AP® Calculus AB
2003 Sample Student Responses
Form B

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Work for problem 5(a)

\[ g(x) = \int_2^x f(t) \, dt \]
\[ g'(x) = f(x) \]
\[ g''(x) = f'(x) \]

\[ g(3) = \int_2^3 f(t) \, dt = F(3) - F(2) = 7 - 4 = 3 \]
\[ g'(3) = f(3) = 2 \]
\[ g''(3) = f'(3) = -2 \]

Work for problem 5(b)

\[ \text{rate of change of } g = g'(x) \]
\[ \frac{1}{3} \int_0^3 g'(x) \, dx = \frac{1}{3} \left[ g(3) - g(0) \right] = \frac{1}{3} \left[ 3 - g(0) \right] = \frac{1}{3} \left( 3 - \int_2^3 f(t) \, dt \right) = \frac{1}{3} \left( 3 + \int_0^2 f(t) \, dt \right) \]
\[ \frac{7}{3} \approx 2.333 \]

Continue problem 5 on page 13.
Work for problem 5(c)

\[ g'(c) = \frac{n}{3} \]

since \( g'(x) = f'(x) \), \( g'(c) = f'(c) \).

\[ g'(c) = f'(c) = \frac{n}{3} \]

The line \( m \) crosses the graph of \( f \) twice at \( g'(c) \) is equal to 2.333 at two values of \( c \).

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Work for problem 5(d)

At points of inflection, \( g''(x) \) should change from (+) to (−), or vice versa.

At \( f(x) \), \( f'(x) \) changes from (−) to (−), and at \( f(5) \), \( f'(x) \) changes from (−) to (+).

Points of inflection exist at \( x = 2 \) and \( x = 5 \).
Work for problem 5(a)

\[ g'(3) = \int_{2}^{3} f(t) \, dt = \boxed{3} \]

\[ f(x) = g'(3) = f(3) = \boxed{2} \]

\[ g''(3) = f'(3) = \text{slope at } 3 = \frac{2-4}{3-2} = -2 = \boxed{2} \]

Work for problem 5(b)

avg rate of change = \[ \frac{g(a) - g(b)}{a - b} \]

\[ g(0) - g(3) = \frac{-4 - 3}{0 - 3} = \boxed{\frac{-7}{3}} \]

Continue problem 5 on page 13.
NO CALCULATOR ALLOWED

Work for problem 5(c)

\[ g'(c) = \frac{7}{3} \Rightarrow \]
\[ f(c) = \frac{7}{3} \text{ at 1 (one) point} \]

because

on (0, 2), \( f(x) = y = 2x \)
\[ 2x = \frac{7}{3} \]
\[ x = \frac{7}{6} \text{ -- only at } x = \frac{7}{6} \]

on (2, 3), \( f(x) = y = 2x + 3 \)
\[ \frac{7}{3} = 2x + 3 \]
\[ \frac{7}{3} - \frac{9}{3} = 2x \Rightarrow -\frac{1}{3} = 2x \]  
\[ x = -\frac{1}{6} \text{ (not on } [0, 3]) \]

Work for problem 5(d)

point of inflection = \( g''(x) = 0 \)

\[ g''(x) = f'(x) \]

\[ f'(x) = 0 \text{ at }\]
\[ x = 2, \]
\[ x = 5 \]

\[ f'(x) \]
\[ \begin{array}{ccccccc}
- & - & - & - & + & + & + \\
0 & 2 & 5 & 7
\end{array} \]