



AP[®] Calculus AB 2003 Sample Student Responses Form B

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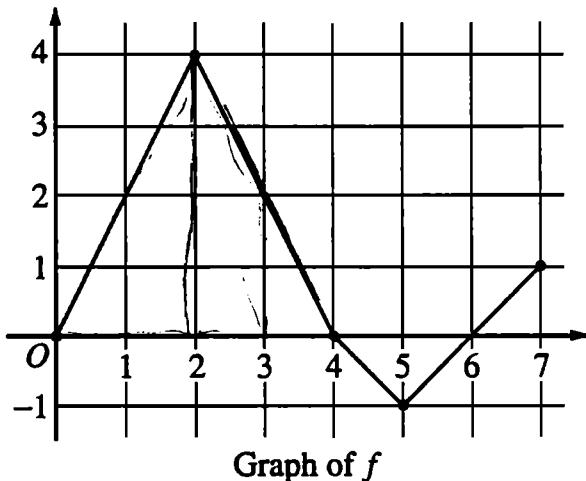
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NO CALCULATOR ALLOWED



Work for problem 5(a)

4+3

$$g(x) = \int_2^x f(t) dt$$

$$g'(x) = f(x)$$

$$g''(x) = f'(x)$$

$$g(3) = \int_2^3 f(t) dt = F(3) - F(2) = 7 - 4 = 3$$

$$g'(3) = f(3) = 2$$

$$g''(3) = f'(3) = -2$$

$$\begin{cases} g(3) = 3 \\ g'(3) = 2 \\ g''(3) = -2 \end{cases}$$

Work for problem 5(b)

rate of change of $g = g'(x)$

$$\begin{aligned} \frac{1}{3-0} \int_0^3 g'(x) dx &= \frac{1}{3} \{g(3) - g(0)\} \\ &= \frac{1}{3} \{3 - g(0)\} \\ &= \frac{1}{3} \left(3 - \int_2^0 f(t) dt\right) \\ &= \frac{1}{3} \left(3 + \int_0^2 f(t) dt\right) \end{aligned}$$

$$\begin{aligned} &= \frac{1}{3} (3 + F(2) - F(0)) \\ &= \frac{1}{3} (3 + 4 - 0) \\ &= \frac{7}{3} \end{aligned}$$

$$\frac{7}{3} \approx 2.333$$

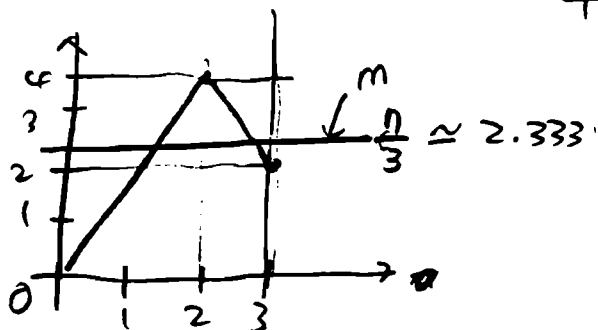
Continue problem 5 on page 13.

Work for problem 5(c)

$$g'(c) = \frac{7}{3}$$

Since $g'(x) = f(x)$, $g'(c) = f(c)$.

$$g'(c) = f(c) = \frac{7}{3}$$



The line m crosses the graph of f twice

→ $g'(c)$ is equal to 2.333 at two values of c .

Work for problem 5(d)

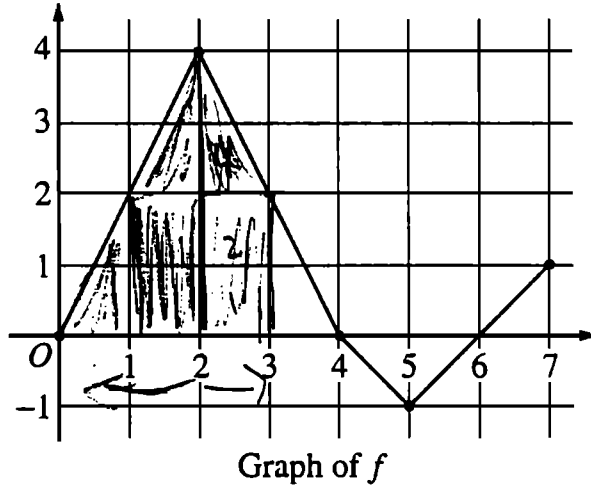
At points of inflection, $g''(x)$ should change from (+) to (-), or vice versa.

At $x=2$, $f'(x)$ changes from (+) to (-), and at $x=5$, $f'(x)$ changes from (-) to (+).

Points of inflection exist at $x=2$ and $x=5$.

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED



Work for problem 5(a)

$$g(3) = \int_2^3 f(t) dt = \boxed{3}$$

$f'(x) = f(x) \Rightarrow g'(3) = f(3) = \boxed{2}$

$$g''(3) = f'(3) = \text{slope at } 3 = \frac{2-4}{3-2} = \frac{-2}{-1} = \boxed{2}$$

Work for problem 5(b)

avg rate of change = $\frac{g(a) - g(b)}{a - b}$

$$g(0) = \int_2^0 f(t) dt = -4$$

$$g(3) = \int_2^3 f(t) dt = 3$$

$$\frac{g(0) - g(3)}{0 - 3} = \frac{-4 - 3}{-3} = \boxed{\frac{7}{3}}$$

Continue problem 5 on page 13.

Work for problem 5(c)

$$g'(c) = 7/3 \Rightarrow$$

$$f(c) = 7/3 \text{ at 1 (one) point}$$

because

$$\text{on } (0, 2), f(x) = y = 2x$$

$$2x = 7/3$$

$$x = 7/6 \leftarrow \text{only at } x = 7/6$$

$$\text{on } (2, 3), f(x) = y = 2x + 8$$

$$7/3 = 2x + 8$$

$$7/3 - 24/3 = 2x \Rightarrow -17/3 = 2x \quad \text{(not on } (0, 3))$$

~~$x = -17/6$~~

Work for problem 5(d)

$$\text{point of inflection} = g''(x) = 0$$

$$g''(x) = f'(x)$$

$$f'(x) = 0 \text{ at } \boxed{\begin{matrix} x = 2, \\ x = 5 \end{matrix}}$$

