AP® Calculus AB
2003 Sample Student Responses

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NO CALCULATOR ALLOWED

Work for problem 6(a)

yes, f(3) is 2, which is the same as the limits as x approaches 3 from either side.

Work for problem 6(b)

$$\frac{\int_0^3 \sqrt{x+1}\,dx + \int_3^5 (5-x)\,dx}{5}$$

$$\frac{\left[\frac{2}{3}(x+1)^{3/2}\right]_0^3 + \left[5x-\frac{1}{2}x^2\right]_3^5}{5}$$

$$\frac{\frac{2}{3}(4)^{3/2} - \frac{2}{3} + 25 - \frac{1}{2}25 - (15+\frac{1}{2}9)}{5}$$

$$\frac{\frac{2}{3}8 - \frac{2}{3} + 10 - \frac{16}{2}}{5}$$

$$\frac{16}{15} - \frac{2}{15} + 2 - \frac{8}{5}$$

$$\frac{14}{15} + 2 - \frac{8}{5}$$

$$\frac{14}{15} + \frac{30}{15} - \frac{24}{15}$$

$$\frac{20}{15}$$

$$\frac{4}{3}$$

Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

Work for problem 6(c)

$g' = k\frac{1}{2\sqrt{x+1}}$

m

$k\frac{1}{2\sqrt{3+1}} = m$ ← slopes must be equal at 3

functions must be equal at 3

$\frac{1}{4}k = m$

$k(2) = m(3) + 2$

$m = \frac{2k-2}{3}$

$\frac{1}{4}k = \frac{2k-2}{3}$

$k = \frac{8k-8}{3}$

$k = \frac{8}{3}k - \frac{8}{3}$

$-\frac{5}{3}k = -\frac{8}{3}$

$k = \frac{24}{15} = \frac{8}{5}$

$\frac{1}{4}\left(\frac{8}{5}\right) = \frac{8}{20}\ \frac{2}{5}$

$k = \frac{8}{5}\quad m = \frac{2}{5}$

END OF EXAMINATION

THE FOLLOWING INSTRUCTIONS APPLY TO THE BACK COVER OF THIS SECTION II BOOKLET.

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Work for problem 6(a)

$$f(x) = \begin{cases} \sqrt{x+1} & 0 \le x \le 3 \\ 5-x & 3 < x \le 5 \end{cases}$$

$$f(3) = \begin{cases} \sqrt{3+1} = \sqrt{4} = 2 \\ 5-3 = 2 = 2 \end{cases}$$

f is continuous at $x=3$ because the $\lim_{x \to 3} f(x) = 2$

and $f(3) = 2$.

Work for problem 6(b)

$$\text{Average value} = \frac{1}{5-0}\int_0^5 f(x)\,dx \rightarrow \begin{array}{l} u = x+1 \\ du = 1 \\ u(3) = 4 \\ u(0) = 1 \end{array}$$

The average value is $4/3$ on $0 \le x \le 5$.

$$\text{value} = \frac{1}{5}\left(\int_0^3 \sqrt{x+1}\,dx + \int_3^5 (5-x)\,dx\right)$$

$$= \frac{1}{5}\left(\int_1^4 \sqrt{u}\,du + \int_3^5 (5-x)\,dx\right)$$

$$= \frac{1}{5}\left(\frac{2}{3}u^{3/2}\Big|_1^4 + 5x - \frac{1}{2}x^2\Big|_3^5\right)$$

$$= \frac{1}{5}\left(\left(\frac{2}{3}(4)^{3/2} - \frac{2}{3}(1)^{3/2}\right) + \left(\left(5(5) - \frac{1}{2}(5)^2 - 5(3) - \frac{1}{2}(3^2)\right)\right)\right)$$

$$= \frac{1}{5}\left(\left(\frac{2}{3}(8) - \frac{2}{3}\right) + \left(\left(25 - \frac{25}{2}\right) - \left(15 - \frac{9}{2}\right)\right)\right)$$

$$= \frac{1}{5}\left(\left(\frac{16}{3} - \frac{2}{3}\right) + \left(\left(\frac{50}{2} - \frac{25}{2}\right) - \left(\frac{30}{2} - \frac{9}{2}\right)\right)\right)$$

$$= \frac{1}{5}\left(\left(\frac{14}{3}\right) + \left(\frac{25}{2} - \frac{21}{2}\right)\right)$$

$$= \frac{1}{5}\left(\frac{14}{3} + \frac{4}{2}\right) = \frac{1}{5}\left(\frac{14}{3} + 2\right) = \frac{1}{5}\left(\frac{14}{3} + \frac{6}{3}\right) = \frac{1}{5}\left(\frac{20}{3}\right) = \frac{20}{15} = \frac{4}{3}$$

Continue problem 6 on page 15.

V_2

NO CALCULATOR ALLOWED

Work for problem 6(c)

$g(x) = \begin{cases} k\sqrt{x+1} & 0 \le x \le 3 \\ mx + 2 & 3 < x \le 5 \end{cases}$

$g(3) = \begin{cases} k\sqrt{3+1} = k\sqrt{4} = 2k \\ 3m+2 \end{cases}$

$\frac{2k}{2} = \frac{3m+2}{2}$

$k = \frac{3m+2}{2}$

$\frac{3m+2}{2}\sqrt{x+1} = 0$

$\frac{3m+2}{2}\sqrt{3+1} = 0$

$\frac{3m+2}{2}\sqrt{4} = 0$ $\quad \frac{3m+2}{\cancel{2}}(\cancel{2}) = 0$ $\quad 3m+2 = 0$

$3m+2 = 0$
$\quad -2 \quad -2$

$\frac{3m}{3} = \frac{-2}{3}$

$m = -2/3$

$k = \frac{3(-2/3)+2}{2}$

$k = \frac{-2+2}{2} = 0$

$k = 0$

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