} \right) \right)$$

$$= \frac{1}{5} \left(\left(\frac{16}{3} - \frac{2}{3} \right) + \left(\frac{50}{2} - \frac{25}{2} - \left(\frac{30}{2} - \frac{9}{2} \right) \right) \right)$$

$$= \frac{1}{5} \left(\left(\frac{14}{3} \right) + \left(\frac{25}{2} - \frac{21}{2} \right) \right)$$

$$= \frac{1}{5} \left(\frac{14}{3} + \frac{4}{2} \right) = \frac{1}{5} \left(\frac{14}{3} + 2 \right) = \frac{1}{5} \left(\frac{14}{3} + \frac{6}{3} \right) = \frac{1}{5} \left(\frac{20}{3} \right) = \frac{20}{15} = \frac{4}{3}$$

Continue problem 6 on page 15.

Work for problem 6(c)

$$g(x) = \begin{cases} k\sqrt{x+1} & 0 \leq x \leq 3 \\ mx+2 & 3 < x \leq 5 \end{cases}$$

$$g(3) = \begin{cases} k\sqrt{3+1} = k\sqrt{4} = 2k \\ 3m+2 \end{cases}$$

$$\frac{2k}{2} = \frac{3m+2}{2}$$

$$k = \frac{3m+2}{2}$$

$$\frac{3m+2}{2} \sqrt{x+1} = 0$$

$$\frac{3m+2}{2} \sqrt{3+1} = 0$$

$$\frac{3m+2}{2} \sqrt{4} = 0 \quad \frac{3m+2}{2} (2) = 0 \quad 3m+2 = 0$$

$$k = \frac{3(-2/3) + 2}{2}$$

$$k = \frac{-2+2}{2} = 0$$

$$k = 0$$

$$3m+2 = 0$$

$$\frac{3m}{3} = \frac{-2}{3}$$

$$m = -2/3$$

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