AP® Calculus AB
2003 Sample Student Responses
Form B

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Work for problem 4(a)

\[ v(t) = -1 - e^{(1-t)} \]

\[ a(t) = v'(t) = -1 (e^{(1-t)}) = -e^{1-t} \]

\[ a(3) = -e^{-3} = -\frac{1}{e^2} \]

Work for problem 4(b)

\[ v(t) \quad + + + + + - - - - \]

\[ a(t) \quad - - - - - \]

-1 + e^{1-t} = 0
-1 + e = 0
1 + e = 0
1 - t = 0

Speed increases when signs are same

Speed is increasing on (1, 0)

t = 3 is within the interval and thus speed is increasing

Continue problem 4 on page 11.
Work for problem 4(c)

\[ v(t) = o = -1 + e^{1-t} \]

\[ e^{1-t} = 1 \]

\[ 1-t = 0 \]

\[ 1 = t \]

The particle changes direction at \( t = 1 \)

Work for problem 4(d)

\[ a(t) = \int_{0}^{1} -1 + e^{1-t} \, dt = -\int_{1}^{3} -1 + e^{1-t} \, dt = \]

\[ -t \bigg|_{0}^{1} + e^{1-t} \bigg|_{0}^{1} + t \bigg|_{1}^{3} + e^{1-t} \bigg|_{1}^{3} = \]

\[ (-1 - 0) + (1 + 1) + (3 - 1) + (e^{-2} - 1) = \]

\[ -1 - 1 + 1 + 2 + e^{-2} - 1 = e + e^{-2} - 1 \]
Work for problem 4(a)

\[ v(t) = \text{velocity} \]
\[ a(t) = \text{accel}. \]

\[ v(t) = -re^{-t} \]
\[ = e^{-t} - 1 \]
\[ v(t) = a(t) = e^{-t}(-1) \]
\[ = -e^{-t} \]

at \( t = 3 \), then, accel. \( a(t) = -e^{-3} = -\frac{1}{e^2} \)

Work for problem 4(b)

at \( t = 3 \) acceleration is negative and velocity is \( -1 + e^{-2} = \frac{1}{e^2} - 1 \) which is clearly negative (since \( \frac{1}{e^2} < 1 \)), thus since acceleration is negative velocity is decreasing (becoming more negative). But speed is \( |\text{velocity}| \) (abs. value of velocity) and thus since velocity becomes more negative speed decreases.

Continue problem 4 on page 11.
Work for problem 4(c)

The particle is going in one direction when \( v(t) \) is positive, and another when \( v(t) \) is neg. So it changes direction at the zeros of \( v(t) \).

\[
0 = -1 + e^t \\
e^{1-t} = 1 \\
\ln (e^{1-t}) = \ln 1 \\
1-t = 0 \\
t = 1 \text{ is a zero.}
\]

Thus it changes direction at \( t = 1 \).

Work for problem 4(d)

Total distance traveled = \( \int_0^3 v(t) \, dt \)

\[
= \int_0^3 1 - e^t \, dt \\
= \left[ e^t \right]_0^3 - \int_0^3 1 \, dt \\
= \left[ e^t \right]_0^3 - [t]_0^3 \\
= (e^3 - 1) - (3 - 0) \\
= e^3 - 1 - 3 = \text{total dist traveled}
\]

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GO ON TO THE NEXT PAGE.