AP® Calculus AB
2003 Sample Student Responses
Form B

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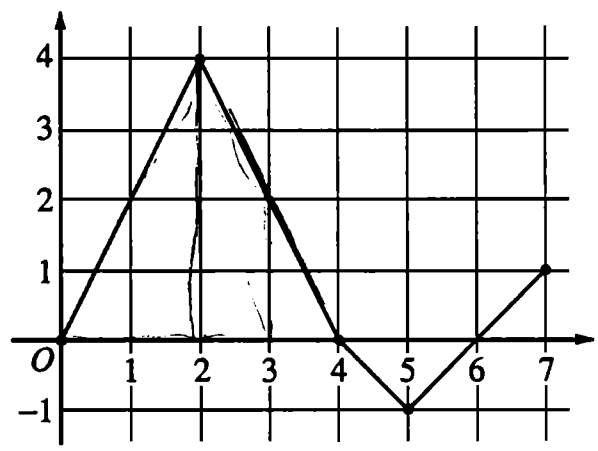
A,

NO CALCULATOR ALLOWED



Graph of f

Work for problem 5(a)

4+3

$g(x) = \int_2^x f(t)\,dt$

$g'(x) = f(x)$

$g''(x) = f'(x)$

$g(3) = \int_2^3 f(t)\,dt = F(3) - F(2) = 7 - 4 = 3$

$g'(3) = f(3) = 2$

$g''(3) = f'(3) = -2$

$g(3) = 3$

$g'(3) = 2$

$g''(3) = -2$

Work for problem 5(b)

rate of change of $g = g'(x)$

$$\frac{1}{3-0}\int_0^3 g'(x)\,dx = \frac{1}{3}\{g(3) - g(0)\}$$

$$= \frac{1}{3}\{3 - g(0)\}$$

$$= \frac{1}{3}\left(3 - \int_2^0 f(t)\,dt\right)$$

$$= \frac{1}{3}\left(3 + \int_0^2 f(t)\,dt\right)$$

$$= \frac{1}{3}\left(3 + F(2) - F(0)\right)$$

$$= \frac{1}{3}(3 + 4 - 0)$$

$$= \frac{7}{3}$$

$\frac{7}{3} \simeq 2.333$

Continue problem 5 on page 13.

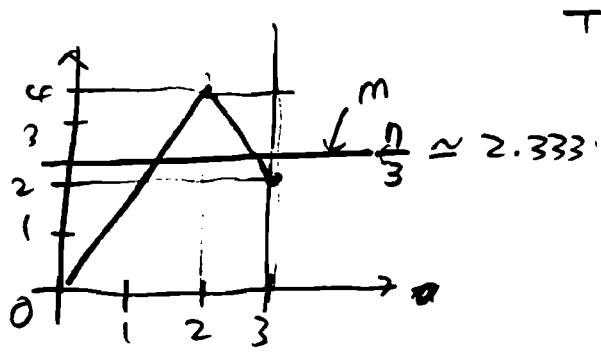
NO CALCULATOR ALLOWED

Work for problem 5(c)

$g'(c) = \frac{\pi}{3}$

since $g'(x) = f(x)$, $g'(c) = f(c)$.

$g'(c) = f(c) = \frac{\pi}{3}$



The line m crosses the graph of f twice

→ g'(c) is equal to 2.333 at two values of c.

Work for problem 5(d)

At points of inflection, g''(x) should change from (+) to (−), or vice versa.

At f(2), f'(x) changes from (+) to (−), and at f(5), f'(x) changes from (−) to (+).

Points of inflection exist at $x=2$ and $x=5$.

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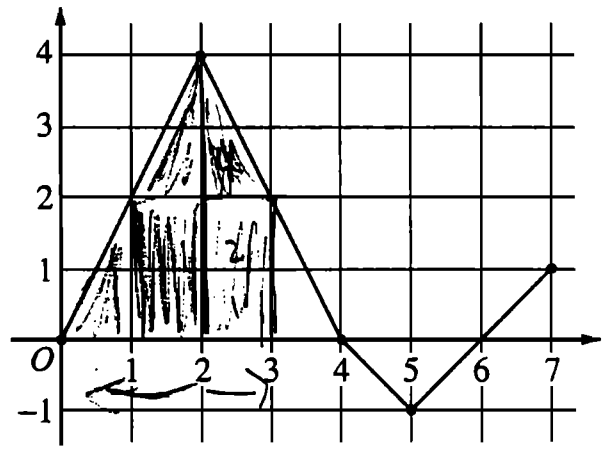
B,

NO CALCULATOR ALLOWED



Graph of f

Work for problem 5(a)

$$g(3) = \int_2^3 f(t)\,dt = \boxed{3}$$

$i'(x) = f(x) \Rightarrow g'(3) = f(3) = \boxed{2}$

$$g''(3) = f'(3) = \text{slope at } 3 = \frac{2-4}{3-2} = \frac{-2}{-1} = \boxed{2}$$

Work for problem 5(b)

$$\text{avg rate of change} = \frac{g(a)-g(b)}{a-b}$$

$$g(0) = \int_2^0 f(t) = -4$$

$$g(3) = \int_2^3 f(t) = 3$$

$$\frac{g(0)-g(3)}{0-3} = \frac{-4-3}{-3} = \boxed{\frac{7}{3}}$$

Continue problem 5 on page 13.

NO CALCULATOR ALLOWED

B_2

Work for problem 5(c)

$g'(c) = 7/3 \Rightarrow$

$f(c) = 7/3$ at 1 (one) point

because

on $(0,2)$, $f(x) = y = 2x$

$2x = 7/3$

$x = 7/6 \leftarrow$ only at $x = 7/6$

on $(2,3)$, $f(x) = y = 2x + 8$

$7/3 = 2x + 8$

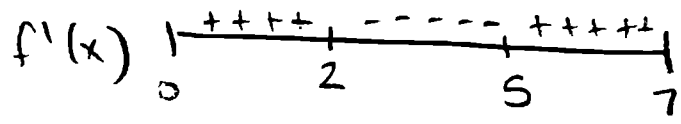
$7/3 - \frac{24}{3} = 2x \Rightarrow -\frac{17}{3} = 2x \Rightarrow x = -17/6$ (not on $(0,3)$)

Work for problem 5(d)

point of inflection $= g''(x) = 0$

$g''(x) = f'(x)$

$f'(x) = 0$ at $x = 2$, $x = 5$



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