AP® Calculus AB
2003 Sample Student Responses

The materials included in these files are intended for use by AP teachers for course and exam preparation; permission for any other use must be sought from the Advanced Placement Program®. Teachers may reproduce them, in whole or in part, in limited quantities for noncommercial, face-to-face teaching purposes. This permission does not apply to any third-party copyrights contained herein. This material may not be mass distributed, electronically or otherwise. These materials and any copies made of them may not be resold, and the copyright notices must be retained as they appear here.

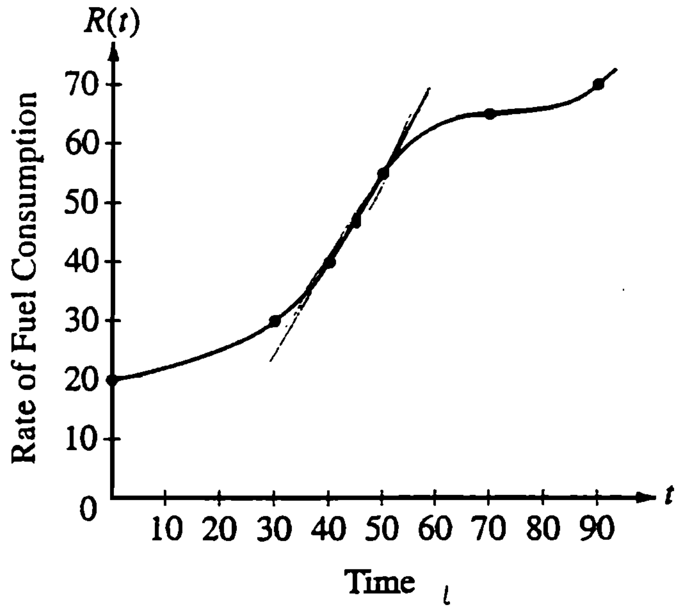
These materials were produced by Educational Testing Service® (ETS®), which develops and administers the examinations of the Advanced Placement Program for the College Board. The College Board and Educational Testing Service (ETS) are dedicated to the principle of equal opportunity, and their programs, services, and employment policies are guided by that principle.

The College Board is a national nonprofit membership association whose mission is to prepare, inspire, and connect students to college and opportunity. Founded in 1900, the association is composed of more than 4,300 schools, colleges, universities, and other educational organizations. Each year, the College Board serves over three million students and their parents, 22,000 high schools, and 3,500 colleges through major programs and services in college admissions, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT®, the PSAT/NMSQT®, and the Advanced Placement Program® (AP®). The College Board is committed to the principles of equity and excellence, and that commitment is embodied in all of its programs, services, activities, and concerns.

For further information, visit www.collegeboard.com

Copyright © 2003 College Entrance Examination Board. All rights reserved. College Board, Advanced Placement Program, AP, AP Vertical Teams, APCD, Pacesetter, Pre-AP, SAT, Student Search Service, and the acorn logo are registered trademarks of the College Entrance Examination Board. AP Central is a trademark owned by the College Entrance Examination Board. PSAT/NMSQT is a registered trademark jointly owned by the College Entrance Examination Board and the National Merit Scholarship Corporation. Educational Testing Service and ETS are registered trademarks of Educational Testing Service. Other products and services may be trademarks of their respective owners.

For the College Board's online home for AP professionals, visit AP Central at apcentral.collegeboard.com.



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

Work for problem 3(a)

$R'(t) = \text{slope}$.

$m = \frac{55-40 \text{ gal/min}}{50-40 \text{ min}} = 1.5 \text{ gallons/minute}^2$

Work for problem 3(b)

$R''(45) = 0$

The value is zero because the rate of increase of fuel consumption is fastest at $t=45$. This would indicat a max point on $R'(45)$. As a result, $R''(45)$ is zero.

Continue problem 3 on page 9.

Copyright © 2003 by College Entrance Examination Board. All rights reserved.
Visit apcentral.collegeboard.com (for AP professionals) and www.collegeboard.com/apstudents (for AP students and parents).

Work for problem 3(c)

$$\int_0^{90} R(t)\,dt = 30(20) + 10(30) + 10(40) + 20(55) + 2c$$

$$= 3700 \text{ gallons}$$

This approximation is less than the actual value because the rectangles are below the curve, and some of the fuel consumed is not included.

Work for problem 3(d)

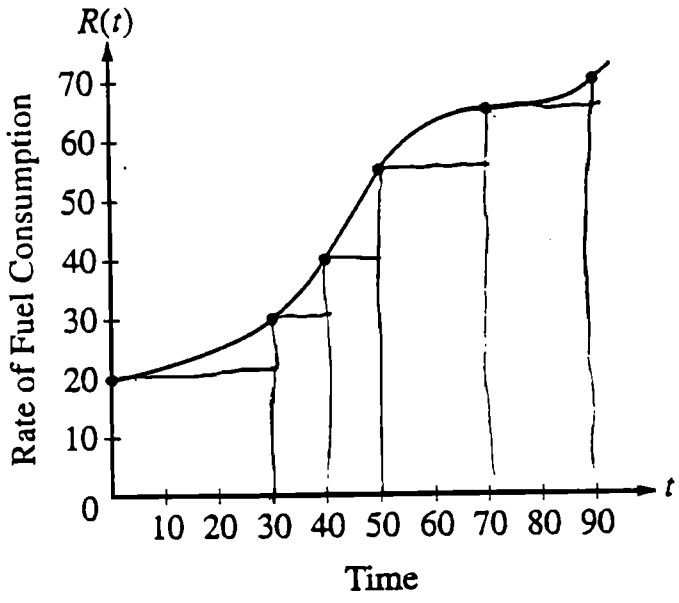
$\int_0^b R(t)\,dt$ represents the number of gallons consumed from 0 to b minutes.

$\frac{1}{b}\int_0^b R(t)\,dt$ is the average value of fuel consumption (in gallons) each minute from 0 to b minutes.

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Copyright © 2003 by College Entrance Examination Board. All rights reserved.
Visit apcentral.collegeboard.com (for AP professionals) and www.collegeboard.com/apstudents (for AP students and parents).



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

Work for problem 3(a)

slope from $t=40$ to $t=50$:

$$m = \frac{55-46}{50-40}$$

$$= \frac{15}{10}$$

$$= \frac{3}{2}$$

$\therefore R'(45) \approx \frac{3}{2}$

Work for problem 3(b)

If at $t=45$, the rate is increasing fastest, the graph of t vs. $R(t)$ is changing concavity,

$\therefore R''(45) = 0$

Continue problem 3 on page 9.

Copyright © 2003 by College Entrance Examination Board. All rights reserved.
Visit apcentral.collegeboard.com (for AP professionals) and www.collegeboard.com/apstudents (for AP students and parents).

Work for problem 3(c)

$$\int_0^{90} R(t)\,dt \approx 30\times 20 + 10\times 30 + 10\times 40 + 20\times 55 + 20$$

$$\approx 3700$$

Since the function R(t) is increasing, this left Riemann sum is less than the actual value

Work for problem 3(d)

for $0 < b \leq 90$ min, $\int_0^b R(t)\,dt$ is the amount of fuel (in gallons) that the plane has consumed in the interval from $t=0$ and $t=b$ minutes

$\frac{1}{b}\int_0^b R(t)\,dt$ is the average amount of fuel consumed per minute by the plane over the interval $t=0$ to $t=b$ minutes. Since this is an average rate, the unit of measure is gallons per minute.

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Copyright © 2003 by College Entrance Examination Board. All rights reserved.
Visit apcentral.collegeboard.com (for AP professionals) and www.collegeboard.com/apstudents (for AP students and parents).